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ON SMALL-SIGNAL AMPLIFICATION IN A GYRO-TWT

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Abstract—The corrected dispersion relation governing the linear interaction of a TE mode in a circular cylindrical wave guide with an annular beam of gyrating electrons in a gyro-TWT configuration is derived. The derivation of the correct dispersion relation no longer involves any integration with respect to the radial coordinate r_o of the electron guiding center as the relevant equilibrium distribution function turns out to be independent of r_o. When the cyclotron resonance condition is satisfied by the TE mode for a positive s-number, the small-signal theory is shown to predict an initial exponential growth of the mode with interaction length over a small but finite band of frequencies around the design frequency.

1. INTRODUCTION

The mechanism of small-signal amplification in gyro-TWTs and cyclotron resonance masers (CRMs) was actively studied by many researchers in the 1980s [1–4] culminating in the derivation of the dispersion relation in the form of an infinite series. The celebrated (Doppler-shifted) cyclotron resonance condition

$$\omega - v_z\beta_{mn}(\omega) - s\Omega_e/\gamma = 0 \quad (1)$$

was identified as a sufficient requirement for small-signal amplification. In (1), ω is the operating (radian) frequency, $\beta_{mn}(\omega)$ is the unperturbed propagation phase constant of the mnth waveguide mode, v_z is the axial speed of the electrons, $\Omega_e = eB_o/m_e$ is the electron cyclotron frequency corresponding to an applied uniform magnetic field $\widehat{z}B_o$ in the axial direction and γ is the relativistic factor. In the expression for Ω_e and in the sequel, $-e$ and m_e are respectively the charge and

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the rest mass of an electron. When the cyclotron resonance condition is satisfied by a particular wave-guide mode for a given $s = s_o$, the dispersion relation may be reduced to an algebraic equation by retaining only the significant contribution from the s_oth term of the infinite series. The resulting biquadratic algebraic equation may be solved for the (complex) propagation phase constant as a function of the operating frequency. The works of Edgcombe [1], Chu et al. [2], Fliflet [3] and Chu and Lin [4] may be cited as being specifically directed towards a derivation of the linear dispersion relation for circular cylindrical wave-guide modes. Edgecombe [1] assumes without adequate justification that the r.f. charge and current densities depend linearly on the electric field of the TE mode in the derivation of the dispersion relation. Also, the need for maximizing

$$I_{mss} \underline{\Delta} \frac{1}{4}(1+\delta_{mo})\left(J_{s+m}^2(k_c r_g) + J_{s-m}^2(k_c r_g)\right),$$

where r_g is the gyro-radius and k_c is the mode cut-off wave number, with respect to $k_c r_g$ to arrive at the biquadratic algebraic equation satisfied by the normalized phase constant k/k_c has not been brought out clearly by him. Although the derivation of the dispersion relation by Chu et al. [2] for TE modes is free from any of these drawbacks, their treatment is too sketchy leaving out many details of analysis. However, a complete derivation with all details filled in has been provided by Chu et al. in a subsequent paper [4]. Fliflet [3], on the other hand, gives a detailed derivation of the linear dispersion relation together with a description of the single-particle quasilinear theory for both TE and TM modes. Finally, Kou et al. [5] have, in the recent past, presented a linear theory, using Laplace transforms, that is applicable to both gyro-TWTs and gyro-BWOs, and used this theory to study the stability of harmonic gyro-TWTs.

All of the above derivations of the dispersion relation make use of kinetic theory based on linearized Vlasov equation and a transformation from the polar coordinates $(r,\, \theta,\, p_t,\, \phi)$ of the transverse position and momentum to the gyro-co-ordinates $(r_o,\, \theta,\, r_L,\, \widetilde{\phi})$ (Edgcombe starts initially with Cartesian coordinates of position and momentum) where $p_t = (p_x^2 + p_y^2)^{\frac{1}{2}} = (p_r^2 + p_\theta^2)^{\frac{1}{2}}$ is the magnitude of the transverse momentum, $\phi = \arctan(p_y/p_x)$, r_o is the distance of the electron guiding center from the waveguide axis, $r_L = p_t/m_e\Omega_e$ is the gyro-radius and $\widetilde{\phi}$ is the gyro-phase (See Figure 1) [3].

It is well known that the functional dependence of the axially symmetric equilibrium distribution function f_o on the position and the momentum variables can only be through the single-particle constants of motion. For an z-directed uniform magnetic field $\widehat{z}B_o$,

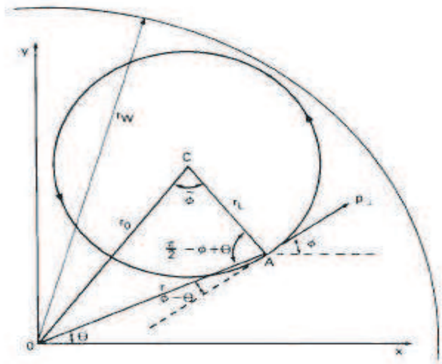

Figure 1. Geometry of Interaction [3].

such constants of motion in absence of space-charge fields are the total energy

$$H = \left[m_e^2 c^4 + c^2 \left(p_t^2 + p_z^2\right)\right]^{\frac{1}{2}} - m_e c^2$$

the canonical angular momentum $P_\theta = rp_\theta - \frac{1}{2}eB_o\frac{r^2}{c}$ and the z-component of the linear momentum p_z [6] where c is the vacuum speed of light. The relation $P_\theta = \frac{1}{2}m_e\Omega_e(r_L^2 - r_o^2)$ implies that the axisymmetric equilibrium distribution function

$$\widetilde{f}_o(r, p_t, \phi, p_z) \triangleq f_o(H(p_t, p_z), P_\theta(p_t, r_o), p_z) \triangleq \widehat{f}_o(r_o, p_t, p_z)$$

is a function only of r_o, p_t (or equivalently r_L) and p_z.

Fliflet's derivation of the dispersion relation is based on the observation that, "for configuration which optimize the cyclotron maser interaction, $\widehat{f}_o$ may be assumed to be independent of r_o with negligible error", and he sets $\partial\widehat{f}_o/\partial r_o$ equal to zero on this ground. However, he contradicts this hypothesis in a subsequent step by assuming a 'delta-function' dependence for $\widehat{f}_o$ on r_o. Chu and Lin [4], on the contrary, do no drop the $\partial\widehat{f}_o/\partial r_o$ terms at any stage of their derivation of the dispersion relation except that they too assume a 'delta-function' dependence for a differentiable function of r_o!

It is shown in Section 2 that the axially symmetric equilibrium distribution function corresponding to an applied axially directed uniform magnetic field will be independent of r_o. As a consequence, the derivation of the correct dispersion relation requires integrations with respect to the momentum variables p_t and p_z only. This is in marked contrast to everyone of the 'derivations' of the dispersion relation, which necessitates an integration with respect to the position variable r_o also, attempted so far in the literature.

2. EQUILIBRIUM DISTRIBUTION FUNCTION

It is quite surprising that every one of the derivations of the dispersion relation attempted so far in the literature has failed to take the following fact into account; whenever an axially-symmetric equilibrium distribution function $\widehat{f}_o$ for an applied axially directed uniform magnetic field $\widehat{z}B_o$ turns out to be a function only of the position variable r_o and the momentum variables p_t and p_z, the steady-state linearized Vlasov equation for $\widehat{f}_o$ reduces to

$$\partial \widehat{f}_o / \partial r_o = 0 \tag{2}$$

Proof of (2): The steady-state linearized Vlasov equation satisfied by an axially-symmetric equilibrium distribution function $\widetilde{f}_o(r, p_t, \phi, p_z)$ for an applied axially directed magnetic field $\widehat{z}B_o$ is

$$\mathbf{v}.\nabla_{\mathbf{r}}\widetilde{f}_o - e\mathbf{v} \times \widehat{z}B_o.\nabla_{\mathbf{p}}\widetilde{f}_o = 0 \tag{3}$$

in absence of any space-charge electric field. In (3), $\mathbf{v}$ is the electron velocity. With the help of the mutually orthogonal unit vectors

$$\widehat{e}_t = \mathbf{p}_t/p_t = \widehat{r}\cos(\phi - \theta) + \widehat{\theta}\sin(\phi - \theta)$$

and

$$\widehat{e}_\phi = \widehat{e}_t \times \widehat{z} = -\widehat{r}\sin(\phi - \theta) + \widehat{\theta}\cos(\phi - \theta),$$

Equation (3) may be expressed as

$$(v_t\widehat{e}_t + v_z\widehat{z}) \cdot \widehat{r}\frac{\partial \widetilde{f}_o}{\partial r} + \frac{ep_t}{m_e\gamma}B_o\widehat{e}_\phi \cdot \left(\widehat{e}_t\frac{\partial}{\partial p_t} + \widehat{e}_\phi\frac{1}{p_t}\frac{\partial}{\partial \phi} + \widehat{z}\frac{\partial}{\partial p_z}\right)\widetilde{f}_o$$

$$= \frac{p_t}{m_e\gamma}\cos(\phi - \theta)\frac{\partial \widetilde{f}_o}{\partial r} + \frac{eB_o}{m_e\gamma}\frac{\partial \widetilde{f}_o}{\partial \phi} = 0 \tag{4}$$

since

$$\frac{\partial \widetilde{f}_o}{\partial \theta} = \frac{\partial \widetilde{f}_o}{\partial z} = 0$$

and

$$\widehat{e}_\phi \cdot \widehat{e}_t = \widehat{e}_\phi \cdot \widehat{z} = 0$$

Under a transformation of coordinates from (r, θ, p_t, ϕ) to $(r_o, \theta, p_t, \widetilde{\phi})$, the left side of (4) becomes

$$\sin\widetilde{\phi}\left[\frac{p_t}{m_e\gamma}\frac{r_o}{r^2}(r_o - r_L\sin\widetilde{\phi}) - \frac{eB_or_L}{m_e\gamma}\right]\frac{\partial \widehat{f}_o}{\partial r_o}$$

since

$$\frac{\partial \widetilde{f}_o}{\partial p_t} = \frac{\partial \widehat{f}_o}{\partial p_t} + \frac{\cos \widetilde{\phi}}{m_e \Omega_e} \frac{\partial \widehat{f}_o}{\partial r_o} - \frac{\sin \widetilde{\phi}}{m_e \Omega_e} \frac{1}{r_o} \frac{\partial \widehat{f}_o}{\partial \widetilde{\phi}}$$
$$\frac{\partial \widetilde{f}_o}{\partial \phi} = -\frac{p_t}{m_e \Omega_e} \sin \widetilde{\phi} \frac{\partial \widehat{f}_o}{\partial r_o} + \frac{(r_o - r_L \cos \widetilde{\phi})}{r_o} \frac{\partial \widehat{f}}{\partial \widetilde{\phi}}$$

and $\frac{\partial \widehat{f}}{\partial \widetilde{\phi}} = 0$ (Recall that $\widehat{f}_o$ is a function of only r_o, p_t and p_z by hypothesis). Canceling, the non-identically zero common factors, the steady-state Vlasov equation for $\widehat{f}_o$ assumes the form

$$-r_L(r_L - r_o \cos \widetilde{\phi}) \frac{\partial \widehat{f}_o}{\partial r_o} = 0$$

that is

$$\frac{\partial \widehat{f}_o}{\partial r_o} = 0$$

since neither r_L nor $r_L - r_o \cos \widetilde{\phi}$ vanishes identically. The proof is complete. Equation (2) implies that *(i)* $\widehat{f}_o$ does not depend on the position variable r_o and *(ii)* there is no restriction on the functional dependence of $\widehat{f}_o$ on the momentum variables.

3. CORRECTED DISPERSION RELATION

In order to arrive at the correct dispersion relation resulting from the linear interaction of the TE_{mn} mode of a circular cylindrical wave guide with an annular beam of gyrating electrons, we follow the approach and the notation of Fliflet [3] to the extent possible. The small-signal assumption permits the electromagnetic field of the TE_{mn} mode in the presence of the electron beam to be represented as

$$\mathbf{E} = \mathrm{Re}\left\{\Pi_o C_{mn}[(jmJ_m(k_{mn}r)/r)\widehat{r} + k_{mn}J'_m(k_{mn}r)\widehat{\theta}]\exp^{j(\omega t - \beta z - m\theta)}\right\} \quad (5)$$

$$\mathbf{B} = \mathrm{Re}\left\{\frac{\Pi_o C_{mn}}{\omega}[-\beta k_{mn}J'_m(k_{mn}r)\widehat{r} + jm\beta(J_m(k_{mn}r)/r)\widehat{\theta}\right\}$$
$$-jk_{mn}^2 J_m(k_{mn}r)\widehat{z}]e^{j(\omega t - \beta z - m\theta)}\} \quad (6)$$

In (5) and (6), the prime superscript denotes differentiation with respect to the argument, Π_o is an amplitude constant, $k_{mn} = x_{mn}/r_w$ is the cut-off wave number of the TE_{mn} mode, $C_{mn} = \{[\pi(x_{mn}^2 - m^2)]^{1/2} J_{mn}(x_{mn})\}^{-1}$ is the normalization constant, x_{mn} is the nth zero of $J_m{}'$, r_w is the waveguide wall radius and β is the a priori unknown value of the perturbed propagation phase constant of the TE_{mn} mode linearly interacting with the electron beam. Closely following the standard procedure adopted by Fliflet for analyzing the linear interaction of a TE mode with an annular electron beam, we arrive at the equation

$$\begin{aligned}&\left(\frac{\omega^2}{c^2}-k_{mn}^2-\beta^2\right)r_w^2\left(1-m^2/x_{mn}^2\right)J_m^2(x_{mn})\\ =&-\frac{e^2\mu_o}{\pi}\sum_q\sum_s\left\{\int_o^\infty p_t^2dp_t\int_{(r_w-b_w)/2}^{(r_w+b_w)/2}rdr\int_o^{2\pi}d\phi\int_o^\infty dp_z\int_o^{2\pi}d\theta\right.\\ &\left[F_{smn}^{TE}(r_o,p_t,p_z)J'_{m+q}(k_{mn}r_L)J_q(k_{mn}r_o)e^{j(q-s+m)\widetilde{\phi}}\right.\\ &\left./m_e\gamma(\omega-\beta v_z-s\Omega_e/\gamma)\right]\Big\}\end{aligned}\tag{7}$$

corresponding to the Equation (30) in [3]. In (7)

$$\begin{aligned}F_{smn}^{TE}(r_o,p_t,p_z)=&J'_s(k_{mn}r_L)J_{s-m}(k_{mn}r_o)[(\omega-\beta p_z/m_e\gamma)\partial\widehat{f}_o(p_t,p_z)/\partial p_t\\ &+(\beta p_t/m_e\gamma)\partial\widehat{f}_o(p_t,p_z)/\partial p_z]\end{aligned}\tag{8}$$

and we have assumed an annular beam of width b_w centered around the cylindrical surface $r = r_w/2$. Making a change of integration variables from (r,θ,p_t,ϕ) to $(r_o,\theta,p_t,\widetilde{\phi})$ with the help of the Jacobian $\partial(r,\theta,p_t,\phi)/\partial(r_o,\theta,p_t,\widetilde{\phi}) = r_op_t/r$, and performing the θ and $\widetilde{\phi}$ integrations of the resulting multiple integral, we have

$$\begin{aligned}&\left(\omega^2/c^2-k_{mn}^2-\beta^2\right)r_w^2\left(1-m^2/x_{mn}^2\right)J_m^2(x_{mn})\\ =&\frac{-4\pi e^2\mu_o}{m_e}\sum_s\int_o^\infty p_t^2dp_t\int_{\frac{r_w-b_w}{2}+r_L}^{\frac{r_w+b_w}{2}-r_L}r_odr_o\int_o^\infty dp_z\\ &\left\{F_{smn}^{TE}(r_o,p_t,p_z)J'_s(k_{mn}r_L)J_{s-m}(k_mr_o)/\left[\gamma(\omega-\beta v_z)-s\Omega_e\right]\right\}\end{aligned}\tag{9}$$

The requirement

$$\frac{r_w+b_w}{2}-r_L>\frac{r_w-b_w}{2}+r_L$$

implies that

$$r_L<b_w/2$$

or equivalently

$$p_t<\sigma_w\,\underline{\Delta}\,m_e\Omega_eb_w/2\tag{10}$$

Substituting for F_{smn}^{TE} from (8) and carrying out the integration with respect to r_o in the triple integral appearing under the summation in

(9), we have

$$\int_o^{\sigma_w}\int_o^{\infty}\left(\int_{b_l+r_L}^{b_u-r_L} J_{s-m}^2(k_m r_o) r_o dr_o\right)\frac{[(\omega\gamma-\beta p_z/m_e)\partial\widehat{f}_o/\partial p_t+(\beta p_t/m_e)\partial\widehat{f}_o/\partial p_z]}{\gamma(\omega\gamma-s\Omega_e-\beta p_z/m_e)}$$
$$p_t^2(J_s'(k_m r_L))^2 dp_z dp_t=\int_o^{\sigma_w}\int_o^{\infty} p_t^2(J_s'(k_{mn}r_L))^2[P_{smn}(k_{mn}(b_u-r_L))$$
$$-P_{smn}(k_{mn}(b_l+r_L))]\left\{\left[(\omega\gamma-\beta p_z/m_e)\partial\widehat{f}_o/\partial p_t+(\beta p_t/m_e)\partial\widehat{f}_o/\partial p_z\right]\right.$$
$$\left./\gamma(\omega\gamma-s\Omega_e-\beta p_z/m_e)\right\} dp_z dp_t \qquad (11)$$

where $b_u \,\underline{\Delta}\, (r_w+b_w)/2$ and $b_l \,\underline{\Delta}\, (r_w-b_w)/2$ are respectively the upper and the lower boundaries of the electron beam and

$$P_{smn}(X)\underline{\Delta}X^2[J_{s-m}^2(X)-J_{s-m-1}(X)J_{s-m+1}(X)]/2k_{mn}^2$$

Performing integration by parts with respect to p_t and p_z in (11), the correct form of the exact dispersion relation works out to be

$$\left(\frac{\omega^2}{c^2}-k_{mn}^2-\beta^2\right)=-\frac{4e^2\pi\mu_o}{m_e K_{mn} r_w^2}\sum_s\int_o^{\sigma_w} p_t dp_t\int_o^{\infty} dp_z$$
$$\left[\frac{(\frac{\omega^2}{c^2}-\beta^2)p_t^2 H_{smn}(r_L)}{m_e^2\gamma^2(\omega\gamma-\frac{\beta p_z}{m_e}-s\Omega_e)^2}-\frac{(\omega\gamma-\frac{\beta p_z}{m_e})Q_{smn}(r_L)}{\gamma(\omega\gamma-\frac{\beta p_z}{m_e}-s\Omega_e)}\right]\widehat{f}_o(p_t,p_z) \quad (12)$$

where

$$K_{mn}=\left(1-\frac{m^2}{x_{mn}^2}\right)J_m^2(x_{mn}),$$
$$H_{smn}(r_L)=[J_s'(k_{mn}r_L)]^2[P_{smn}(k_{mn}(b_u-r_L))-P_{smn}(k_{mn}(b_l+r_L))],$$
$$Q_{smn}(r_L)=2\left[(s^2-k_{mn}^2r_L^2)J_s(k_{mn}r_L)/k_{mn}r_L J_s'(k_{mn}r_L)\right]H_{smn}(r_L)$$
$$-k_{mn}r_L(J_s'(k_{mn}r_L))^2[P_{smn}'(k_{mn}(b_u-r_L))+P_{smn}'(k_{mn}(b_l+r_L))],$$

and we have assumed that $\widehat{f}_o(p_t,p_z)$ has compact support within the open rectangle $(0,\sigma_w)\times(0,\infty)$ in order to make the integrated parts vanish. It is to be emphasized that the coefficients H_{smn} and Q_{smn} are functions only of the gyro-radius r_L. In order to evaluate the double integral appearing in (12) in closed form, we choose the functional dependence of the equilibrium distribution function $\widehat{f}_o$ on p_t and p_z to be

$$\widehat{f}_o(p_t,p_z)=n_o\delta_N(p_t-p_{to})\delta_N(p_z-p_{zo})/2\pi p_{to}$$

where $\delta_N(x)$ is a smooth (C^1 at least) function with support contained within the interval $[-1/N,1/N]$ tending to the Dirac delta "function" in the sense of distribution theory [7] as $N \to \infty$, n_o is the number density of the electrons in the beam, and $p_{to} \in (0, \sigma_w)$ and $p_{zo} \in (0, \infty)$ are respectively the mean values of the magnitudes of the transverse momentum and the axial momentum with which the electrons enter the interaction region. For the above choice of $\widehat{f}_o$, the double integral in (12) is well approximated by its limit as $N \to \infty$ provided N is sufficiently large. Evaluating the double integral in closed form in this fashion for a sufficiently large value of N, the dispersion relation (12) takes on a familiar look:

$$\left(\frac{\omega^2}{c^2} - k_{mn}^2 - \beta^2\right)$$
$$= -\frac{2e^2\mu_o n_o}{m_e\gamma_o K_{mn} r_w^2} \sum_s \left[p_{to}^2(\omega^2/c^2 - \beta^2) H_{smn}(r_{Lo}) \Big/ \left(\omega\gamma_o - \frac{\beta p_{zo}}{m_e} - s\Omega_e\right)^2 \right.$$
$$\left. -(\omega\gamma_o - \beta p_{zo}/m_e) Q_{smn}(r_{Lo}) / (\omega\gamma_o - \beta p_{zo}/m_e - s\Omega_e)\right] \qquad (13)$$

where

$$\gamma_o = \left\{1 + (p_{to}^2 + p_{zo}^2)/c^2\right\}^{1/2}$$

A closer look, however, reveals that (13) is fundamentally different from what is available in the literature as the coefficients H_{smn} and Q_{smn} are no longer dependent on the radial coordinate r_o of the electron guiding center.

It is clear from the form of the dispersion relation that the principal contribution to the infinite sum in (13) arises from that value s_o of s for which

$$\omega\gamma_o - \beta p_{zo}/m_e - s_o\Omega_e \approx 0$$

which may be identified as the cyclotron resonance condition (1) since the magnitude of the fractional deviation $|\Delta\beta/\beta_{mn}| \underline{\Delta} |(\beta - \beta_{mn})/\beta_{mn}|$ of β from β_{mn} is $\ll 1$ in small-signal interactions. Retaining only this contribution, the dispersion relation simplifies to

$$\left(\omega^2/c^2 - k_{mn}^2 - \beta^2\right)(\omega\gamma_o - \beta p_{zo}/m_e - s_o\Omega_e)^2 + \sigma_o\left[p_{to}^2\left(\omega^2/c^2 - \beta^2\right) H_{smn}\right.$$
$$\left. -(\omega\gamma_o - \beta p_{zo}/m_e)(\omega\gamma_o - \beta p_{zo}/m_e - s_o\Omega_e) Q_{smn}\right] = 0 \qquad (14)$$

where, in terms of the beam current

$$I_b = e n_o \pi r_w b_w v_{zo},$$
$$\sigma_o = 2e^2\mu_o n_o / m_e\gamma_o K_{mn} r_w^2 = 2e\mu_o I_b / \pi m_e r_w^3 b_w v_{zo}\gamma_o K_{mn},$$

$H_{smn} = H_{smn}(r_{Lo})$, $Q_{smn} = Q_{smn}(r_{Lo})$ and $r_{Lo} = p_{to}/m_e\Omega_e$.

The one-term approximation (14) to the dispersion relation (13) may be recast as a biquadratic algebraic equation for $Z \underline{\Delta} \beta/\beta_{mn}$:

$$Z^4 + 2Z^3 + (\Lambda + \Omega)Z^2 + 2(\Lambda - \sigma\Omega/2)Z - (k_{mn}^2/\beta_{mn}^2)\Lambda = 0 \qquad (15)$$

where the non-dimensional coefficients Λ and Ω and the parameter σ are defined in terms of the non-dimensional quantities

$$\widehat{k}_{mn} = \widehat{\omega}_{mn} = k_{mn}/k_{11}, \quad \widehat{\omega} = \omega/k_{11}c, \quad \widehat{\beta}_{mn} = \beta_{mn}/k_{11}$$

by

$$\Lambda \underline{\Delta} \, \sigma_o(p_{to}/p_{zo})^2 H_{smn}/k_{11}, \quad \Omega \underline{\Delta} \, \sigma_o Q_{smn}/k_{11}^2 \widehat{\beta}_{mn}^2,$$

$$\sigma = c\widehat{\omega}/v_{zo}\widehat{\beta}_{mn} - 1$$

From the wave-guide dispersion relation we have $\widehat{\beta}_{mn} = (\widehat{\omega}^2 - \widehat{k}_{mn}^2)^{1/2}$ which is real for $\widehat{\omega} \geq \widehat{\omega}_{mn} = \widehat{k}_{mn}$ (normalized cut-off frequency (wave number)) of the TE_{mn} mode. In arriving at (15) we have not resorted to the standard approximation of dropping the second term within the square brackets of (14) in comparison with the first term. This approximation is equivalent to setting $\Omega = 0$ in (15). Such a step cannot be justified a priori. The biquadratic equation (15) has always a pair of real roots and a second pair complex conjugate roots if the ratio p_{to}/p_{zo} is not too small. Thus, the interacting TE_{mn} mode splits into a growing wave, a decaying wave, and a pair of waves not subjected to any attenuation or amplification. However, all four waves undergo a shift in their propagation phase constant relative to that of the unperturbed wave-guide mode as a result of the interaction with the electron beam.

4. ILLUSTRATIVE DESIGN EXAMPLE

We now illustrate the small-signal theory developed in this paper by indicating the steps involved in the preliminary design of a typical gyro-TWT amplifier. The design specifications are collected together in Table 1.

For the data in Table 1, we compute

$$\omega_{11} = ck_{11} = cx_{11}/r_w = 1.0221 \times 10^{11} \,\text{rad/sec}$$
$$\widehat{\omega}_d = 2\pi f_d/\omega_{11} = 5.778508$$

and the required values of the axial electron speed v_{zo} and magnetic field strength B_o to be

$$v_{zo} = c\sqrt{1 - (x_{02}c/2\pi r_w f_d)^2} = 0.751737c$$

and

$$B_o = m_e c k_{02}/e\sqrt{s_o^2 - (x_{02}/8)^2} = 1.23127T$$

The following performance curves are plotted in Figs. 2–5 against the normalized frequency variable $\widehat{\omega} = \omega/\omega_{11}$:

Table 1. Design Specifications.

Beam current	500 A
Wave-guide radius	$r_w = 0.54\,\mathrm{cm}$
Operating (center) frequency	$f_d = 94\,\mathrm{GHz}$
Annular beam width	$b_w = r_w/2$
Beam upper boundary	$b_u = 3r_w/4$
Beam lower boundary	$b_l = r_w/4$
Gyro radius at design frequency	$r_{Lo} = b_w/4 = r_w/8$
Operating mode	TE_{02}
s-number	$s_o = 2$

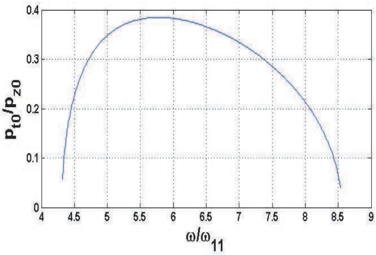

Figure 2. Variation of (p_{t0}/p_{z0}) required for cyclotron resonance.

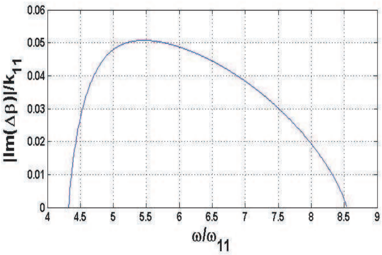

Figure 3. Variation of the normalized initial growth rate.

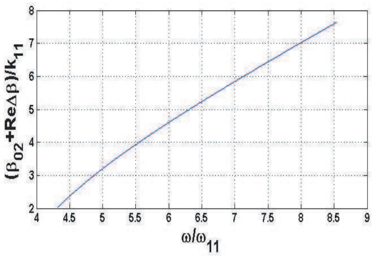

Figure 4. Variation of the perturbed non-dimensional propagation constant.

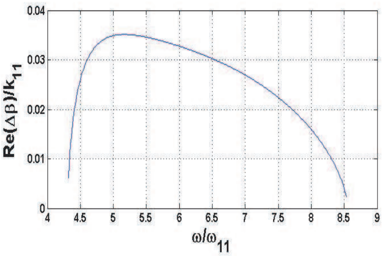

Figure 5. Variation of the normalized deviation of the perturbed propagation constant from β_{02}.

(i) Value of the ratio p_{to}/p_{zo} required for cyclotron resonance as function of $\widehat{\omega}$.

(ii) Variation of the normalized initial growth rate $|\text{Im}\Delta\beta|/k_{11}$ with respect to the normalized frequency $\widehat{\omega}$.

(iii) Perturbed non-dimensional propagation phase constant $(\beta_{02} + \text{Re}\Delta\beta)/k_{11}$ of the exponentially growing wave as a function of $\widehat{\omega}$.

(iv) Normalized deviation $\text{Re}\Delta\beta/k_{11}$ of the propagation phase constant $\text{Re}\beta$ from the unperturbed propagation phase constant β_{02} of the TE_{02} mode as a function of $\widehat{\omega}$.

5. CONCLUDING COMMENTS

It may be seen from Figs. 2 and 3 that the drop in gain around the design frequency, which corresponds to the flat maximum in the p_{to}/p_{zo} plot, arising from the slight mismatch is not significant. This means that the amplifier is capable of a reasonable operating bandwidth around the design frequency. Since, the perturbation of the electron-beam characteristics arising out of the interaction is neglected in a small-signal theory, it does not make sense to discuss about the performance indices like power gain, efficiency, optimum interaction length etc. on the basis of a small-signal analysis of the gyro-TWT amplifier.

Work on a large signal field theory for a gyro-TWT amplifier incorporating space-charge effects is in progress and will be reported in due course.

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