EDDY CURRENT PHENOMENA IN LAMINATED STRUCTURES DUE TO TRAVELLING ELECTROMAGNETIC FIELDS

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Abstract—The distribution of fields travelling in the laminated structure with assumed values for the tangential components of the magnetic field intensities on the top and bottom surfaces of the structure, has been obtained using linear electromagnetic field theory. The treatment takes cognizance of interlaminar capacitance inherently present in a laminated structure. Analysis presented in this paper assumes identical field distribution in each lamination. It has been concluded that convection currents are developed at the interface between iron and insulator regions.

1. INTRODUCTION

In dynamo electric machines, poles are usually made of solid iron. Excitation windings on these poles carry dc current. The iron surface across the air gap is slotted. Air gap permeance therefore varies periodically in the peripheral direction. This causes a periodic variation of excitation field in the peripheral direction. Travelling electromagnetic fields result when the machine is rotating. Consequently, eddy currents are induced on the pole surface. To reduce eddy currents, solid poles are often fitted with laminated pole-shoes.

Eddy currents are induced in conductors subjected to time varying electromagnetic fields [1–3]. Eddy current phenomena in laminated cores subjected to sinusoidally time varying electromagnetic fields have been studied by many authors [4–7]. Theoretical and experimental investigations of eddy currents in laminated pole-shoes are reported in literature [8–12]. Bondi and Mukherji [10] have

analyzed electromagnetic fields in the laminated poleshoe using a model with negligible thickness of insulation on iron surfaces of each pole-shoe lamination and infinite air gap length. Greig and Sathirakul [11] in their treatment based on finite air-gap length, conclude that the length of the air gap in a practical machine has little effect on the eddy current loss. Both treatments [10, 11] assume that the normal component of eddy current density vanishes at the iron-insulator interfaces. Clearly, these treatments ignore the presence of distributed capacitance in the path of eddy currents [3, 7]. As a result, the continuity condition for the tangential component of electric field intensity at the boundary between adjacent laminations [10, 11] is violated. Analyses presented in these two references are quite involved. A simplified version of the eddycurrent loss equation for laminated pole-shoes is presented by Greig and Freeman [12].

The laminated armature core of any common type of rotating electrical machine is subjected to travelling electromagnetic fields [13]. This paper attempts to provide a simple treatment for eddy currents in laminated structures with assumed values for the tangential components of the magnetic field intensities on the top and bottom surfaces of the laminated structure. The analysis takes cognizance of the presence of distributed capacitance in the path of eddy currents. It is believed that the treatment can be adapted for the study of eddy currents in smooth laminated pole-shoes as well as in the laminated armature core of rotating electrical machines.

2. FIELD EQUATIONS

The rectangular Cartesian co-ordinates as shown in Fig. 1, are defined as follows: X_1 and X_2 are parallel to the axial, Y to the peripheral and Z to the radial directions The co-ordinate system is fixed on
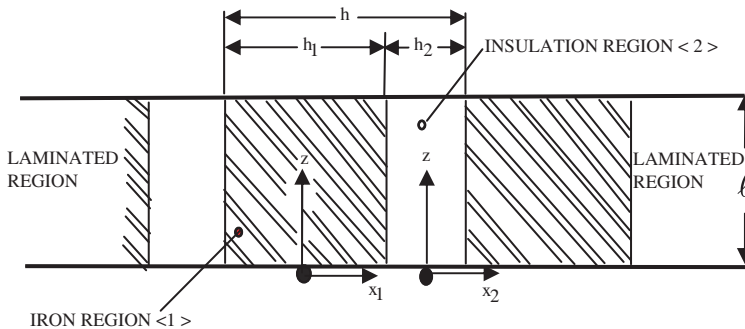

Figure 1. Laminated structure.

the laminated structure. The space, $0 < z < \ell$, is occupied by the laminated structure consisting of iron and insulator regions laid alternately along the axial direction. Let the positive direction of Y be taken along the direction of motion of the travelling electromagnetic field. If U is the relative peripheral velocity and λ the wavelength, the electromagnetic fields will be functions of $e^{j(\omega \cdot t - k \cdot y)}$, where $k = 2\pi/\lambda$, and ω is the product of U and k. The exponential factor has, however, been suppressed to present field expressions in the phasor form.

With constant values of permeability μ, permittivity ε, and conductivity σ, Maxwell's equations for harmonic fields in source-free regions of the laminated structure, are:

$$\nabla \cdot \mathbf{B} = \nabla \cdot \mathbf{H} = 0 \tag{1a}$$

$$\nabla \cdot \mathbf{D} = \nabla \cdot \mathbf{E} = 0 \tag{1b}$$

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H} \tag{1c}$$

$$\nabla \times \mathbf{H} = \hat{\sigma}\mathbf{E} \tag{1d}$$

where the complex conductivity $\hat{\sigma}$, is defined as:

$$\hat{\sigma} \triangleq (\sigma + j\omega\varepsilon) \tag{2}$$

From the above, following field equations are found:

$$\nabla^2\mathbf{E} = \eta^2\mathbf{E} \tag{3}$$

$$\nabla^2\mathbf{H} = \eta^2\mathbf{H} \tag{4}$$

where
$$\eta^2 = j\omega\mu\hat{\sigma} \tag{5}$$

3. SOLUTION OF FIELD EQUATIONS

Let the suffixes 1 and 2 indicate respectively, the iron and insulator regions in the laminated structure, as shown in Fig. 1. In view of the geometric symmetry about the plane $x_1 = 0$, the magnetic field H_{1x} is an odd function of x_1. It is assumed that H_{1x} vanishes at $z = 0$ as well as at $z = \ell$. The solution of Eq. (4), for H_{1x} can therefore be expressed as:

$$H_{1x} = \sum_{q=1}^{\infty} b_{1q} \cdot \frac{\sinh(\delta_{1q} \cdot x_1)}{\sinh(\delta_{1q} \cdot \frac{1}{2}h_1)} \sin\left(\frac{q\pi}{\ell} \cdot z\right) \tag{6}$$

where,

$$\delta_{1q} = \sqrt{\left(\frac{q\pi}{\ell}\right)^2 + \eta_1^2 + k^2} \tag{6a}$$

$$\eta_1^2 = j\omega\mu_1\hat{\sigma}_1 \tag{6b}$$

$$\hat{\sigma}_1 = (\sigma_1 + j\omega\varepsilon_1) \tag{6c}$$

while, b_{1q} indicates a set of arbitrary constants.

The magnetic field H_{1y} is an even function of x_1, and it is assumed that this field component vanishes at $z = \ell$. Let the magnetic field, H_{1y} on the surface $z = 0$ be given as $(-H_0)$. Therefore, a tentative solution of Eq. (4), gives:

$$H_{1y} = H_0 \cdot \frac{\sinh \delta_{10}(z - \ell)}{\sinh (\delta_{10} \cdot \ell)} + \sum_{q=1}^{\infty} c_{1q} \cdot \frac{\cosh (\delta_{1q} \cdot x_1)}{\sinh \left(\delta_{1q} \cdot \frac{1}{2} h_1\right)} \sin \left(\frac{q\pi}{\ell} \cdot z\right) \quad (7)$$

where, c_{1q} indicates a set of arbitrary constants.

Using Eqs. (1a), (6) and (7), one obtains:

$$H_{1z} = -H_0 \cdot \frac{jk}{\delta_{10}} \cdot \frac{\cosh \delta_{10}(z - \ell)}{\sinh (\delta_{10} \cdot \ell)} + \sum_{q=1}^{\infty} d_{1q} \cdot \frac{\cosh (\delta_{1q} \cdot x_1)}{\sinh \left(\delta_{1q} \cdot \frac{1}{2} h_1\right)}$$

$$\cdot \cos \left(\frac{q\pi}{\ell} \cdot z\right) + d_{10} \frac{\cosh (\delta_{10} \cdot x_1)}{\sinh \left(\delta_{10} \cdot \frac{1}{2} h_1\right)} \quad (8)$$

where, d_{1q} indicates a set of arbitrary constants.

In view of Eq. (1a), for regions 1 and 2:

$$b_{1q} \cdot \delta_{1q} + c_{1q} \cdot jk - d_{1q} \cdot \frac{q\pi}{\ell} = 0 \quad (1a1)$$

$$b_{2q} \cdot \delta_{2q} + c_{2q} \cdot jk - d_{2q} \cdot \frac{q\pi}{\ell} = 0 \quad (1a2)$$

for $q = 1, 2, 3, \ldots$.

Using Eqs. (6)–(8) and (1d), the components of electric field intensity found are as follows:

$$E_{1x} = -\frac{1}{\hat{\sigma}_1} \cdot \left[H_0 \cdot \frac{\eta_1^2}{\delta_{10}} \cdot \frac{\cosh \delta_{10} \cdot (z - \ell)}{\sinh(\delta_{10} \cdot \cdot \ell)} + d_{10} \cdot jk \cdot \frac{\cosh (\delta_{10} \cdot x_1)}{\sinh \left(\delta_{10} \cdot \frac{1}{2} h_1\right)} \right.$$

$$\left. + \sum_{q=1}^{\infty} \left\{ c_{1q} \cdot \frac{q\pi}{\ell} + d_{1q} \cdot jk \right\} \cdot \frac{\cosh (\delta_{1q} \cdot x_1)}{\sinh \left(\delta_{1q} \cdot \frac{1}{2} h_1\right)} \cdot \cos \left(\frac{q\pi}{\ell} \cdot z\right) \right] \quad (9)$$

$$E_{1y} = \frac{1}{\hat{\sigma}_1} \cdot \left[\sum_{q=1}^{\infty} \left\{ b_{1q} \cdot \frac{q\pi}{\ell} - d_{1q} \cdot \delta_{1q} \right\} \cdot \frac{\sinh (\delta_{1q} \cdot x_1)}{\sinh \left(\delta_{1q} \cdot \frac{1}{2} h_1\right)} \cdot \cos \left(\frac{q\pi}{\ell} \cdot z\right) \right.$$

$$\left. - d_{10} \cdot \delta_{10} \cdot \frac{\sinh (\delta_{10} \cdot x_1)}{\sinh \left(\delta_{10} \cdot \frac{1}{2} h_1\right)} \right] \quad (10)$$

$$E_{1z} = \frac{1}{\hat{\sigma}_1} \cdot \left[\sum_{q=1}^{\infty} \{ b_{1q} \cdot jk + c_{1q} \cdot \delta_{1q} \} \cdot \frac{\sinh (\delta_{1q} \cdot x_1)}{\sinh \left(\delta_{1q} \cdot \frac{1}{2} h_1\right)} \cdot \sin \left(\frac{q\pi}{\ell} \cdot z\right) \right] \quad (11)$$

Considering Fig. 1, the field distributions for the insulation region, $-\frac{1}{2}h_2 < x_2 < \frac{1}{2}h_2$, can be obtained if the suffix-1, in all equations of this section is replaced by suffix-2. As a result, new sets of arbitrary constants are introduced.

4. BOUNDARY CONDITIONS

4.1. Boundary Conditions at $x_1 = \frac{1}{2}h_1$ (or $x_2 = -\frac{1}{2}h_2$)

Boundary conditions for the tangential components of electric field intensity are as follows:

$$E_{1y}\big|_{x_1=\frac{1}{2}\cdot h_1} = E_{2y}\big|_{x_2=-\frac{1}{2}\cdot h_2} \quad \text{over } 0 < z < \ell. \tag{12a}$$

$$E_{1z}\big|_{x_1=\frac{1}{2}\cdot h_1} = E_{2z}\big|_{x_2=-\frac{1}{2}\cdot h_2} \quad \text{over } 0 < z < \ell. \tag{12b}$$

The boundary condition for the continuity of current is:

$$\hat{\sigma_1} \cdot E_{1x}\big|_{x_1=\frac{1}{2}\cdot h_1} = \hat{\sigma_2} \cdot E_{2x}\big|_{x_2=-\frac{1}{2}\cdot h_2}, \quad \text{over } 0 < z < \ell. \tag{13}$$

while, the boundary condition for the normal component of magnetic field intensity is:

$$\mu_1 H_{1x}\big|_{x_1=\frac{1}{2}\cdot h_1} = \mu_2 H_{2x}\big|_{x_2=-\frac{1}{2}\cdot h_2}, \quad \text{over } 0 \leq z \leq \ell. \tag{14}$$

4.2. Boundary Conditions at $Z = 0$

The magnetic field, H_y on the $z = 0$ surface, is assumed to be given as $(-H_o)$. Further, it is assumed that the magnetic field, H_x on this surface will vanish. Expressions for these field components are chosen to satisfy these boundary conditions identically.

4.3. Boundary Conditions at $Z = \ell$

It is further assumed that the tangential components of magnetic field intensity take zero value on the surface $z = \ell$. Expressions for these field components are chosen to satisfy this boundary condition identically.

5. EVALUATION OF ARBITRARY CONSTANTS

With the help of the orthogonal property of Fourier series and all boundary conditions stated in Section 4.1 above, linear simultaneous algebraic equations are found between various arbitrary constants (vide Appendix A). Solution of these equations, together with Eqs. (1a1) and (1a2), determines all arbitrary constants involved in the field

expressions. These arbitrary constants are complex functions of various parameters (geometric as well as electromagnetic), of the laminated structure. For instance, expressions for d_{10} and d_{20} found from Eqs. (A1) and (A2) are:

$$d_{10} = -H_0 \cdot \frac{\frac{jk}{\ell} \cdot \hat{\sigma}_1 \cdot \delta_{20} \cdot \left\{ \frac{1}{\delta_{10}^2} - \frac{1}{\delta_{20}^2} \right\}}{\hat{\sigma}_2 \cdot \delta_{10} \cdot \coth\left(\delta_{20} \cdot \frac{1}{2} h_2 \right) + \hat{\sigma}_1 \cdot \delta_{20} \cdot \coth\left(\delta_{10} \cdot \frac{1}{2} h_1 \right)} \quad (15)$$

$$d_{20} = H_0 \cdot \frac{\frac{jk}{\ell} \cdot \hat{\sigma}_2 \cdot \delta_{10} \cdot \left\{ \frac{1}{\delta_{10}^2} - \frac{1}{\delta_{20}^2} \right\}}{\hat{\sigma}_2 \cdot \delta_{10} \cdot \coth\left(\delta_{20} \cdot \frac{1}{2} h_2 \right) + \hat{\sigma}_1 \cdot \delta_{20} \cdot \coth\left(\delta_{10} \cdot \frac{1}{2} h_1 \right)} \quad (16)$$

The general solutions of simultaneous Eqs. (A3)–(A6), (1a1) and (1a2), can be readily found giving expressions for remaining arbitrary constants. These, however, are of little use as the arbitrary constants are not simple functions of various parameters. On the basis of practical values of geometric and electromagnetic parameters of the laminated structure, using simplified albeit approximate expressions for various arbitrary constants are found as given in the Appendix B.

6. CONVECTION CURRENT DENSITY

The distributions of current density components, K_y and K_z, on the surface $x_1 = \frac{1}{2} \cdot h_1$ (or $x_2 = -\frac{1}{2} \cdot h_2$), are given by:

$$K_y = H_{1z}\big|_{x_1 = \frac{1}{2} \cdot h_1} - H_{2z}\big|_{x_2 = -\frac{1}{2} \cdot h_2}, \quad \text{over} \quad 0 < z < \ell. \quad (17)$$

and

$$K_z = H_{1y}\big|_{x_2 = \frac{1}{2} \cdot h_2} - H_{1y}\big|_{x_1 = -\frac{1}{2} \cdot h_1}, \quad \text{over} \quad 0 < z < \ell. \quad (18)$$

Thus, using Eqs. (7) and (8)

$$
\begin{aligned}
K_y = {} & -H_0 \cdot \left\{ \frac{jk}{\delta_{10}} \cdot \frac{\cosh \delta_{10}(z - \ell)}{\sinh(\delta_{10} \cdot \ell)} - \frac{jk}{\delta_{20}} \cdot \frac{\cosh \delta_{20}(z - \ell)}{\sinh(\delta_{20} \cdot)} \right\} \\
& + \left\{ d_{10} \cdot \coth\left(\delta_{10} \cdot \frac{1}{2} h_1 \right) - d_{20} \cdot \coth\left(\delta_{20} \cdot \frac{1}{2} h_2 \right) \right\} \\
& + \sum_{q=1}^{\infty} \left\{ d_{1q} \cdot \coth\left(\delta_{1q} \cdot \frac{1}{2} h_1 \right) - d_{2q} \cdot \coth\left(\delta_{2q} \cdot \frac{1}{2} h_2 \right) \right\} \\
& \cdot \cos\left(\frac{q\pi}{\ell} \cdot z \right) \quad \text{over} \quad 0 < z < \ell. \quad (19)
\end{aligned}
$$

and

$$K_z = -H_0 \cdot \left\{ \frac{\sinh \delta_{10}(z-\ell)}{\sinh (\delta_{10} \cdot \ell)} - \frac{\sinh \delta_{20}(z-\ell)}{\sinh (\delta_{20} \cdot \ell)} \right\} - \sum_{q=1}^{\infty} \left\{ c_{1q} \cdot \coth \left(\delta_{1q} \cdot \frac{1}{2} h_1 \right) \right. $$

$$\left. -c_{2q} \cdot \coth \left(\delta_{2q} \cdot \frac{1}{2} h_2 \right) \right\} \cdot \sin \left(\frac{q\pi}{\ell} \cdot z \right) \quad \text{over } 0 < z < \ell. \tag{20}$$

7. DISCUSSION

The solution of the boundary-value problem is based on a set of continuity conditions on the boundary between two adjacent regions. In the present case it was found that the continuity of the two tangential components of the vector $\mathbf{E}$ and the normal component of the vector $\mathbf{B}$ at the boundary do not provide sufficient equations for the determination of various arbitrary constants. Similar situation has been reported earlier as well [14, 15]. The additional boundary condition used is the well known continuity of the total current crossing the boundary surface $(x_1 = \frac{1}{2} \cdot h_1)$, at every point.

Discontinuity in the normal component of the vector $\mathbf{D}$ at the boundary surface $(x_1 = \frac{1}{2} \cdot h_1)$, indicates deposition of electric charges on this surface due to the capacitive effect of the interlaminar insulation.

Discontinuity in the two tangential components of the vector $\mathbf{H}$ at this boundary surface determines the convection current density with components K_y and K_z on the boundary surface. It has been noticed that, since:

$$K_y = H_{1z}|_{x_1 = \frac{1}{2} \cdot h_1} - H_{2z}|_{x_2 = -\frac{1}{2} \cdot h_2} \quad \text{and} \quad K_z = H_{2y}|_{x_2 = -\frac{1}{2} \cdot h_2} - H_{1y}|_{x_1 = \frac{1}{2} \cdot h_1}$$

hence on the surface $x_1 = \frac{1}{2} \cdot h_1$ (or, $x_2 = -\frac{1}{2} \cdot h_2$),

$$\boldsymbol{\nabla} \cdot \mathbf{K} = \frac{\partial K_y}{\partial y} + \frac{\partial K_z}{\partial z}$$

$$= \left[\frac{\partial H_{1z}}{\partial y} \bigg|_{x_1 = \frac{1}{2} \cdot h_1} - \frac{\partial H_{2z}}{\partial y} \bigg|_{x_2 = -\frac{1}{2} \cdot h_2} \right] + \left[\frac{\partial H_{2y}}{\partial z} \bigg|_{x_2 = -\frac{1}{2} \cdot h_2} - \frac{\partial H_{1y}}{\partial z} \bigg|_{x_1 = \frac{1}{2} \cdot h_1} \right]$$

$$= \left[\frac{\partial H_{1z}}{\partial y} - \frac{\partial H_{1y}}{\partial z} \right] \bigg|_{x_1 = \frac{1}{2} \cdot h_1} - \left[\frac{\partial H_{2z}}{\partial y} - \frac{\partial H_{2y}}{\partial z} \right] \bigg|_{x_2 = -\frac{1}{2} \cdot h_2}$$

$$= \hat{\sigma_1} \cdot E_{1x}|_{x_1 = \frac{1}{2} \cdot h_1} - \hat{\sigma_2} \cdot E_{2x}|_{x_2 = -\frac{1}{2} \cdot h_2}$$

In view of Eq. (13), the right hand side of this equation is zero. Therefore,

$$\boldsymbol{\nabla} \cdot \mathbf{K} = 0 \tag{21}$$

Power loss per unit surface area of iron-insulator interface is obtained from:

$$\wp_s = Re\left[\frac{1}{2}\mathbf{E}\cdot\mathbf{K}^*\bigg|_{x_1=\frac{1}{2}h_1}\right] = Re\left[\frac{1}{2}\left(E_{1y}\cdot K_y^* + E_{1z}\cdot K_z^*\right)\bigg|_{x_1=\frac{1}{2}h_1}\right] \quad (22)$$

While the power loss per unit volume of the iron in each lamination is given as:

$$\wp_v = Re\left[\frac{1}{2}\mathbf{E}_1\cdot\mathbf{J}_1^*\right] = Re\left[\frac{1}{2}\sigma_1\left(E_{1x}\cdot E_{1x}^* + E_{1y}\cdot E_{1y}^* + E_{1z}\cdot E_{1z}^*\right)\right] \quad (23)$$

Therefore, total eddy current loss per unit peripheral length, $\wp_e$, for each lamination will be:

$$\wp_e = 2\int_0^{\ell}\wp_s\,dz + \int_0^{\ell}\int_{-\frac{1}{2}\cdot h_1}^{\frac{1}{2}\cdot h_1}\wp_v dx_1 dz \quad (24)$$

In the design of large electrical machines efficiency and cooling are important considerations. Last three equations could be useful in the study of machine efficiency and thermal circuits.

8. CONCLUSION

In view of Uniqueness theorem [16] for the solution of Maxwell's equations, Fourier series based field expressions are developed in Section 2. These solutions satisfy assumed values for the tangential components of the magnetic field intensities on the surfaces, $z = 0$ and $z = \ell$. Therefrom it is possible to infer following features of the eddy current phenomena associated with travelling electromagnetic field in laminated structures.

(1) Because of capacitive effects, the axial component of eddy current density in the iron J_{1x}, does not vanish on the iron-insulator interface. Consequently, charges are deposited on these surfaces (i.e., $x_1 = \pm\frac{1}{2}\cdot h_1$ or, $x_2 = \pm\frac{1}{2}\cdot h_2$).

(2) These surface-charge distributions, travelling in the peripheral direction constitute a surface distribution of convection currents with density K_y.

(3) Since K_y varies sinusoidally along the peripheral direction-y, radial component of convection currents with density K_z is also present on these surfaces.

(4) The presence of these convection currents is one of the deciding factors for shaping the distributions of electromagnetic fields in laminated structures.

(5) From approximate calculations it appears that the axial component of magnetic field H_x in both conducting as well as non-conducting regions, is negligible for practical values of geometric and electromagnetic parameters of the laminated structure.

(6) From the approximate expressions for arbitrary constants given in the Appendix B, it may be inferred that the various Fourier series describing field distributions converge satisfactorily.

APPENDIX A.

Equations between various arbitrary constants found using boundary conditions are as follows:

$$d_{10} \cdot \frac{\delta_{10}}{\hat{\sigma}_1} + d_{20} \cdot \frac{\delta_{20}}{\hat{\sigma}_2} = 0 \tag{A1}$$

$$d_{10} \cdot \coth\left(\delta_{10} \cdot \frac{1}{2} h_1\right) - d_{20} \cdot \coth\left(\delta_{20} \cdot \frac{1}{2} h_2\right) = -H_0 \cdot \frac{jk}{\ell} \cdot \left\{ \frac{1}{\delta_{10}^2} - \frac{1}{\delta_{20}^2} \right\} \tag{A2}$$

Other equations, for $q = 1, 2, 3, \ldots$, are as follows:

$$\frac{1}{\hat{\sigma}_1} \cdot \left\{ b_{1q} \cdot \frac{q\pi}{\ell} - d_{1q} \cdot \delta_{1q} \right\} + \frac{1}{\hat{\sigma}_2} \cdot \left\{ b_{2q} \cdot \frac{q\pi}{\ell} - d_{2q} \cdot \delta_{2q} \right\} = 0 \tag{A3}$$

$$\frac{1}{\hat{\sigma}_1} \left\{ b_{1q} \cdot jk + c_{1q} \cdot \delta_{1q} \right\} + \frac{1}{\hat{\sigma}_2} \cdot \left\{ b_{2q} \cdot jk + c_{2q} \cdot \delta_{2q} \right\} = 0 \tag{A4}$$

$$\mu_1 \cdot b_{1q} + \mu_2 \cdot b_{2q} = 0 \tag{A5}$$

$$\left\{ c_{1q} \cdot \frac{q\pi}{\ell} + d_{1q} \cdot jk \right\} \cdot \coth\left(\delta_{1q} \cdot \frac{1}{2} h_1\right) - \left\{ c_{2q} \cdot \frac{q\pi}{\ell} + d_{2q} \cdot jk \right\}$$

$$\cdot \coth\left(\delta_{2q} \cdot \frac{1}{2} h_2\right) = -H_0 \cdot \frac{2}{\ell} \cdot \left\{ \frac{\eta_1^2}{\delta_{1q}^2} - \frac{\eta_2^2}{\delta_{2q}^2} \right\} \tag{A6}$$

APPENDIX B.

Approximate expressions for various arbitrary constants found are as follows:

$$b_{1q} \cong b_{2q} \cong 0 \tag{B1}$$

$$c_{1q} \cong \left[-H_0 \cdot \frac{j\omega\mu_1\sigma_1^2}{(\sigma_1 + j\omega\varepsilon_2)} \right] \cdot \frac{2\ell^2/\pi^3}{q \cdot \left[q^2 - (k\ell/\pi)^2 \right]} \triangleq \frac{C_1}{q \cdot \left[q^2 - (k\ell/\pi)^2 \right]} \tag{B2}$$

$$c_{2q} \cong \left[-H_0 \cdot \frac{\omega^2 \mu_1 \varepsilon_2 \sigma_1}{(\sigma_1 + j\omega\varepsilon_2)} \right] \cdot \frac{2\ell^2/\pi^3}{q \cdot \left[q^2 - (k\ell/\pi)^2 \right]} \triangleq \frac{C_2}{q \cdot \left[q^2 - (k\ell/\pi)^2 \right]} \quad \text{(B3)}$$

$$d_{1q} \cong \left[H_0 \cdot \frac{k\omega\mu_1\sigma_1^2}{(\sigma_1 + j\omega\varepsilon_2)} \right] \cdot \frac{2\ell^3/\pi^4}{q^2 \cdot \left[q^2 - (k\ell/\pi)^2 \right]} \triangleq \frac{D_1}{q^2 \cdot \left[q^2 - (k\ell/\pi)^2 \right]} \quad \text{(B4)}$$

$$d_{2q} \cong \left[-H_0 \cdot \frac{jk \cdot \omega^2 \mu_1 \varepsilon_2 \sigma_1}{(\sigma_1 + j\omega\varepsilon_2)} \right] \cdot \frac{2\ell^3/\pi^4}{q^2 \cdot \left[q^2 - (k\ell/\pi)^2 \right]} \triangleq \frac{D_2}{q^2 \cdot \left[q^2 - (k\ell/\pi)^2 \right]} \quad \text{(B5)}$$

for $q = 1, 2, 3, \ldots$, where, C_1, C_2, D_1 and D_2 are known constants as defined above.

For the remaining arbitrary constants, approximate expressions are:

$$d_{10} \cong H_0 \cdot \frac{j}{k\ell} \quad \text{(B6)}$$

$$d_{20} \cong 0 \quad \text{(B7)}$$

Therefore, over $0 < z < \ell$,

$$K_y \cong -H_0 \cdot \left\{ \frac{jk}{\delta_{10}} \cdot \frac{\cosh \delta_{10}(z-\ell)}{\sinh(\delta_{10} \cdot \ell)} - \frac{jk}{\delta_{20}} \cdot \frac{\cosh \delta_{20}(z-\ell)}{\sinh(\delta_{20} \cdot \ell)} + \frac{j}{k\ell} \right\}$$

$$+ \sum_{q=1}^{\infty} \left\{ \frac{D_1 - D_2}{q^2 \cdot \left[q^2 - (k\ell/\pi)^2 \right]} \right\} \cos\left(\frac{q\pi}{\ell} \cdot z \right) \quad \text{(B8)}$$

$$K_z \cong -H_0 \cdot \left\{ \frac{\sinh \delta_{10}(z-\ell)}{\sinh(\delta_{10} \cdot \ell)} - \frac{\sinh \delta_{20}(z-\ell)}{\sinh(\delta_{20} \cdot \ell)} \right\}$$

$$- \sum_{q=1}^{\infty} \left\{ \frac{C_1 - C_2}{q \cdot \left[q^2 - (k\ell/\pi)^2 \right]} \right\} \cdot \sin\left(\frac{q\pi}{\ell} \cdot z \right) \quad \text{(B9)}$$

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