Progress In Electromagnetics Research, Vol. 115, 243–257, 2011

NEGATIVE REFRACTION IN AN ANISOTROPIC METAMATERIAL WITH A ROTATION ANGLE BETWEEN THE PRINCIPAL AXIS AND THE PLANAR INTERFACE

S.-H. Liu and L.-X. Guo

School of Science
Xidian University, Xi'an 710071, China

Abstract—The propagation characteristics of electromagnetic waves at the interface between an isotropic regular medium and an anisotropic metamaterial for arbitrary orientation of principal axis are investigated. In terms of the different sign combinations of the tensor components along principal axes, the anisotropic media are divided into four classes. The existence conditions of negative refraction are discussed in different cases, indicating that the conditions for the existence of negative refraction are closely dependent on the principal components and the rotation angle. Furthermore, the influence of the rotation of the principal axes on the incident angle region is analyzed for each case, and the optimal material parameters are attained for the maximum area of the incident angle region of negative refraction occurrence.

1. INTRODUCTION

Artificial metamaterials have received much attention in the past few years [1–10]. The concept of such materials with negative permittivity and permeability in a certain band of frequency was first proposed by Veselago in 1968 [1], who anticipated many unique electromagnetic properties exhibited by the media such as negative refraction, reversed Doppler effect and reversed Cerenkov radiation. It was not until 2001 that the phenomenon of the negative refraction was experimentally verified first by using artificial metamaterials realized by periodic arrays of split ring resonators (SRRs) and wire strips [4]. Based on the negative refraction, the metamaterials have found several potential

Received 20 February 2011, Accepted 28 March 2011, Scheduled 1 April 2011
Corresponding author: Song-Hua Liu (shliu@mail.xidian.edu.cn).

applications in subwavelength image reconstruction [5, 6] and wave guiding [7].

However, the isotropic three-dimensional metamaterials are difficult to be prepared. Thus, the electromagnetic characteristics of anisotropic metamaterial, in which the components of the permittivity and permeability tensors are not all of the same sign, have been discussed theoretically and experimentally in many literatures [11–23]. Lindell et al. have demonstrated that the occurrence of negative refraction at the interface associated with uniaxially anisotropic media requires only one tensor component with a negative value [14]. Hu and Chui compared the characteristics of anisotropic metamaterial with that of isotropic metamaterial further. They have found that in a uniaxially anisotropic metamaterial it is the approximately left-handed waves that may be able to propagate ($\vec{E}$, $\vec{H}$, and $\vec{k}$ form an approximately left-handed triplet of vectors and the direction of energy flow is in the backward but not exactly opposite direction of wave vector), and besides, the left-handed waves are not necessarily tight to the negative refraction [15]. Ding et al. simulated the negative refraction of the continuous-wave Gaussian Beam passing from free space into biaxially anisotropic metamaterial by the finite difference time domain (FDTD) method [16]. In addition, the negative refraction has also been demonstrated experimentally at the interface associated with a wedge composed of anisotropic metamaterial cut along nonprincipal axes [17], indicating that certain classes of anisotropic media have identical refractive properties as isotropic negative index materials [18]. Such materials can be readily constructed using different resonant inclusions such as SRRs and rods to demonstrate negative refraction, and they have potential applications in imaging, polarizing beam splitter, and wave guiding [19–21].

To the authors' knowledge, however, most of the theoretical studies reported previously are limited in special cases that the planar interface coincides with a principal axis. Recently, some characteristics of wave propagation in anisotropic media for arbitrary orientation of principal axis, such as group delay [24], Goos-Hänchen shift [25], and amphoteric refraction [26], have been discussed. Nevertheless, the property of negative refraction is closely dependent on the parameters of the anisotropic metamaterial, and a detailed investigation on the effects of the constitutive parameters and rotation angle has not yet been accomplished.

In this paper, we discuss the approximately left-handed behavior and negative refraction in an arbitrarily oriented anisotropic metamaterial. Taking TE-polarized waves as an example, we derive the existence conditions of negative refraction. Furthermore, we present our numerical results and analyze the influence of the principle components and rotation angle on negative refraction for each case of the anisotropic medium graphically. For TM waves, the negative refraction properties can be analyzed similarly.

2. CONDITIONS FOR THE OCCURRENCE OF NEGATIVE REFRACTION

A plane wave is incident from region 1 at an incident angle θ_i into region 2, as shown in Fig. 1. Region 1 is filled with isotropic regular media with the permittivity ε_1 and permeability μ_1. Region 2 is occupied by anisotropic metamaterial. The interface formed by the two media is in the xy plane, and the normal direction is the z axis. We assume that the permittivity and permeability tensors can be simultaneously diagonalizable in the principal coordinate system (X, Y, Z):

$$\bar{\bar{\varepsilon}}_2 = \begin{bmatrix} \varepsilon_X & & \\ & \varepsilon_Y & \\ & & \varepsilon_Z \end{bmatrix}, \quad \bar{\bar{\mu}}_2 = \begin{bmatrix} \mu_X & & \\ & \mu_Y & \\ & & \mu_Z \end{bmatrix} \tag{1}$$

where not all of the principal components have the same sign. Without loss of generality, we present a discussion of negative refraction at the interface associated with an arbitrarily oriented anisotropic metamaterial. Thus, in the coordinate system (x, y, z),

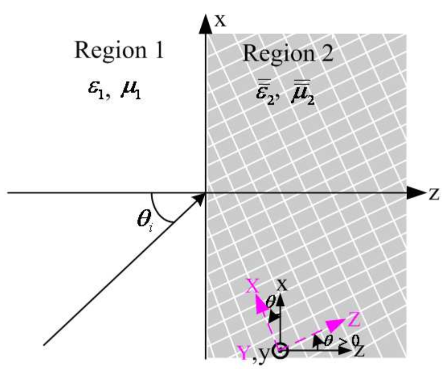

Figure 1. Geometrical structure of the problem. The rotation angle between the principal axis X of the anisotropic metamaterial and the planar surface is θ.

the permittivity and permeability tensors become

$$\bar{\bar{\varepsilon}}_2 = \begin{bmatrix} \varepsilon_{xx} & & \varepsilon_{xz} \\ & \varepsilon_y & \\ \varepsilon_{zx} & & \varepsilon_{zz} \end{bmatrix}, \quad \bar{\bar{\mu}}_2 = \begin{bmatrix} \mu_{xx} & & \mu_{xz} \\ & \mu_y & \\ \mu_{zx} & & \mu_{zz} \end{bmatrix} \tag{2}$$

with

$$\varepsilon_{xx} = \varepsilon_X \cos^2\theta + \varepsilon_Z \sin^2\theta \tag{3a}$$
$$\varepsilon_{zz} = \varepsilon_X \sin^2\theta + \varepsilon_Z \cos^2\theta \tag{3b}$$
$$\varepsilon_{xz} = \varepsilon_{zx} = (\varepsilon_Z - \varepsilon_X)\sin\theta\cos\theta \tag{3c}$$
$$\varepsilon_y = \varepsilon_Y \tag{3d}$$

where the rotation angle θ is made by rotating one of the principal axes X on the fixed Y axis, and $0 \leq \theta \leq \pi/2$ is considered for the convenience in the following analysis. The tensor permeability components can be obtained from the expressions above by substitution of $\varepsilon \to \mu$.

For TE waves, the wave vector $\vec{k}$ is assumed in the xz plane, so the electric field vectors of the incident and reflected waves in region 1 are expressed as

$$\vec{E}_{1i} = \hat{y}E_1^+ e^{-j(k_x x + k_{1z} z)} \tag{4}$$
$$\vec{E}_{1r} = \hat{y}RE_1^+ e^{-j(k_x x - k_{1z} z)} \tag{5}$$

In region 2, the electric field vectors of the refracted waves can be written as

$$\vec{E}_{2t} = \hat{y}TE_1^+ e^{-j(k_x x + k_{2z} z)} \tag{6}$$

The magnetic field vectors in region 2 are governed by equation $\vec{k} \times \vec{E} = \omega\bar{\bar{\mu}} \cdot \vec{H}$, which gives

$$\vec{H}_{2t} = \left[\frac{-(k_{2z}\mu_{zz} + k_x\mu_{xz})TE_1^+}{\omega\mu_X\mu_Z}\hat{x} + \frac{(k_x\mu_{xx} + k_{2z}\mu_{zx})TE_1^+}{\omega\mu_X\mu_Z}\hat{z}\right] e^{-j(k_x x + k_{2z} z)} \tag{7}$$

where R and T are the reflection and transmission coefficients at the interface respectively, and k_{2z} can be determined by

$$\alpha k_x^2 + \beta k_{2z}^2 + \gamma k_x k_{2z} = \omega^2 \tag{8}$$

where

$$\alpha = \frac{\mu_{xx}}{\varepsilon_Y\mu_X\mu_Z}, \quad \beta = \frac{\mu_{zz}}{\varepsilon_Y\mu_X\mu_Z}, \quad \gamma = \frac{\mu_{xz} + \mu_{zx}}{\varepsilon_Y\mu_X\mu_Z} \tag{9}$$

The time-averaged Poynting vectors of the refracted waves can be written as

$$\vec{S}_{2t} = \mathrm{Re}\left[\frac{k_x\mu_{xx} + k_{2z}\mu_{zx}}{2\omega\mu_X\mu_Z}\left|TE_1^+\right|^2\hat{x} + \frac{k_x\mu_{xz} + k_{2z}\mu_{zz}}{2\omega\mu_X\mu_Z}\left|TE_1^+\right|^2\hat{z}\right] \tag{10}$$

Then, the product $\vec{k}_2 \cdot \vec{S}_{2t}$ reduces to

$$\vec{k}_2 \cdot \vec{S}_{2t} = \mathrm{Re}\left[\frac{\omega\varepsilon_Y}{2}\left|TE_1^+\right|^2\right] \tag{11}$$

From these expressions, we can see that the condition of approximately left-handed wave propagation for TE waves is $\varepsilon_Y < 0$, and other tensor components do not need to be negative.

Here, we will discuss the conditions of negative refraction at the interface in the case of approximately left-handed wave propagation by following principles. Plus, a restricted range of incident angles $0 < \theta_i < \pi/2$ is assumed. To ensure the approximately left-handed behavior, $\varepsilon_Y < 0$ is a precondition.

(i). The occurrence of refraction requires that the z component of the refracted wave vectors must be real. It follows from Eq. (8) that

$$k_{2z} = \frac{-\gamma k_x + \sigma\sqrt{A}}{2\beta} \tag{12}$$

where $\sigma = \pm 1$. That is, the refraction will occur if the following inequality is satisfied:

$$A = \frac{-4\omega^2}{\varepsilon_Y \mu_X \mu_Z}\left(\frac{\mu_1\varepsilon_1 \sin^2\theta_i}{\varepsilon_Y} - \mu_{zz}\right) > 0 \tag{13}$$

When $A \leq 0$, k_{2z} is complex. The normal component of the energy current density of the refracted waves is equal to zero and the total reflection phenomenon will occur. The critical angle can be obtained from $A = 0$, which gives

$$\theta_c = \sin^{-1}\sqrt{\frac{\varepsilon_Y \mu_{zz}}{\varepsilon_1 \mu_1}} \tag{14}$$

where $\mu_{zz} < 0$. If $\varepsilon_Y \mu_{zz} > \varepsilon_1 \mu_1$, the critical angle θ_c will be equal to $\pi/2$. The condition of the total reflection occurrence at such interface is quite different from that at the general anisotropic metamaterial interface along a principal axis, which requires k_{2z} must be imaginary [15].

(ii). Energy conservation defines one of the two possible solutions for the refracted wave vector k_{2z}. When $\varepsilon_Y < 0$, we take $\sigma = -1$.

(iii). The negative refraction occurs when the Poynting vectors of the incident and refracted waves appear on the same side of the normal, which requires $\vec{k}_x \cdot \vec{S}_{2t} < 0$. According to Eqs. (10) and (12), the last condition is obtained as

$$\vec{k}_x \cdot \vec{S}_{2t} = \mathrm{Re}\left[\frac{k_x(2k_x - \sqrt{A}\mu_{zx}\varepsilon_Y)}{4\omega\beta\varepsilon_Y\mu_X\mu_Z}\left|TE_1^+\right|^2\right] < 0 \tag{15}$$

3. ANALYSIS OF NEGATIVE REFRACTION IN FOUR CLASSES

In terms of the signs of the principal components of the permittivity and permeability tensors, the existence conditions of negative refraction for TE waves are discussed under the condition for approximately left-handed wave propagation. For simplicity, we define $t_1 = \varepsilon_Y \mu_Z$, $t_2 = \varepsilon_Y \mu_X$ in later analysis and classify them into the following four classes. The obtained results are summarized in Table 1. Moreover, the dispersion of anisotropic metematerial is taken into account in the numerical example. The tensor components along principal axes in Eq. (1) can be expressed by the cold plasma media model as follows [5, 12]:

$$\varepsilon(\omega) = \varepsilon_0 \left(1 - \frac{\omega_{ep}^2}{\omega^2}\right), \quad \mu(\omega) = \mu_0 \left(1 - \frac{\omega_{mp}^2}{\omega^2}\right) \tag{16}$$

where ω_{ep} and ω_{mp} are electric and magnetic plasma frequencies. The required principal components can be realized by changing ω_{ep}, ω_{mp} and the operating frequency ω_0.

3.1. $t_1 > 0$, $t_2 > 0$ ($\varepsilon_Y < 0$, $\mu_X < 0$, $\mu_Z < 0$)

From Eq. (13), we can find that when the incident angle $\theta_i < \theta_c$, the wave vector k_{2z} becomes real and the refraction can occur. From Eq. (15), we know that the conditions of negative refraction are dependent on the sign of μ_{zx}. Thus, we shall consider the problem in two cases. (i) $\mu_X < \mu_Z$. The inequality (15) is satisfied for any incident angles, the negative refraction can thus occur for any refracted waves. (ii) $\mu_X > \mu_Z$. From the inequality (15), we can see that if the following condition is satisfied:

$$\theta_i > \theta_a = \sin^{-1}\left(|\mu_{xz}| \sqrt{\frac{\varepsilon_Y}{\varepsilon_1 \mu_1 \mu_{xx}}}\right) \tag{17}$$

the negative refraction will occur. The angle θ_a is the particular incident angle when the refraction angle of Poynting vector is equal to zero, and it can be solved from $\vec{k}_x \cdot \vec{S}_{2t} = 0$. If $\mu_1 \varepsilon_1 < \mu_{zx}^2 \varepsilon_Y / \mu_{xx}$, θ_a will be equal to $\pi/2$. The negative refraction phenomenon thus can't exist for any incident waves. It should be noted that the inequality $\theta_c > \theta_a$ is always valid. From these restrictions mentioned above, the conditions of the negative refraction occurrence in this section, with special cases of $\theta_c = \pi/2$ and $\theta_a = \pi/2$ included, are listed in Table 1.

Wave vector surface can be used to describe the refractive characteristics at the interface between two media efficiently. In this

Table 1. The existence conditions of negative refraction for $\varepsilon_Y < 0$.

	Media conditions	Incident angle conditions
$\varepsilon_Y < 0,\ \mu_X < 0,\ \mu_Z < 0$	$\mu_X < \mu_Z$	$\lvert\theta_i\rvert < \theta_c$
	$\mu_X > \mu_Z$	$\theta_a < \lvert\theta_i\rvert < \theta_c$
$\varepsilon_Y < 0,\ \mu_X < 0,\ \mu_Z > 0$	$\tan^2\theta < -\mu_Z/\mu_X$	Non-existence
	$\tan^2\theta > -\mu_Z/\mu_X$	$\lvert\theta_i\rvert > \theta_c$
$\varepsilon_Y < 0,\ \mu_X > 0,\ \mu_Z < 0$	$\tan^2\theta < \min(-\mu_Z/\mu_X, -\mu_X/\mu_Z)$	$\lvert\theta_i\rvert > \theta_c$
	$-\mu_X/\mu_Z < \tan^2\theta < -\mu_Z/\mu_X$	$\theta_c < \lvert\theta_i\rvert < \theta_a$
	$-\mu_Z/\mu_X < \tan^2\theta < -\mu_X/\mu_Z$	All incident angles
	$\tan^2\theta > \max(-\mu_Z/\mu_X, -\mu_X/\mu_Z)$	$\lvert\theta_i\rvert < \theta_a$
$\varepsilon_Y < 0,\ \mu_X > 0,\ \mu_Z > 0$	Non-existence	Non-existence

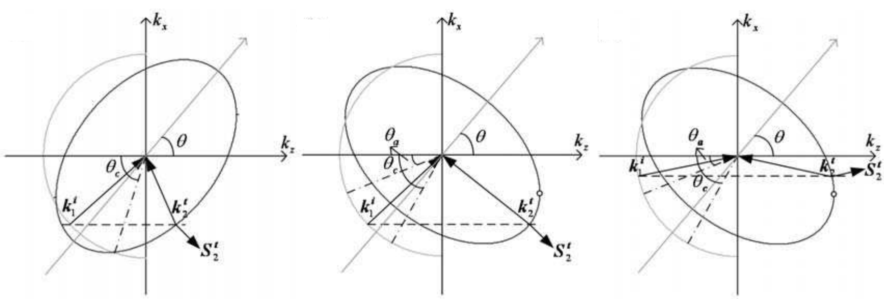

Figure 2. Diagram for wave-vector surfaces corresponding to the isotropic media and anisotropic metamaterial with $\varepsilon_Y < 0$, $\mu_X < 0$, $\mu_Z < 0$. The incident, refracted wave vectors and energy current of the refracted waves are indicated by the solid arrow. The angles θ_c and θ_a are indicated by the dashed dotted line. The refracted waves with the refraction angle of Poynting vector zero are determined by the small circle on the ellipse. (a) $\mu_X < \mu_Z$, (b) $\mu_X > \mu_Z$, with the wave incident in the range of $\theta_a < \theta_i < \theta_c$, (c) $\mu_X > \mu_Z$, with the wave incident in the range of $\theta_i < \theta_a$.

section, the dispersion curve for anisotropic media is an ellipse. When $\mu_X < \mu_Z$, the long axis of the ellipse is the Z axis. We take the wave incident in the range of $\theta_i < \theta_c$. Then the wave vector and Poynting vector are negatively refracted, as shown in Fig. 2(a). When $\mu_X > \mu_Z$, the long axis of the ellipse becomes the X axis, as we can see from Fig. 2(b). The incident angle in the range of $\theta_a < \theta_i < \theta_c$

is employed, and the refraction angles of wave vector and Poynting vector are always negative. Note that because of the existence of the angle θ_a, the refraction of Poynting vector may be positive or negative depending on the range of incident angles. However, there is no such amphoteric refraction phenomenon at regular interfaces. We can further give physical insight into the appearance of amphoteric refraction from the wave vector surface. At the noted point shown in Fig. 2(b), $dk_z/dk_x = 0$ is satisfied. By solving the equation we find that, when the refracted wave vector and the ellipse intersect at this point, the corresponding incident angle is just the angle θ_a in Eq. (17). In Fig. 2(c), the incident angle in the range of $\theta_i < \theta_a$ is employed, and it yields that the refraction of Poynting vector becomes positive in this case.

Next, we will focus our attention on the effect of the rotation angle on this phenomenon. In order to observe the incident angle region of negative refraction occurrence for $\mu_X > \mu_Z$, we choose $\omega_{epY} = \omega_{mpX} = \sqrt{2}\omega_0$ and $\omega_{mpZ} = \sqrt{7}\omega_0$ in Eq. (16). Then the component parameters $\varepsilon_Y = -\varepsilon_0$, $\mu_X = -\mu_0$ and $\mu_Z = -6\mu_0$ can be obtained. The graphs of θ_c and θ_a versus θ are shown in Fig. 3(a). It is seen that the incident angle region (shade part) becomes narrow initially, and then becomes wide with the further increasing rotation angle θ in this case. In our numerical example, by simple derivations we know that when $\theta > \theta_1 = 63.4°$, where $\theta_1 = \sin^{-1}\sqrt{(\varepsilon_1\mu_1 - \varepsilon_Y\mu_Z)/[\varepsilon_Y(\mu_X - \mu_Z)]}$, the total reflection occurs. Moreover, the phenomenon of negative refraction vanishes when $\mu_1\varepsilon_1 < \mu_{zx}^2\varepsilon_Y/\mu_{xx}$, so the corresponding rotation angle range is $26.6° < \theta < 39.2°$. Thus, the ampoteric refraction that the refraction can be either positive or negative depending on the incident angles can only occur for the rotation angle chosen properly. The shaded area can be expanded by changing the material parameters. To obtain the maximum of the area of region S_1, the optimal value $\mu_X = \mu_Z$ and $\varepsilon_Y\mu_X > \varepsilon_1\mu_1$ should be taken.

3.2. $t_1 < 0$, $t_2 > 0$ ($\varepsilon_Y < 0$, $\mu_X < 0$, $\mu_Z > 0$)

The sign of μ_{zz} will have effect both on the occurrence of refraction and the conditions of negative refraction, so we further discuss them in two cases. (i) $\mu_{zz} > 0$. In such a case the critical angle does not exist. Electromagnetic waves can always be transmitted into the second medium. From Eq. (15), we know that the positive refraction can occur for any incident waves. (ii) $\mu_{zz} < 0$. According to Eq. (13), the occurrence of refraction requires $\theta_i > \theta_c$. If $\theta_i < \theta_c$, the incident waves will be totally reflected. This phenomenon is called anomalous total reflection. If $\varepsilon_Y\mu_{zz} > \varepsilon_1\mu_1$, θ_c will be equal to $\pi/2$. Thus, k_{2z} will be complex and the total reflection can exist for any incident waves. This is the so-called omnidirectional total reflection [23]. Besides, the inequality (15) is always satisfied and the negative refraction will occur for any refracted waves.

In this section, the dispersion curve is a hyperbola with its real axis Z. When $\tan^2\theta < -\mu_Z/\mu_X$, we take the wave incident at an arbitrary angle to the interface. Fig. 4(a) shows that the wave vector is negatively refracted, while the Poynting vector is always positively refracted. Fig. 4(b) illustrates the dispersion curves for the two media in the case of $\tan^2\theta > -\mu_Z/\mu_X$. The incident angle in the range of $\theta_i > \theta_c$ is employed, and the refraction of Poynting vector is negative even if the wave vector refraction is positive. Note

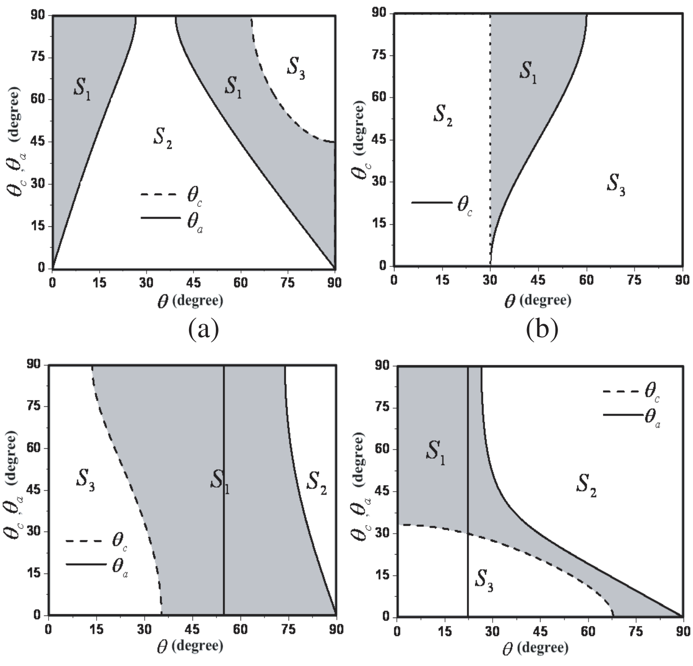

Figure 3. θ_c and θ_a as a function of the rotation angle with $\varepsilon_1 = \varepsilon_0$, $\mu_1 = 2\mu_0$. (a) $\omega_{epY} = \sqrt{2}\omega_0$, $\omega_{mpX} = \sqrt{2}\omega_0$, $\omega_{mpZ} = \sqrt{7}\omega_0$; (b) $\omega_{epY} = \sqrt{3}\omega_0$, $\omega_{mpX} = \sqrt{2.5}\omega_0$, $\omega_{mpZ} = \sqrt{0.5}\omega_0$; (c) $\omega_{epY} = \sqrt{7}\omega_0$, $\omega_{mpX} = \sqrt{0.2}\omega_0$, $\omega_{mpZ} = \sqrt{1.4}\omega_0$; (d) $\omega_{epY} = \sqrt{2}\omega_0$, $\omega_{mpX} = \sqrt{0.9}\omega_0$, $\omega_{mpZ} = \sqrt{1.6}\omega_0$. The negative refraction, positive refraction and total reflection can exist respectively in incident angle regions S_1, S_2 and S_3 for different rotation angles.

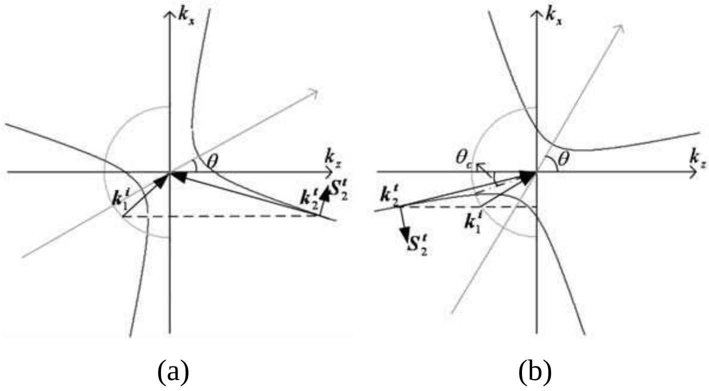

Figure 4. Diagram for wave-vector surfaces corresponding to the isotropic media and anisotropic metamaterial with $\varepsilon_Y < 0$, $\mu_X < 0$, $\mu_Z > 0$. The incident, refracted wave vectors and energy current of the refracted waves are indicated by the solid arrow. The angle θ_c is indicated by the dashed dotted line. (a) $\tan^2\theta < -\mu_Z/\mu_X$, (b) $\tan^2\theta > -\mu_Z/\mu_X$.

that the wave vector k_{2z} chosen in this case is the lager one since $\beta < 0$. From the analysis above, it is concluded that the conditions of negative refraction are strongly dependent on the rotation angle θ, which can further be explicitly seen in Fig. 3(b). As the rotation angle increases, the incident angle region of shade part becomes narrow. When $\theta > \theta_2 = 30°$, where $\theta_2 = \tan^{-1}\sqrt{|\mu_Z/\mu_X|}$, it is interesting to observe the existence of anomalous total reflection phenomenon, which indicates the qualitative characteristics of reflection are altered with the rotation angle changing. When $\theta > \theta_1 = 60°$, the omnidirectional total reflection occurs. Thus, the smaller the anisotropy parameters $|\varepsilon_Y|$ and μ_Z, the larger the shaded area, and the maximum value is limited by the artificial metamaterial constructed in practice.

3.3. $t_1 > 0$, $t_2 < 0$ ($\varepsilon_Y < 0$, $\mu_X > 0$, $\mu_Z < 0$)

The occurrence of refraction depends on the sign of μ_{zz}, and the conditions of negative refraction are determined by the sign μ_{xx}. For $\mu_{zz} < 0$, we can find that when $\theta_i > \theta_c$, the refraction occurs. Otherwise, the critical angle does not exist and the refraction can thus occur for any incident angles. Then we discuss the existence conditions of negative refraction in two cases. (i) $\mu_{xx} > 0$. The negative refraction can exist for any refracted waves because the inequality (15) is always satisfied. (ii) $\mu_{xx} < 0$. It is found from Eq. (15) that the negative

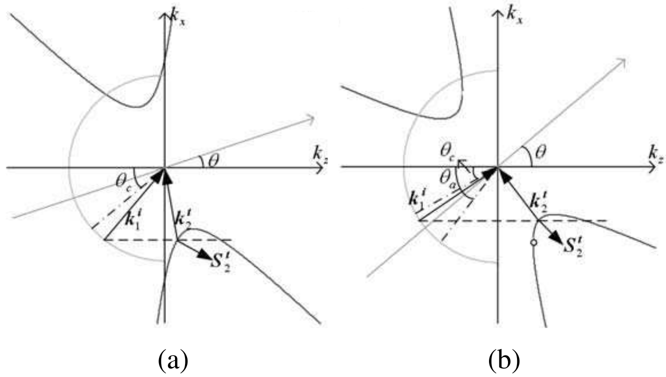

Figure 5. Diagram for wave vector surfaces corresponding to the isotropic media and anisotropic metamaterial with $\varepsilon_Y < 0$, $\mu_X > 0$, $\mu_Z < 0$. The incident, refracted wave vectors and energy current of the refracted waves are indicated by the solid arrow. The angles θ_c and θ_a are indicated by the dashed dotted line. (a) $\tan^2\theta < \min(-\mu_Z/\mu_X, -\mu_X/\mu_Z)$, (b) $-\mu_X/\mu_Z < \tan^2\theta < -\mu_Z/\mu_X$.

refraction only occurs for the branch $\theta_i < \theta_a$. The inequality $\theta_c < \theta_a$ is always true in this case and the obtained results are listed in Table 1.

The dispersion curve is a hyperbola with its real axis X in this section. When $\tan^2\theta < \min(-\mu_Z/\mu_X, -\mu_X/\mu_Z)$, we take the wave incident in the range of $\theta_i > \theta_c$, and the refraction angles of wave vector and Poynting vector are always negative, as shown in Fig. 5(a). When $-\mu_X/\mu_Z < \tan^2\theta < -\mu_Z/\mu_X$, the incident angle in the range of $\theta_c < \theta_i < \theta_a$ is employed, and the Poynting vector is negatively refracted, as we can see from Fig. 5(b). If the wave is incident in the range of $\theta_i > \theta_a$, the refraction of Poynting vector will become positive. This phenomenon of amphoteric refraction has also been described in Section 3.1. The wave vector diagram in the case of $\mu_{zz} > 0$ is omitted here for the sake of brevity. Considering the cases of $|\mu_X| > |\mu_Z|$ and $|\mu_X| < |\mu_Z|$, we find it necessary to discuss the influence of the rotation angle on negative refraction properties in two cases, as shown in Figs. 3(c) and 3(d). In the first case, the incident angle region S_1 is broadened first, goes through a maximum width and then becomes small. When $\theta < \theta_2 = 35.3°$, the phenomenon of anomalous total reflection occurs. As the rotation angle increases further, the qualitative characteristics of reflection are altered. When $\theta > \theta_3 = 54.7°$, where $\theta_3 = \tan^{-1}\sqrt{|\mu_X/\mu_Z|}$, the ampoteric refraction may occur for the rotation angle chosen large enough. Thus, the smaller $|\mu_Z|$ and $1/|\mu_X|$, the larger the area of region S_1. Similar to the analysis in the corresponding Section 3.2, the maximum value

Table 2. The existence conditions of negative refraction for $\varepsilon_Y > 0$.

	Media conditions	Incident angle conditions
$\varepsilon_Y > 0,\ \mu_X > 0,\ \mu_Z > 0$	$\mu_X < \mu_Z$	Non-existence
	$\mu_X > \mu_Z$	$\theta_i < \theta_a$
$\varepsilon_Y > 0,\ \mu_X > 0,\ \mu_Z < 0$	$\tan^2\theta < \min(-\mu_Z/\mu_X, -\mu_X/\mu_Z)$	$\theta_i > \theta_a$
	$-\mu_X/\mu_Z < \tan^2\theta < -\mu_Z/\mu_X$	Non-existence
	$-\mu_Z/\mu_X < \tan^2\theta < -\mu_X/\mu_Z$	$\theta_i > \theta_a$
	$\tan^2\theta > \max(-\mu_Z/\mu_X, -\mu_X/\mu_Z)$	Non-existence
$\varepsilon_Y > 0,\ \mu_X < 0,\ \mu_Z > 0$	$\tan^2\theta < -\mu_Z/\mu_X$	Non-existence
	$\tan^2\theta > -\mu_Z/\mu_X$	All incident angles
$\varepsilon_Y > 0,\ \mu_X < 0,\ \mu_Z < 0$	Non-existence	Non-existence

is limited by the material parameters in practice. In the other case, with the increase in θ the incident angle region also becomes wide first, and then becomes narrow for smaller rotation angle because of the occurrence of θ_a.

3.4. $t_1 < 0$, $t_2 < 0$ ($\varepsilon_Y < 0$, $\mu_X > 0$, $\mu_Z > 0$)

The inequality (13) can't be satisfied for any incident angles and the refraction can never occur in such a case.

It is worth pointing out that when $-\pi/2 < \theta_i < 0$, the existence conditions of negative refraction for the four classes mentioned can be obtained from the results shown in Table 1 by substitution of Sections 3.2 $\leftrightarrow$ 3.3, the signs "$>$" $\leftrightarrow$ "$<$" and "min" $\leftrightarrow$ "max" in the expressions of media conditions.

When $\varepsilon_Y > 0$, the existence conditions of negative refraction can be analyzed in a similar way, and they are summarized in Table 2. It is found that, by substitution of Sections 3.2 $\leftrightarrow$ 3.3, the signs "$>$" $\leftrightarrow$ "$<$" and "min" $\leftrightarrow$ "max" in the expressions of media conditions in Sections 3.2 and 3.3, the incident angle conditions in Table 1 are just the conditions of positive refraction occurrence for $\varepsilon_Y > 0$.

4. CONCLUSION

In this paper, we present an investigation on the negative refraction properties of electromagnetic waves at the interface containing arbitrarily oriented anisotropic metamaterials. We analyze the effect of the rotation angle on the negative refractive incident angle region under the condition for approximately left-handed wave propagation in different cases. In contrast to an isotropic medium and a general anisotropic metamaterial, we obtain some peculiar properties of this medium. (i) The existence conditions of negative refraction are determined not only by the physical parameters of the anisotropic medium but also by the rotation angle. (ii) The existence of total reflection requires the normal component of the refracted wave vectors must be complex, and the qualitative characteristics of reflection are altered by the rotation of the principal axes under the hyperbolic dispersion. (iii) Under suitable media conditions, the amphoteric refraction may occur, which has also been found in this kind of anisotropic medium with elliptical dispersion relation [26].

The investigation may be helpful for understanding the complicating phenomenon at the anisotropic metamaterial interface along a nonprincipal axis. It should be pointed out further that when the rotation angle is equal to 0 or $\pi/2$, arbitrary oriented anisotropic media can be reduced to general anisotropic media as presented in [14, 15], which makes the conclusions in this paper more general and practical.

ACKNOWLEDGMENT

This work is supported by the Fundamental Research Funds for the Central Universities.

REFERENCES

1. Veselago, V. G., "The electrodynamics of substances with simultaneously negative values of ε and μ," *Soviet Physics USPEKHI*, Vol. 10, No. 4, 509–514, 1968.
2. Duan, Z. Y., B.-I. Wu, S. Xi, H. S. Chen, and M. Chen, "Research progress in reversed cherenkov radiation in double-negative metamaterials," *Progress In Electromagnetics Research*, Vol. 90, 75–87, 2009.
3. Kuo, C. W., S. Y. Chen, Y. D. Wu, and M. H. Chen, "Analyzing the multilayer optical planar waveguides with double-negative metamaterial," *Progress In Electromagnetics Research*, Vol. 110, 163–178, 2010.
4. Shelby, R. A., D. R. Smith, and S. Schultz, "Experimental verification of a negative index of refraction," *Science*, Vol. 292, 77–79, 2001.
5. Pendry, J. B., "Negative refraction makes a perfect lens," *Phys. Rev. Lett.*, Vol. 85, No. 18, 3966–3969, 2000.

6. Fang, N., H. S. Lee, C. Sun, and X. Zhang, "Sub-diffraction-limited optical imaging with a silver superlens," *Science*, Vol. 308, 534–537, 2005.
7. Bai, Q., J. Chen, N. H. Shen, C. Cheng, and H. T. Wang, "Controllable optical black hole in left-handed materials," *Optics Express*, Vol. 18, No. 3, 2106–2115, 2010.
8. Huang, Y. and L. Gao, "Effective negative refraction in anisotropic layered composites," *Journal of Applied Physics*, Vol. 105, 013532, 2009.
9. Mirza, I. O., J. N. Sabas, S. Shi, and D. W. Prather, "Experimental demonstration of metamaterial-based phase modulation," *Progress In Electromagnetics Research*, Vol. 93, 1–12, 2009.
10. Sabah, C. and S. Uckun, "Multilayer system of Lorentz/Drude type metamaterials with dielectric slabs and its application to electromagnetic filters," *Progress In Electromagnetics Research*, Vol. 91, 349–364, 2009.
11. Khalilpour, J. and M. Hakkak, "S-shaped ring resonator as anisotropic uniaxial metamaterial used in waveguide tunneling," *Journal of Electromagnetic Waves and Applications*, Vol. 23, No. 13, 1763–1772, 2009.
12. Liu, S. H., C. H. Liang, W. Ding, L. Chen, and W. T. Pan, "Electromagnetic wave propagation through a slab waveguide of uniaxially anisotropic dispersive metamaterial," *Progress In Electromagnetics Research*, Vol. 76, 467–475, 2007.
13. Woodley, J. and M. Mojahedi, "Backward wave propagation in left-handed media with isotropic and anisotropic permittivity tensors," *J. Opt. Soc. Am. B*, Vol. 23, No. 11, 2377–2382, 2006.
14. Lindell, I. V., S. A. Tretyakov, K. I. Nikoskinen, and S. Ilvonen, "BW media-media with negative parameters, capable of supporting backward waves," *Microwave and Optical Technology Letters*, Vol. 31, No. 2, 129–133, 2001.
15. Hu, L. and S. T. Chui, "Characteristics of electromagnetic wave propagation in uniaxially anisotropic left-handed materials," *Phys. Rev. B*, Vol. 66, 085108, 2002.
16. Ding, W., L. Chen, and C. H. Liang, "Characteristics of electromagnetic wave propagation in biaxially anisotropic left-handed materials," *Progress In Electromagnetics Research*, Vol. 70, 37–52, 2007.
17. Parazzoli, C. G., R. B. Greegor, K. Li, B. E. C. Koltenbah, and M. Tanielian, "Experimental verification and simulation of negative index of refraction using Snell's law," *Phys. Rev. Lett.*,

Vol. 90, No. 10, 107401, 2003.

18. Smith, D. R., P. Kolinko, and D. Schurig, "Negative refraction in indefinite media," *J. Opt. Soc. Am. B*, Vol. 21, No. 5, 1032–1043, 2004.
19. Luo, H., Z. Ren, W. Shu, and F. Li, "Construct a polarizing beam splitter by an anisotropic metamaterial slab," *Applied Physics B*, Vol. 87, No. 2, 283–287, 2007.
20. Degiron, A., D. R. Smith, J. J. Mock, B. J. Justice, and J. Gollub, "Negative index and indefinite media waveguide couplers," *Applied Physics A*, Vol. 87, No. 2, 321–328, 2007.
21. Liu, Y., G. Bartal, and X. Zhang, "All-angle negative refraction and imaging in a bulk medium made of metallic nanowires in the visible region," *Optics Express*, Vol. 16, No. 20, 15439–15448, 2008.
22. Smith, D. R. and D. Schurig, "Electromagnetic wave propagation in media with indefinite permittivity and permeability tensors," *Phys. Rev. Lett.*, Vol. 90, No. 7, 077405, 2003.
23. Xiang, Y., X. Dai, and S. Wen, "Total reflection of electromagnetic waves propagating from an isotropic medium to an indefinite metamaterial," *Optics Communications*, Vol. 274, 248–253, 2007.
24. Xu, G., L. Su, T. Pan, and T. Zang, "Group delay in indefinite media," *Physica B*, Vol. 403, 3417–3423, 2008.
25. Wang, Z. P., C. Wang, and Z. H. Zhang, "Goos-Hänchen shift of the uniaxially anisotropic left-handed material film with an arbitrary angle between the optical axis and the interface," *Optics Communications*, Vol. 281, No. 11, 3019–3024, 2008.
26. Luo, H., W. Hu, X. Yi, H. Liu, and J. Zhu, "Amphoteric refraction at the interface between isotropic and anisotropic media," *Optics Communications*, Vol. 254, 353–360, 2005.