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FORWARD AND BACKWARD WAVES IN HIGH-FREQUENCY DIFFRACTION BY AN ELONGATED SPHEROID

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Abstract—The asymptotics of induced current of forward and backward waves on a strongly elongated spheroid is constructed by matching the asymptotic representations to the exact solution valid in the vicinity of the rear tip of the spheroid. These asymptotic results are compared with numerical computations.

1. INTRODUCTION

Diffraction of high-frequency electromagnetic waves by smooth convex bodies generates creeping rays in the shadow zone of the body. These creeping rays contributions to the field diffracted by the body can be computed in UTD format [1], but provide a valid description of the field only in the shadow. A way of including both shadow and light shadow transition zone is given by Fock asymptotics [2]. This asymptotics is written as decomposition in inverse powers of the parameter

$$m = \left(\frac{k\rho}{2}\right)^{1/3},$$

where k is the wave number of the incident electromagnetic wave, ρ is the radius of curvature of the geodesics on the surface taken at the light-shadow boundary. Fock asymptotics gives an accurate

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approximation for high frequencies. Moreover, the error remains small for diffraction by a cylinder down to $k\rho \approx 3$ (see [3], Figure 111d). For diffraction by a sphere, this low bound increases to $k\rho \approx 10$ [4]. For elongated objects it grows further and may reach several hundreds. Several approaches to diffraction by these objects has been proposed, some supposing that the body is slender [5, 6], others studying specific effects, such as diffraction of creeping waves by an edge [7]. To deal with strongly elongated objects, an asymptotic approach, in the spirit of the work of Engineer and al [8], was developed in [9, 10]. This asymptotics is derived under the supposition that the parameter

$$\chi = k\rho_t^{3/2}\rho^{-1/2},$$

where ρ_t is the transverse radius of curvature of the surface, is of order one, namely that large wavelength is compensated by small ratio ρ_t/ρ of the surface curvatures. It is based on the solution of the problem of diffraction of a plane wave by a spheroid [11, 12], that can be written in closed form, for the specific case of elongated spheroid.

Though the agreement with extensive numerical computations of the solution of [10] is satisfactory close to the light shadow separatrix, this solution only includes the forward going wave, propagating from the separatrix towards the rear tip of the spheroid, and does not address the problem of reflection of creeping waves at the rear tip of the object. This reflection generates a backward wave that interferes with the forward going wave.

It is known that in a problem of diffraction by a cylindrical surface the field in a vicinity of light shadow separatrix contains contributions of creeping wave that envelop the shadowed side of the cylindrical surface. Though theses waves exponentially attenuate, at not so high frequencies their contribution may be significant (see, e.g., [13]). On a strongly elongated object, the attenuation of the creeping waves is smaller than on a cylinder and the contribution of backward wave is more noticeable. As the result of forward and backward waves interference, oscillations, which are larger in the vicinity of the rear tip, appear in the amplitude of the current.

In this paper, we present a complete solution of the problem by adding to the solution of [10] the asymptotics of backward going wave.

In Section 2 we present the solution derived in [10] and suggest a general form of the backward wave asymptotics in the form of an integral of Whittaker functions with unknown amplitude. The determination of this amplitude requires the solution of the canonical problem of the paraboloid, and we use the results of Fock [14]. We show in part 3, that waves on this object appear as combination of a wave propagating towards the tip and a wave reflected by a tip, and compute the reflection coefficient. We match in part 4 this solution to the forward and backward wave on the spheroid, and, as a result, obtain a complete and explicit solution. In Section 5, we use this solution to compute the current on the surface. We check the validity of our approach by comparing the results with numerical computations in Section 6.

The relations between Whittaker functions useful to the derivations are provided in appendices.

2. FORWARD AND BACKWARD WAVES

2.1. Previous Results

The asymptotics of [10] is derived under the assumptions that the body is strongly elongated, that is the parameter χ introduced in the introduction is of order one, that the body has rotational symmetry and the incident wave runs along the axis of the body. The surface is approximated by the surface of an appropriately chosen spheroid (See geometry of the problem on Figure 1). Namely the semi-axes are

$$b = \sqrt{\rho \rho_t}, \quad a = \rho_t,$$

where the radii ρ and ρ_t are taken at the light-shadow boundary. The solution uses angular spheroidal [15] coordinate η, while the radial spheroidal coordinate ξ is replaced with a stretched coordinate ν according to the formula

$$\xi = 1 - \frac{\nu}{4m^2}.$$

The derivations of [10] result in the following asymptotics representations of electric and magnetic fields:

$$\begin{aligned} E_\varphi &= e^{ikb\eta} \frac{e^{-i\pi/4}}{\pi} \frac{\cos\varphi}{\sqrt{\alpha\nu}\sqrt{1-\eta^2}} \int_{-i\infty}^{+i\infty} \Gamma\left(\frac{1}{2}-\mu\right)\Gamma\left(\frac{1}{2}+\mu\right) \\ &\quad \times \left(\frac{1-\eta}{1+\eta}\right)^{\mu} \Big(M_{\mu,0}(-i\alpha\nu) + \omega_0(\mu) W_{\mu,0}(-i\alpha\nu) \\ &\quad + \omega_1(\mu) W_{\mu,1}(-i\alpha\nu)\Big) d\mu, \end{aligned} \tag{1}$$

$$\begin{aligned} H_\varphi &= e^{ikb\eta} \frac{e^{-i\pi/4}}{\pi} \frac{\sin\varphi}{\sqrt{\alpha\nu}\sqrt{1-\eta^2}} \int_{-i\infty}^{+i\infty} \Gamma\left(\frac{1}{2}-\mu\right)\Gamma\left(\frac{1}{2}+\mu\right) \\ &\quad \times \left(\frac{1-\eta}{1+\eta}\right)^{\mu} \Big(M_{\mu,0}(-i\alpha\nu) + \omega_0(\mu) W_{\mu,0}(-i\alpha\nu) \\ &\quad - \omega_1(\mu) W_{\mu,1}(-i\alpha\nu)\Big) d\mu. \end{aligned} \tag{2}$$

Here $\alpha = (\chi/2)^{1/3}$ is a parameter, $M_{\mu,0}$, $W_{\mu,0}$ and $W_{\mu,1}$ are Whittaker functions [16] and the amplitudes ω_0 and ω_1 should be chosen such that the boundary conditions on a perfect conductor

$$E_\varphi|_{\nu=2\alpha^2} = 0, \quad E_\eta|_{\nu=2\alpha^2} = 0$$

are satisfied. The latter condition reduces to

$$4\alpha^2 \frac{\partial H_\varphi}{\partial \nu}\bigg|_{\nu=2\alpha^2} + H_\varphi|_{\nu=2\alpha^2} = 0$$

and we get the following system of equations for the amplitudes ω_0 and ω_1

$$\begin{cases} \omega_0 W_{\mu,0}(-i\chi) + \omega_1 W_{\mu,1}(-i\chi) = -M_{\mu,0}(-i\chi), \\ \omega_0 \dot{W}_{\mu,0}(-i\chi) - \omega_1 \dot{W}_{\mu,1}(-i\chi) = -\dot{M}_{\mu,0}(-i\chi). \end{cases} \tag{3}$$

Here and below dot denotes derivative of a function. The solution is given by the formulae

$$\omega_0 = -\frac{M_{\mu,0}(-i\chi)\dot{W}_{\mu,1}(-i\chi) + \dot{M}_{\mu,0}(-i\chi)W_{\mu,1}(-i\chi)}{W_{\mu,0}(-i\chi)\dot{W}_{\mu,1}(-i\chi) + \dot{W}_{\mu,0}(-i\chi)W_{\mu,1}(-i\chi)}, \tag{4}$$

$$\omega_1 = \frac{1}{\Gamma(1/2-\mu)} \frac{1}{W_{\mu,0}(-i\chi)\dot{W}_{\mu,1}(-i\chi) + \dot{W}_{\mu,0}(-i\chi)W_{\mu,1}(-i\chi)}. \tag{5}$$

The terms containing Whittaker function $M_{\mu,0}$ in formulae (1) and (2) represent the incident plane wave of unit amplitude.

The results obtained in [10] and presented in this section provide a good approximation of the mean current on the spheroid. However, as can be seen on Figures 2–5 below, oscillations in the amplitude of the current appear on the numerical solution. These oscillations are generated by the interference of the forward going wave, with a wave, that we shall call backward, generated by the partial reflection of this forward going wave by the tip of the spheroid. These oscillations are, as expected, stronger close to the tip, where the amplitude of this backward wave is maximal, and nearly disappear close to the light-shadow boundary, because of the attenuation of the wave. In the next section, we derive the asymptotic for the backward wave.

2.2. Asymptotic Formula for Backward Wave

The asymptotics of backward going wave can be constructed by the same method as the asymptotics of forward going wave. However, we can guess it by simple manipulations with formulae (1) and (2). Firstly, we remove the incident wave, that is exclude Whittaker function $M_{\mu,0}$. Secondly, we change the direction of wave propagation, that is replace η by $-\eta$ and change sign of H_φ component. Finally we replace the amplitudes ω_0 and ω_1 by some other amplitudes ω_0^b and ω_1^b. This results in the following formulae for the backward wave

$$E_\varphi^b = e^{-ikb\eta}\frac{e^{-i\pi/4}}{\pi}\frac{\cos\varphi}{\sqrt{\alpha\nu}\sqrt{1-\eta^2}}\int_{-i\infty}^{+i\infty}\Gamma\left(\frac{1}{2}-\mu\right)\Gamma\left(\frac{1}{2}+\mu\right)$$
$$\times\left(\frac{1+\eta}{1-\eta}\right)^\mu\left\{\omega_0^b(\mu)W_{\mu,0}(-i\alpha\nu)+\omega_1^b(\mu)W_{\mu,1}(-i\alpha\nu)\right\}d\mu, \quad (6)$$

$$H_\varphi^b = e^{-ikb\eta}\frac{e^{-i\pi/4}}{\pi}\frac{\sin\varphi}{\sqrt{\alpha\nu}\sqrt{1-\eta^2}}\int_{-i\infty}^{+i\infty}\Gamma\left(\frac{1}{2}-\mu\right)\Gamma\left(\frac{1}{2}+\mu\right)$$
$$\times\left(\frac{1+\eta}{1-\eta}\right)^\mu\left\{-\omega_0^b(\mu)W_{\mu,0}(-i\alpha\nu)+\omega_1^b(\mu)W_{\mu,1}(-i\alpha\nu)\right\}d\mu \quad (7)$$

The amplitudes ω_0^b and ω_1^b should be chosen such that the boundary conditions on the surface are satisfied. These boundary conditions yield the system with the left-hand side as in (3), but homogeneous due to lack of incident wave

$$\begin{cases}\omega_0^b(\mu)W_{\mu,0}(-i\chi)+\omega_1^b(\mu)W_{\mu,1}(-i\chi)=0,\\ \omega_0^b(\mu)\dot{W}_{\mu,0}(-i\chi)-\omega_1^b(\mu)\dot{W}_{\mu,1}(-i\chi)=0.\end{cases}$$

In order that homogeneous system has a nonzero solution, the determinant of its matrix should be equal to zero. This defines values of the parameter $\mu=\mu_j$, $j=1,2,\ldots$. As a result, the integrals in (6), (7) reduce to sums of terms by all that solutions μ_j. Each of these terms is a creeping wave on a strongly elongated spheroid. Instead of having the representation of the field as the sum of creeping waves, we continue to use the integral form and introduce there the determinant of the system in the denominator in such a way that if the integral is computed by residue theorem, it gives back the sum of creeping waves. Finally, (6) and (7) can be rewritten as

$$E_\varphi^b = e^{-ikb\eta}\frac{e^{-i\pi/4}\cos\varphi}{\pi\sqrt{\alpha\nu}\sqrt{1-\eta^2}}\int_{-i\infty}^{+i\infty}\Gamma\left(\frac{1}{2}-\mu\right)\Gamma\left(\frac{1}{2}+\mu\right)\omega^b(\mu)$$
$$\times\left(\frac{1+\eta}{1-\eta}\right)^\mu\frac{-\frac{W_{\mu,0}(-i\alpha\nu)}{W_{\mu,0}(-i\chi)}+\frac{W_{\mu,1}(-i\alpha\nu)}{W_{\mu,1}(-i\chi)}}{W_{\mu,0}(-i\chi)\dot{W}_{\mu,1}(-i\chi)+\dot{W}_{\mu,0}(-i\chi)W_{\mu,1}(-i\chi)}d\mu, \quad (8)$$

$$H_\varphi^b = e^{-ikb\eta}\frac{e^{-i\pi/4}\sin\varphi}{\pi\sqrt{\alpha\nu}\sqrt{1-\eta^2}}\int\limits_{-i\infty}^{+i\infty}\Gamma\left(\frac{1}{2}-\mu\right)\Gamma\left(\frac{1}{2}+\mu\right)\omega^b(\mu)$$
$$\times\left(\frac{1+\eta}{1-\eta}\right)^\mu\frac{\frac{W_{\mu,0}(-i\alpha\nu)}{W_{\mu,0}(-i\chi)}+\frac{W_{\mu,1}(-i\alpha\nu)}{W_{\mu,1}(-i\chi)}}{W_{\mu,0}(-i\chi)\dot{W}_{\mu,1}(-i\chi)+\dot{W}_{\mu,0}(-i\chi)W_{\mu,1}(-i\chi)}d\mu. \quad (9)$$

Here ω^b is the amplitude of the back-going wave which should be determined by matching to the solution valid in a vicinity of the shadowed ending of the spheroid.

3. SOLUTION NEAR THE SHADOWED END OF SPHEROID

We approximate the surface of the spheroid in a vicinity of shadowed end with a surface of paraboloid and use exact solution derived by Fock [14].

3.1. Paraboloidal Coordinates and Debye Potentials

According to [14], covariant components of an electromagnetic field in the paroboloidal coordinate system (u, v, φ) where

$$z = \frac{1}{2k}(u-v), \quad r = \frac{1}{k}\sqrt{uv},$$

can be represented via Debye potentials U, V by the formulae

$$\begin{aligned} E_u &= \frac{1}{4}(u+v)U + \frac{\partial(MU)}{\partial u} - \frac{u+v}{4u}\frac{\partial V}{\partial\varphi}, \\ E_v &= \frac{1}{4}(u+v)U + \frac{\partial(MU)}{\partial v} + \frac{u+v}{4v}\frac{\partial V}{\partial\varphi}, \\ E_\varphi &= \frac{\partial(MU)}{\partial\varphi} + uv\left(\frac{\partial V}{\partial u} - \frac{\partial V}{\partial v}\right), \end{aligned} \quad (10)$$

where

$$MU = u\frac{\partial U}{\partial u} + v\frac{\partial U}{\partial v} + U.$$

The components iH_u, iH_v and iH_φ of magnetic field are expressed by the above formulae with U and V interchanged.

The field of an electromagnetic wave that runs to the extremity of the spheroid depends on the angular coordinate by means of $\sin\varphi$ or $\cos\varphi$. It produces the field which has E_u, E_v and H_φ components proportional to $\cos\varphi$ and E_φ, H_u and H_v components proportional to $\sin\varphi$. Thus we set

$$U(u,v,\varphi) = U(u,v)\cos\varphi, \quad V(u,v,\varphi) = V(u,v)\sin\varphi.$$

Functions U and V satisfy Helmholtz equation, so

$$\left(L_u - \frac{1}{4u} + L_v - \frac{1}{4v}\right) U, V = 0,$$

where

$$L_x = x\frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial x} + \frac{1}{4}.$$

Further, following [14], we introduce P and Q such that

$$U = \frac{\sqrt{uv}}{u+v}\left(\frac{\partial P}{\partial u} + \frac{\partial P}{\partial v}\right), \quad V = \frac{\sqrt{uv}}{u+v}\left(\frac{\partial Q}{\partial u} + \frac{\partial Q}{\partial v}\right).$$

Then P and Q satisfy the equations

$$(L_u + L_v)P, Q = 0$$

This equation allows variables separation

$$P = \frac{p}{\sqrt{uv}} M_{i\mu,0}(iu) W_{i\mu,0}(-iv),$$
$$Q = \frac{q}{\sqrt{uv}} M_{i\lambda,0}(iu) W_{i\lambda,0}(-iv),$$

where M and W are Whittaker functions chosen in such a way that P and Q are finite at $u = 0$ and satisfy radiation condition at $v \to +\infty$, μ and λ are parameters of variables separation and p and q are arbitrary amplitudes.

Using (10) and noting that

$$L_u P = -\mu P, \quad L_u Q = -\lambda Q$$

the components of electric field can be written via functions P and Q in the following way

$$E_u = \frac{\sqrt{uv}}{4u}\left\{-4\mu\frac{\partial P}{\partial v} - 2\frac{\partial^2 P}{\partial u \partial v} - \frac{1}{2}P - \frac{\partial Q}{\partial u} - \frac{\partial Q}{\partial v}\right\},$$
$$E_v = \frac{\sqrt{uv}}{4v}\left\{4\mu\frac{\partial P}{\partial u} - 2\frac{\partial^2 P}{\partial u \partial v} - \frac{1}{2}P + \frac{\partial Q}{\partial u} + \frac{\partial Q}{\partial v}\right\}, \quad (11)$$
$$E_\varphi = \sqrt{uv}\left\{-\frac{\partial^2 P}{\partial u \partial v} + \frac{1}{4}P - \lambda Q - \frac{1}{2}\frac{\partial Q}{\partial u} + \frac{1}{2}\frac{\partial Q}{\partial v}\right\}.$$

The components of magnetic field can be obtained by the given above rule of interchanging P and Q and division by i. In particular for H_φ we get

$$H_\varphi = -i\sqrt{uv}\left\{-\frac{\partial^2 Q}{\partial u \partial v} + \frac{1}{4}Q - \lambda P - \frac{1}{2}\frac{\partial P}{\partial u} + \frac{1}{2}\frac{\partial P}{\partial v}\right\}. \quad (12)$$

3.2. Boundary Conditions

Consider the boundary conditions. We define the surface of paraboloid by the formula

$$v = v_0 = k\rho_e,$$

where ρ_e is the curvature of the spheroid surface at the tip. The boundary conditions $E_u = 0$, $E_\varphi = 0$ on the surface of perfect conductor will be satisfied if $\lambda = \mu$ and

$$\begin{cases} 2\frac{\partial P(u,v_0)}{\partial v} + Q(u, v_0) = 0, \\ P(u, v_0) + 2\frac{\partial Q(u,v_0)}{\partial v} - 4i\mu Q(u, v_0) = 0 \end{cases}$$

This gives a homogeneous system of equations for the coefficients p and q. In order it has a nontrivial solution, such that p is arbitrary and

$$q = \left[2i\frac{\dot{W}_{\mu,0}(-iv_0)}{W_{\mu,0}(-iv_0)} + \frac{1}{v_0} \right] p, \tag{13}$$

its determinant should be equal to zero, which defines μ as a solution of the equation

$$-4\dot{W}^2_{\mu,0}(-iv_0) + 4i\left[\frac{1}{v_0} + 2i\mu\right] \dot{W}_{\mu,0}(-iv_0)W_{\mu,0}(-iv_0)$$
$$+\left[\frac{1}{v_0^2} + \frac{4i\mu}{v_0} - 1\right] W^2_{\mu,0}(-iv_0) = 0. \tag{14}$$

4. MATCHING

We shall match the asymptotic formulae (1), (2) expressing the forward going wave with the solution (11), (12). By this we can define the amplitude p which remained arbitrary in the solution valid near the tip. Then we match this solution with the asymptotic representations (6), (7) of the reflected wave and define the amplitude ω^b in the formulae for backward wave.

4.1. Relations between Coordinates

We shall use Cartesian coordinate z for matching the amplitudes of waves. Actually, there are two different z coordinates. When dealing with spheroidal coordinates, we direct z coordinate along the wave vector of the incident wave and place its origin in the center of the spheroid. When working with parabolic coordinates, we direct z in the opposite direction and shift the origin to the focal point of the paraboloid. To distinguish these coordinates we shall refer to z coordinate associated with spheroidal coordinate system as z_s and refer to z coordinate of parabolic system as z_p. The following relation between these coordinates is evident

$$z_s + z_p = const.$$

To find this constant, we consider coordinates of the shadowed far end point of the body. We have

$$z_p = -\frac{v_0}{2k} = -\frac{\rho_e}{2} = \frac{a^2}{2b}$$

and $z_s = b$, which gives

$$z_s + z_p = b - \frac{a^2}{2b}.$$

The domain of matching is such that the argument η of the asymptotic representations of forward and backward waves is close to one and that the argument u in the solution of Section 3 is asymptotically large. As the spheroid is strongly elongated we can accept, in this intermediate domain

$$\eta = \frac{z_s}{b} = 1 - \frac{a^2}{2b^2} - \frac{z_p}{b}, \quad u = 2kz_p.$$

4.2. Dispersion Equations

The electric and magnetic components in all three solutions depend on the normal coordinate in the form of a combination of Whittaker functions. The arguments of these functions can be found to be asymptotically coincident. In this section we show that the parameters μ also coincide. For that we consider the dispersion Equation (14) and using relation (see Appendix A)

$$\left(\mu - \frac{1}{2}\right) W_{\mu,1}(x) = \dot{W}_{\mu,0}(x) + \left(\mu - \frac{1}{2x}\right) W_{\mu,0}(x) \tag{15}$$

rewrite it as

$$\dot{W}_{\mu,0}(-iv_0) W_{\mu,1}(-iv_0) + W_{\mu,0}(-iv_0) \dot{W}_{\mu,1}(-iv_0) = 0, \tag{16}$$

which coincides with the form of the denominators in (1), (2), (8) and (9). Thus, both dispersion equations has the same form. To check that they coincide completely requires to see that $\chi = kv_0$ which is evident as $\rho_e = a^2/b$.

Thus, we see that μ parameters in the asymptotics and in the solution of Section 3 are the same and for matching it is sufficient to match the amplitudes at Whittaker functions $W_{\mu,0}(-i\chi)$ and $W_{\mu,1}(-i\chi)$ of the solutions.

4.3. Components of Electromagnetic Field

First, we rewrite the amplitude factor in the asymptotics (1) and (2)

$$\frac{e^{ikp\eta}}{\sqrt{1-\eta^2}}\left(\frac{1-\eta}{1+\eta}\right)\sim\frac{1}{2}\exp\left(ikb-ikz_p-i\frac{ka^2}{2b}\right)\left(\frac{z_p}{2b}\right)^{\mu-1/2}. \quad (17)$$

The amplitude factor in the asymptotics (8), (9) can be written as

$$e^{-ikpz_s}\sqrt{1-\eta^2}\left(\frac{1+\eta}{1-\eta}\right)^{\mu}\sim\frac{1}{2}\exp\left(ikz_p-ikb+i\frac{ka^2}{2b}\right)\left(\frac{2b}{z_p}\right)^{\mu+1/2}. \quad (18)$$

In the solution, valid near the tip, covariant components of the field can be represented in the following form (we use (11) and (12) and represent derivatives of Whittaker functions W by v with the help of relation (15))

$$E_\varphi = C_0(u)W_{\mu,0}(-iv)+C_1(u)W_{\mu,1}(-iv), \quad (19)$$

$$H_\varphi = -iD_0(u)W_{\mu,0}(-iv)-iD_1(u)W_{\mu,1}(-iv), \quad (20)$$

where

$$\begin{aligned}
C_0 &= -\frac{2\mu+1}{4u}\Big\{(2i\mu p+q)\left(M_{\mu+1,0}(iu)-M_{\mu,0}(iu)\right)\\
&\quad +(iq-p)uM_{\mu,0}(iu)\Big\},\\
C_1 &= \frac{2\mu-1}{4u}\Big\{(2\mu+1)ip\left(M_{\mu+1,0}(iu)-M_{\mu,0}(iu)\right)\\
&\quad -(iq+p)uM_{\mu,0}(iu)\Big\},\\
D_0 &= -\frac{2\mu+1}{4u}\Big\{(-2i\mu q+p)\left(M_{\mu+1,0}(iu)-M_{\mu,0}(iu)\right)\\
&\quad +(ip+q)uM_{\mu,0}(iu)\Big\},\\
D_1 &= -\frac{2\mu-1}{4u}\Big\{(2\mu+1)iq\left(M_{\mu+1,0}(iu)-M_{\mu,0}(iu)\right)\\
&\quad +(ip-q)uM_{\mu,0}(iu)\Big\}.
\end{aligned}$$

Here amplitudes p and q are related to each other by the formula (13) which in view of (15) transforms to

$$q=ip\left((2\mu-1)\frac{W_{\mu,1}(-iv_0)}{W_{\mu,0}(-iv_0)}-2\mu\right). \quad (21)$$

The components of physical vectors are

$$\mathcal{E}_\varphi=\frac{k}{2\sqrt{uv}}E_\varphi, \quad \mathcal{H}_\varphi=\frac{k}{2\sqrt{uv}}H_\varphi. \quad (22)$$

Now, using the asymptotics [16] of Whittaker function

$$M_{\mu,\ell}(x) \sim e^{-x/2} x^{\mu} e^{i\pi(\ell-\mu+1/2)} \frac{\Gamma(1+2\ell)}{\Gamma(\ell+\mu+1/2)} + e^{x/2} x^{-\mu} \frac{\Gamma(1+2\ell)}{\Gamma(\ell-\mu+1/2)}, \tag{23}$$

we find (see Appendix C)

$$C_0 \sim -\frac{2\mu-1}{4} e^{-ikz_p} (2kz_p)^{\mu-1/2} \frac{e^{-i\pi\mu/2}}{\Gamma(\mu+1/2)} (ip-q) - \frac{2\mu+1}{4} e^{ikz_p} (2kz_p)^{-\mu-1/2} \frac{e^{-i\pi\mu/2}}{\Gamma(1/2-\mu)} (iq-p). \tag{24}$$

We represent this asymptotics in the form

$$C_0 \sim c_0 \left(e^{-ikz_p} z_p^{\mu-1/2} + R_1 e^{ikz_p} z_p^{-\mu-1/2} \right), \tag{25}$$

with

$$c_0 = -\frac{2\mu-1}{4} (2k)^{\mu-1/2} \frac{e^{-i\pi\mu/2}}{\Gamma(\mu+1/2)} (ip-q) \tag{26}$$

and

$$R_1 = \frac{2\mu+1}{2\mu-1} (2k)^{-2\mu} \frac{\Gamma(1/2+\mu)}{\Gamma(1/2-\mu)} \frac{iq-p}{ip-q}.$$

It can be shown (see Appendix B) that for μ satisfying the dispersion Equation (16) Whittaker functions $W_{\mu,1}(-iv_0)$ and $W_{\mu,0}(-iv_0)$ satisfy a simple relation

$$W_{\mu,1}(-iv_0) = \sqrt{\frac{2\mu+1}{2\mu-1}} W_{\mu,0}(-iv_0). \tag{27}$$

This allows to simplify the expression for the reflection coefficient R_1. Substituting relation (21) and using relation (27) we get

$$R_1 = i \sqrt{\frac{2\mu+1}{2\mu-1}} \frac{\Gamma(1/2+\mu)}{\Gamma(1/2-\mu)}.$$

In a similar manner we can represent the asymptotics of C_1, D_0 and D_1. however we shall define the reflection coefficient by matching the amplitude factor at $W_{\mu,0}(-i\chi)$ and show in Appendix C that matching of other amplitudes gives the same result.

4.4. Amplitude of Backward Going Wave

We see that the first exponent in (25) matches to the incoming wave (17), while the second exponent matches to outgoing wave (18). Using the relation between z_p and z_s coordinates it is a simple matter to find the amplitude of backward wave. We compare the asymptotic expansions (17), (18) and the amplitude coefficient c_0 and find

$$\omega^b = -\exp\left(2ikb - i\frac{ka^2}{b}\right)(2b)^{-2\mu}\tilde{\omega}_0 W_{\mu,0}(-i\chi)R_1, \tag{28}$$

where $\tilde{\omega}_0$ is the numerator of ω_0 which for μ satisfying dispersion equation can be simplified. For that from dispersion equation we express

$$\dot{W}_{\mu,1}(-iv_0) = \frac{W_{\mu,1}(-iv_0)}{W_{\mu,0}(-iv_0)}\dot{W}_{\mu,0}(-iv_0)$$

and substituting it to the expression

$$\tilde{\omega}_0 = -\dot{M}_{\mu,0}(-iv_0)W_{\mu,1}(-iv_0) - M_{\mu,0}(-iv_0)\dot{W}_{\mu,1}(-iv_0)$$

get

$$\tilde{\omega}_0 = \frac{W_{\mu,1}(-iv_0)}{W_{\mu,0}(-iv_0)}\left\{M_{\mu,0}(-iv_0)\dot{W}_{\mu,0}(-iv_0) - \dot{M}_{\mu,0}(-iv_0)W_{\mu,0}(-iv_0)\right\}.$$

With the use of relation (27) and expression for the Wronskian we can simplify the formula for the numerator to

$$\tilde{\omega}_0 = -\sqrt{\frac{2\mu+1}{2\mu-1}}\frac{1}{\Gamma(1/2-\mu)}.$$

Substituting it into (28) and using once again relation (27) we finally get for the amplitude of back going wave the following expression

$$\omega^b = i\exp\left(2ikb - \frac{ika^2}{b}\right)(4kb)^{-2\mu}\frac{\Gamma(1/2+\mu)}{\Gamma^2(1/2-\mu)}\frac{2\mu+1}{2\mu-1}W_{\mu,0}(-i\chi). \tag{29}$$

5. CURRENT ON THE SURFACE

5.1. Special Functions

Substituting expressions (4) and (5) into representation (2) and setting $\nu = \kappa$, we can find the current on the surface. We represent it by introducing a special function

$$A(\eta;\chi) = \frac{2e^{-i\pi/4}}{\pi}\frac{e^{-i\chi\eta/2}}{\sqrt{\chi}\sqrt{1-\eta^2}}\int_{-i\infty}^{+i\infty}\left(\frac{1-\eta}{1+\eta}\right)^{\mu} \times\frac{\Gamma(1/2+\mu)W_{\mu,1}(-i\chi)}{W_{\mu,0}(-i\chi)\dot{W}_{\mu,1}(-i\chi)+\dot{W}_{\mu,0}(-i\chi)W_{\mu,1}(-i\chi)}d\mu \tag{30}$$

of two variables η and χ. The current of forward going wave is given in the leading order approximation by the formula

$$J = e^{ikz}\sin(\varphi)A\left(\frac{z}{\sqrt{\rho\rho_t}}; k\rho_t\sqrt{\frac{\rho_t}{\rho}}\right). \tag{31}$$

Substituting expression (29) into (9) and letting $\nu = \kappa$ yields

$$\begin{aligned} J^b &= e^{-ikp\eta}\frac{2e^{i\pi/4}}{\pi}\frac{\sin\varphi}{\sqrt{\chi}\sqrt{1-\eta^2}}\exp\left(2ikb - i\frac{ka^2}{b}\right) \\ &\quad\times\int_{-i\infty}^{+i\infty}\Gamma^2\left(\mu+\frac{1}{2}\right)\Gamma^{-1}\left(\frac{1}{2}-\mu\right)\frac{2\mu+1}{2\mu-1}(4kb)^{-2\mu}\left(\frac{1+\eta}{1-\eta}\right)^{\mu} \\ &\quad\times\frac{W_{\mu,1}(-i\chi)}{W_{\mu,1}(-i\chi)\dot{W}_{\mu,0}(-i\chi)+\dot{W}_{\mu,1}(-i\chi)W_{\mu,0}(-i\chi)}d\mu. \end{aligned} \tag{32}$$

We can represent (32) in the way similar to (31)

$$J^b = e^{2ikb-i\chi-ikz}\sin\varphi B\left(\frac{z}{\sqrt{\rho\rho_t}}; k\rho_t\sqrt{\frac{\rho_t}{\rho}}; R(\mu)\right), \tag{33}$$

where

$$\begin{aligned} B(\eta;\chi;kb) &= \frac{2}{\pi}e^{-i\pi/4}\frac{e^{i\chi\eta/2}}{\sqrt{\chi}\sqrt{1-\eta^2}}\int_{-i\infty}^{+i\infty}\left(\frac{1+\eta}{1-\eta}\right)^{\mu}R(\mu) \\ &\quad\times\frac{\Gamma(1/2+\mu)W_{\mu,1}(-i\chi)}{W_{\mu,1}(-i\chi)\dot{W}_{\mu,0}(-i\chi)+\dot{W}_{\mu,1}(-i\chi)W_{\mu,0}(-i\chi)}d\mu, \\ R &= i\frac{\Gamma(1/2+\mu)}{\Gamma(1/2-\mu)}\sqrt{\frac{2\mu+1}{2\mu-1}}(4kb)^{-2\mu}. \end{aligned}$$

Note that the expression (34) defining special function $B(\eta;\chi;kb)$ differs from the definition (30) of special function $A(\eta;\chi)$ by the replacement of η by $-\eta$, which simply means that backward wave propagates in the opposite direction, and by an additional multiplier $R(\mu)$ under the sign of the integral.

The exponential term $e^{2ikb-i\chi}$ in the formula for the current of backward wave describes the phase which appears because backward wave is the result of forward wave which runs up to the shadowed tip of the spheroid and returns back.

5.2. Whittaker and Coulomb Wave Functions

To compute Whittaker function involved in the definition of the special functions A and B we can use the program developed in [18]. For that we introduce new variable of integration $t = i\mu$ and use the relation [18]

$$W_{-it,\,\lambda+\frac{1}{2}}(-i\chi) = \exp\left(i\frac{\pi}{2}\lambda - \frac{\pi}{2}t\right)\sqrt{\frac{\Gamma(1+\lambda-it)}{\Gamma(1+\lambda+it)}}H_\lambda^+\left(t,\frac{\chi}{2}\right)$$

which expresses Whittaker function via Coulomb wave function H_λ^+. After applying the symmetry formula for Gamma function we get

$$A(\eta,\chi) = \frac{4}{\sqrt{\pi}}\frac{e^{-i\chi\eta/2}}{\sqrt{\chi}\sqrt{1-\eta^2}}\int_{-\infty}^{+\infty}\left(\frac{1+\eta}{1-\eta}\right)^{it}\Omega(t)dt, \tag{34}$$

$$B(\eta,\chi,R) = \frac{4}{\sqrt{\pi}}\frac{e^{i\chi\eta/2-i\chi}}{\sqrt{\chi}\sqrt{1-\eta^2}}\int_{-\infty}^{+\infty}\left(\frac{1-\eta}{1+\eta}\right)^{it}\Omega(t)\mathcal{R}(t)dt, \tag{35}$$

where

$$\Omega(t) = \frac{H^+_{1/2}(t,\chi/2)}{H^+_{1/2}(t,\chi/2)\dot{H}^+_{-1/2}(t,\chi/2)+\dot{H}^+_{1/2}(t,\chi/2)H^+_{-1/2}(t,\chi/2)} \times \frac{e^{\pi t/2}}{\sqrt{\cosh(\pi t)}}$$

$$\mathcal{R}(t) = \frac{\Gamma(1/2-it)}{\Gamma(1/2+it)}\sqrt{\frac{1-2it}{1+2it}}(4kb)^{2it}.$$

6. NUMERICAL RESULTS

We consider four spheroids at two frequencies. Geometry of the problem is presented on Figure 1. Numerical results by finite elements method are provided by M. Duruflé (Institute of Mathematics, Bordeaux, France). The parameters are presented in Table 1. The problems for spheroid No. 1 at frequency of 1 GHz is equivalent to the problem for spheroid No. 2 at frequency of 2 GHz.

The backward going wave is, as was already mentioned, a contribution of residues, and the main contribution is due to the residue in the pole with minimal imaginary part of t. Nevertheless we compute the integrals along the real axis of t to avoid finding zeros of the dispersion equation in complex plane.

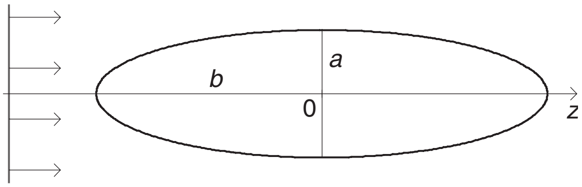

Figure 1. Geometry of the problem.

Table 1. Test problems parameters.

No.	f (GHz)	a (m)	b (m)	χ	kb
1	1	1.0	2.5	8.38338	52.39613
1	2	1.0	2.5	16.76676	104.79225
2	1	0.5	1.25	4.19169	26.19806
2	2	0.5	1.25	8.38338	52.39613
3	1	0.5	1.76776695	2.96397	37.04966
3	2	0.5	1.76776695	5.92795	74.09931
4	1	0.3125	1.39754249	1.46452	29.29032
4	2	0.3125	1.39754249	2.92903	58.58065

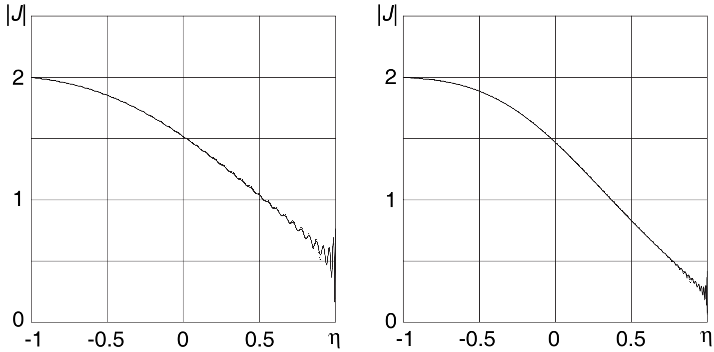

Figure 2. Absolute values of test current and current computed asymptotically on spheroid No. 1.

The integrals in the definitions of special functions A and B rapidly converge at $\pm\infty$. We use numerical integration and restrict the domain to the segment $[-5, 10 + \chi/2]$.

The fields of backward waves increase with η because waves propagate with some attenuation in the opposite direction. This

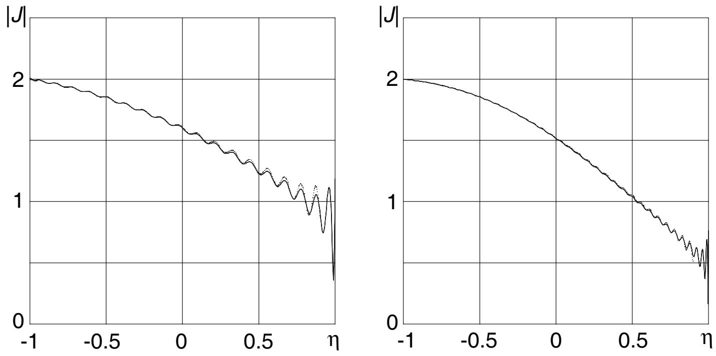

Figure 3. Absolute values of test current and current computed asymptotically on spheroid No. 2.

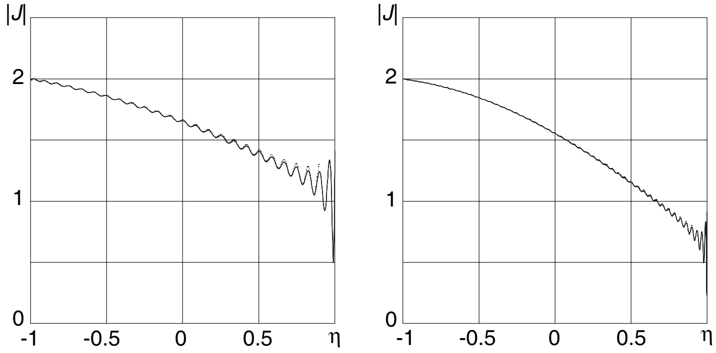

Figure 4. Absolute values of test current and current computed asymptotically on spheroid No. 3.

attenuation is significant for large values of kb when waves travel a long distance around the shadowed end of the spheroid and the attenuation is smaller for small kb.

We present result of comparison for the total current $J + J^b$ at Figures 2–5. Results computed by finite elements method are given by solid lines and asymptotic results are given by dotted lines. We see that adding backward going wave allows to reproduce the oscillating character of the current amplitude. The agreement is good. The amplitude of backward wave is correct, indeed we see that the oscillations are of approximately the same amplitude as computed numerically. However, some phase shift can be noticed. This phase shift can be found small compared to the total wave distance kb.

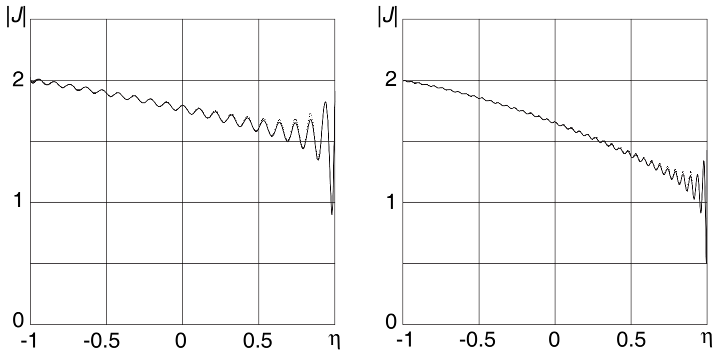

Figure 5. Absolute values of test current and current computed asymptotically on spheroid No. 4.

We can also note some peculiarities. The agreement is better for more elongated spheroids. This was expected, because, when deriving our asymptotic formulae, the fact that spheroid is strongly elongated was used in all our derivations and terms that are small for elongated spheroids were neglected.

7. CONCLUSION

In this paper we derived complete asymptotic approximation for diffraction of a plane wave by an elongated spheroid. The current on the spheroid is composed of two travelling waves. The first propagates forward; the second is generated by reflection of the first one at the end of the body and propagates backward. The currents of these travelling waves are given by special functions, that can be considered as generalizations of Fock function, taking into account the effect of transverse curvature of the surface. Large transverse curvature decreases the attenuation in the shadow zone. The comparison with numerical results shows good accuracy of the approach.

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APPENDIX A. SOME RELATIONS FOR WHITTAKER FUNCTIONS

We start with relations [17]

$$W_{\mu+1/2,\ell+1/2}(z) - \sqrt{z}W_{\mu,\ell}(z) + \left(\mu - \ell - \frac{1}{2}\right)W_{\mu-1/2,\mu+1/2} = 0, \quad \text{(A1)}$$

$$W_{\mu+1/2,\ell-1/2}(z) - \sqrt{z}W_{\mu,\ell}(z) + \left(\mu + \ell - \frac{1}{2}\right)W_{\mu-1/2,\mu-1/2} = 0, \quad \text{(A2)}$$

$$\left(\mu + \ell - \frac{1}{2}\right)\sqrt{z}W_{\mu-1/2,\ell-1/2}(z) - (z+2\ell)W_{\mu,\ell}(z) + \sqrt{z}W_{\mu+1/2,\ell+1/2} = 0, \quad \text{(A3)}$$

$$\left(\mu - \ell - \frac{1}{2}\right)\sqrt{z}W_{\mu-1/2,\ell+1/2}(z) - (z-2\ell)W_{\mu,\ell}(z) + \sqrt{z}W_{\mu+1/2,\ell-1/2} = 0. \quad \text{(A4)}$$

We combine (A1) and (A2) excluding Whittaker function $W_{\mu,\ell}(z)$ and get

$$W_{\mu+1/2,\ell+1/2}(z) + \left(\mu - \ell - \frac{1}{2}\right)W_{\mu-1/2,\ell+1/2}(z) - W_{\mu+1/2,\ell-1/2}(z) - \left(\mu + \ell - \frac{1}{2}\right)W_{\mu-1/2,\ell-1/2}(z) = 0. \quad \text{(A5)}$$

Similarly from (A3) and (A4) we get

$$W_{\mu+1/2,\ell+1/2}(z) - \frac{z+2\ell}{z-2\ell}W_{\mu+1/2,\ell-1/2}(z) - \frac{z+2\ell}{z-2\ell}\left(\mu + \ell - \frac{1}{2}\right)W_{\mu-1/2,\ell+1/2}(z) + \left(\mu + \ell - \frac{1}{2}\right)W_{\mu-1/2,\ell-1/2}(z) = 0. \quad \text{(A6)}$$

Excluding $W_{\mu+1/2,\ell+1/2}(z)$ from (A5) and (A6) we get

$$\frac{z}{z-2\ell}\left(\mu - \ell - \frac{1}{2}\right)W_{\mu-1/2,\ell+1/2}(z) + \frac{2\ell}{z-2\ell}W_{\mu+1/2,\ell-1/2}(z) - \left(\mu + \ell - \frac{1}{2}\right)W_{\mu-1/2,\ell-1/2}(z) = 0.$$

Now we change μ into $\mu+1/2$ and ℓ into $\ell+1/2$ and express $W_{\mu,\ell+1}(z)$ via Whittaker functions with other indices

$$W_{\mu,\ell+1}(z) = \frac{2\mu+2\ell+1}{2\mu-2\ell-1}\left(1-\frac{2\ell+1}{z}\right)W_{\mu,\ell}(z) - \frac{4\ell+2}{2\mu-2\ell-1}\frac{1}{z}W_{\mu+1,\ell}(z) \tag{A7}$$

Finally with the help of the formula [16]

$$\dot{W}_{\mu,\ell}(z) = \left(\frac{1}{2}-\frac{\mu}{z}\right)W_{\mu,\ell}(z) - \frac{1}{z}W_{\mu+1,\ell}(z) \tag{A8}$$

we exclude $W_{\mu+1,\ell}(z)$ from (A7) and get

$$\left(\mu-\ell-\frac{1}{2}\right)W_{\mu,\ell+1}(z) = (2\ell+1)\dot{W}_{\mu,\ell}(z) + \left(\mu-\frac{2\ell+1)^2}{2z}\right)W_{\mu,\ell}(z). \tag{A9}$$

For $\ell = 0$ formula (A9) simplifies to

$$\left(\mu-\frac{1}{2}\right)W_{\mu,1}(z) = \dot{W}_{\mu,0}(z) + \left(\mu-\frac{1}{2z}\right)W_{\mu,0}(z). \tag{A10}$$

APPENDIX B. DIFFERENT FORM OF DISPERSION RELATION

Here we rewrite the dispersion relation

$$\Delta = \frac{d}{dz}\left(W_{\mu,1}(z)W_{\mu,0}(z)\right) = W_{\mu,1}(z)\dot{W}_{\mu,0}(z) + \dot{W}_{\mu,1}(z)W_{\mu,0}(z)$$

in another form. For that we use formula (A10) to express the derivative

$$\dot{W}_{\mu,0}(z) = \left(\mu-\frac{1}{2}\right)W_{\mu,1}(z) - \left(\mu-\frac{1}{2z}\right)W_{\mu,0}(z). \tag{B1}$$

A similar formula expressing $\dot{W}_{\mu,1}(z)$ via functions $W_{\mu,1}(z)$ and $W_{\mu,0}(z)$ can be obtained from (A8) in which we need to exclude Whittaker function $W_{\mu+1,1}(z)$. We do that by using formula

$$W_{\mu+1,\ell+1}(z) + \left(\mu-\ell-\frac{1}{2}\right)W_{\mu,\ell+1}(z) - W_{\mu+1,\ell}(z) - \left(\mu+\ell+\frac{1}{2}\right)W_{\mu,\ell}(z) = 0$$

which is formula (A5) with the indices shifted by 1/2. This gives expression

$$\dot{W}_{\mu,1}(z) = -\frac{1}{z}W_{\mu+1,0}(z) - \left(\mu-\frac{1}{2}\right)\frac{1}{z}W_{\mu,0}(z).$$

Further with the use of (A7) which gives expression

$$W_{\mu+1,0}(z) = -z\left(\mu - \frac{1}{2}\right)W_{\mu,1}(z) + (z-1)\left(\mu + \frac{1}{2}\right)W_{\mu,0}(z)$$

we get

$$\dot{W}_{\mu,1}(z) = \left(\mu - \frac{1}{2z}\right)W_{\mu,1}(z) - \left(\mu + \frac{1}{2}\right)W_{\mu,0}(z). \qquad \text{(B2)}$$

Substituting (B1) and (B2) into dispersion relation we get

$$\Delta = \left(\mu - \frac{1}{2}\right)W^2_{\mu,1}(z) - \left(\mu + \frac{1}{2}\right)W^2_{\mu,0}(z).$$

APPENDIX C. DIFFERENT FORMS OF REFLECTION COEFFICIENTS

Using (23) one can get the following asymptotics

$$\frac{M_{\mu+1,0}(iu)}{u^{3/2}} \sim e^{-ikz_p}(2kz_p)^{\mu-1/2}\frac{e^{-i\pi\mu/2}}{\Gamma(\mu+3/2)}$$
$$+e^{ikz_p}(2kz_p)^{-\mu-5/3}\frac{e^{-i\pi(\mu+1)/2}}{\Gamma(-\mu-1/2)},$$
$$\frac{M_{\mu,0}(iu)}{u^{3/2}} \sim e^{-ikz_p}(2kz_p)^{\mu-3/2}\frac{e^{i\pi(1-\mu)/2}}{\Gamma(\mu+1/2)}$$
$$+e^{ikz_p}(2kz_p)^{-\mu-3/2}\frac{e^{-i\pi\mu/2}}{\Gamma(1/2-\mu)},$$
$$\frac{M_{\mu,0}(iu)}{u^{1/2}} \sim e^{-ikz_p}(2kz_p)^{\mu-1/2}\frac{e^{i\pi(1-\mu)/2}}{\Gamma(\mu+1/2)}$$
$$+e^{ikz_p}(2kz_p)^{-\mu-1/2}\frac{e^{-i\pi\mu/2}}{\Gamma(1/2-\mu)}.$$

With the help of these formulae we get

$$C_0 \sim -\frac{2\mu+1}{4}e^{-ikz_p}(2kz_p)^{\mu-1/2}$$
$$\times\left\{(2i\mu\alpha+\beta)\frac{e^{-i\pi\mu/2}}{\Gamma(\mu+3/2)} + (i\beta-\alpha)\frac{e^{i\pi(1-\mu)/2}}{\Gamma(\mu+1/2)}\right\}$$
$$-\frac{2\mu+1}{4}e^{ikz_p}(2kz_p)^{-\mu-1/2}(iq-p)\frac{e^{-i\pi\mu/2}}{\Gamma(1/2-\mu)}$$

which with the use of Gamma function properties can be reduced to (24).

Analogously we find

$$C_1 \sim c_1 \left(e^{-ikz_p} z_p^{\mu-1/2} + R_2 e^{ikz_p} z_p^{-\mu-1/2} \right)$$

where

$$\begin{aligned} c_1 &= \frac{2\mu-1}{4}(2k)^{\mu-1/2}\frac{e^{-i\pi\mu/2}}{\Gamma(\mu+1/2)}(ip+q), \\ R_2 &= -(2k)^{-2\mu}\frac{\Gamma(1/2+\mu)}{\Gamma(1/2-\mu)}\frac{iq+p}{ip+q}. \end{aligned}$$

For the coefficients of iH_φ we get

$$\begin{aligned} D_0 &\sim d_0 \left(e^{-ikz_p} z_p^{\mu-1/2} + R_3 e^{ikz_p} z_p^{-\mu-1/2} \right), \\ D_1 &\sim d_1 \left(e^{-ikz_p} z_p^{\mu-1/2} + R_4 e^{ikz_p} z_p^{-\mu-1/2} \right), \end{aligned}$$

with

$$\begin{aligned} d_0 &= \frac{2\mu-1}{4}(2k)^{\mu-1/2}\frac{e^{-i\pi\mu/2}}{\Gamma(\mu+1/2)}(p+iq), \\ d_1 &= \frac{2\mu-1}{4}(2k)^{\mu-1/2}\frac{e^{-i\pi\mu/2}}{\Gamma(\mu+1/2)}(p-iq), \\ R_3 &= -\frac{2\mu+1}{2\mu-1}(2k)^{-2\mu}\frac{\Gamma(1/2+\mu)}{\Gamma(1/2-\mu)}\frac{ip+q}{p+iq} \end{aligned}$$

and

$$R_4 = -(2k)^{-2\mu}\frac{\Gamma(1/2+\mu)}{\Gamma(1/2-\mu)}\frac{ip-q}{p-iq}.$$

We are mainly interested in the formulae for the reflection coefficients R_1, R_2, R_3 and R_4 which all should either coincide or differ only in sign. It is a simple matter to see that

$$R_3 = -R_1, \quad R_4 = R_2,$$

which is consistent with the representations for forward and backward going waves on an elongated spheroid.

To check that $R_1 = R_2$ we use the formula (21) and take into account that μ parameter satisfies the dispersion equation. Substituting expression (21) we get

$$R_1 = i(2\mu+1)\frac{\Gamma(1/2+\mu)}{\Gamma(1/2-\mu)}\frac{W_{\mu,1}(-iv_0)-W_{\mu,0}(-iv_0)}{(2\mu+1)W_{\mu,0}(-iv_0)-(2\mu-1)W_{\mu,1}(-iv_0)}$$

and

$$R_2 = \frac{-i}{2\mu-1}(2k)^{-2\mu} \times \frac{\Gamma(1/2+\mu)}{\Gamma(1/2-\mu)} \frac{(2\mu-1)W_{\mu,1}(-iv_0)-(2\mu+1)W_{\mu,0}(-iv_0)}{W_{\mu,1}(-iv_0)-W_{\mu,0}(-iv_0)}.$$

Now we consider the ratio of these two coefficients

$$\frac{R_2}{R_1} = \frac{[(2\mu-1)W_{\mu,1}(-iv_0)-(2\mu+1)W_{\mu,0}(-iv_0)]^2}{(2\mu-1)(2\mu+1)\,[W_{\mu,1}(-iv_0)-W_{\mu,0}(-iv_0)]^2}.$$

Exploiting relation (27) we get

$$\frac{R_2}{R_1} = \frac{1}{(2\mu-1)(2\mu+1)} \frac{\left[\sqrt{2\mu-1}\sqrt{2\mu+1}-(2\mu+1)\right]^2}{\left[\sqrt{\frac{2\mu+1}{2\mu-1}}-1\right]^2} = 1.$$

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