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IMPLICIT SPACE MAPPING APPLIED TO THE SYNTHESIS OF ANTENNA ARRAYS

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Abstract—This paper introduces a novel technique for efficiently combining implicit space mapping (ISM) with method of moments (MoM) for the synthesis of antenna arrays and explores several example applications of the ISM approach. The antenna arrays geometric parameters are extracted to be optimized by ISM, and a fitness function is evaluated by MoM simulations to represent the performance of each candidate design. A coarse-mesh MoM and a fine-mesh MoM solver are used for the coarse and the fine models, respectively. To achieve the parameter extraction, the auxiliary parameter is selected and the approximation between the two models is accomplished by particle swarm optimization (PSO). The results show that the running time of the ISM algorithm is $2 \sim 3$ times faster than that of other optimization algorithms (e.g., PSO).

1. INTRODUCTION

As the communication technology develops rapidly, requirements for the antenna radiation performance increase greatly. The pattern synthesis technology, an effective method of antenna arrays pattern control, has received more and more attention in the electromagnetic community. Many optimization algorithms have been successfully applied to the array synthesis, and these methods can be categorized into 1) gradient-based (e.g., Woodward, steepest-descent), 2) stochastic (e.g., differential evolution, simulated-annealing, genetic algorithm (GA), and particle-swarm (PSO)) [1–3], 3) neural-based approaches. The accurate calculation and fast optimization for array

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synthesis can be made as a result of the rapid improvement of the computer's processing performance. In order to calculate the radiation more accurately and take into account the element coupling (mutual coupling between elements), some numerical methods such as the method of moments (MoM) [4–6], the finite difference time domain (FDTD) and the finite element method (FEM) [7] are combined with optimization algorithms (e.g., GA, PSO) [8–12]. In spite of this combination, problems for which great computational time is needed to accurately simulate each possible solution remain yet excessively costly. However, with the development of space mapping (SM) [13], the contradiction between optimal efficiency and accuracy has been relieved.

Space mapping was proposed by Bandler for the first time in 1994, and has been successfully applied to RF devices, particularly the design of filters [13, 17–21]. Space mapping strategy has been previously used to optimize antennas in recent years [14, 15]. The main idea of SM is to establish the so-called coarse model (not accurate but fast) and fine model (very accurate but expensive to evaluate) for optimization. Then, the coarse model would be optimized by updating the mapping between the two models to find the global optimal solution. In the SM process, the optimization for the coarse model ensures the efficiency of the calculation, while the approximation for the fine model guarantees the precision.

The SM-based method is rarely used in array synthesis. That's because the conventional SM algorithm requires an obvious mapping relation between the coarse model and the fine model. As the implicit space mapping (ISM) [16] does not have this restriction. In this paper, a novel integration of ISM and MoM for the array synthesis is presented. A coarse-mesh MoM solver is used for the coarse model and a fine-mesh MoM solver is employed for the fine model. The PSO algorithm is applied to achieve the optimization for the coarse model and the approximation between the two models. The optimization results show the feasibility of the ISM algorithm. It allows improved optimization efficiency, higher simulation accuracy and less computational time.

2. BASICS OF IMPLICIT SPACE MAPPING

The SM can be classified into the explicit (or original) SM and the implicit SM. The explicit SM includes the original SM [17], aggressive SM [18] and neural SM [19], etc. In the explicit SM, a clear mapping relationship between the coarse model and the fine model should be extracted. In each iteration, the coarse model keeps fixed while the mapping is updated continuously, and the optimal solution of the fine model can be obtained if the inverse mapping is available.

2.1. Implicit Space Mapping

If the mapping relationship is not obvious, it may be hidden in the coarse model. It is necessary to add an extra mapping to match the coarse model and the fine model. And the extra mapping is the characteristic of the implicit space mapping.

In ISM, our final purpose is to find the optimal solution of the fine model, which can be defined as

$$x_f^* = \arg\min_{x_f} U(R_f(x_f)) \tag{1}$$

where U is the so-called minimax objective function, x_f^* and $R_f(x_f)$ are the optimal solution and responses vector of the fine model, respectively. $R_f(x_f)$ is evaluated by fine-mesh MoM solver.

The optimal solution of the coarse model $x_c^{*(0)}$ is

$$x_c^{*(0)} = \arg\min_{x_c} U(R_c(x_c, x_a^{(0)})) \tag{2}$$

where $x_a^{(0)}$ denotes the initial auxiliary parameter. The solution of (2) is an initial process in the SM optimization.

$x_c^{*(i)}$ denotes the optimal solution of the ith iteration of the coarse model. The corresponding coarse model response vector is $R_c(x_c^{*(i)},\ x_a^{(i)})$, and it can be evaluated by coarse-mesh MoM solver. To establish a mapping between the coarse and the fine models by x_f, x_c and x_a, it sets

$$x_f^{(i)} = x_c^{*(i-1)} \tag{3}$$

and $x_a^{(i)}$ needs to satisfy (4).

$$\left\| R_f(x_f^{(i)}) - R_c(x_f^{(i)}, x_a^{(i)}) \right\| \leq \varepsilon \tag{4}$$

The process above is called as parameter extraction (PE).

Then the prediction could be obtained by optimizing the coarse model through PSO.

$$x_c^{(i)} = \arg\min_{x_c} U\left(R_c\left(x_c, x_a^{(i)}\right)\right) \tag{5}$$

and the fine model parameters are updated as

$$x_f^{(i+1)} = x_c^{(i)} \tag{6}$$

2.2. Parameter Extraction Based on PSO

Parameter extraction (PE) is the key to the space mapping. Many methods are proposed in recent years, such as multipoint parameter extraction (MPE) [20], statistical PE [20] and penalty PE [21], etc. The selection of auxiliary (preassigned) parameter is very important in PE. For example, if the excitation magnitudes of array elements need to be optimized, the excitation phases can be used as the auxiliary (preassigned) parameters [22]. Should be noted that the auxiliary parameter is used to calibrate the coarse model, it is not a design parameter. While the auxiliary parameter is fixed, the PSO is used in this paper to accomplish the process of parameter extraction. The key to the PE is to solve (4), and the following fitness function can be used in the PE

$$\mathit{fitness} = \mathit{norm}\left(R_f\left(x_f^{(i)}\right) - R_c\left(x_f^{(i)}, x_a^{(i)}\right)\right) \tag{7}$$

In general, the final goal of the PE is to realize an infinite approximation between the coarse and fine models. After PE, with fixed the auxiliary parameter, the calibrated coarse model is reoptimized. The mapping from the coarse to the fine models can be established by the optimized auxiliary parameter. Then assign the optimized design parameter to the fine model and repeat this process until the fine model response is sufficiently close to the target response.

2.3. Coarse Model and Fine Model

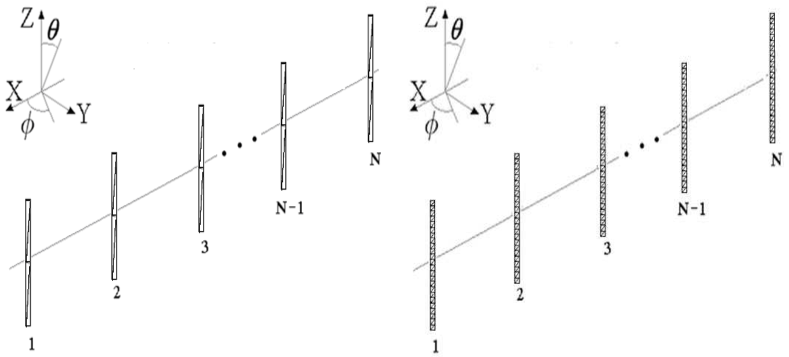
The fine model utilizes a fine mesh satisfying mesh convergence so that the results are accurate. The fine model evaluation is computationally costly. The coarse model utilizes a coarse MoM mesh which can not achieve the accurate response.

To illustrate the validity of ISM, a linear half-wave dipole array optimization is considered as examples. Fig. 1 depicts the dipole array geometry, the dipole element spacing is half-wavelength in which has 16(20) equally spaced elements along the x axis and oriented the z axis. The coarse and the fine models of each element are subdivided into 4 and 48 triangular patches, respectively. The cost time of calculation of the coarse and the fine models are about 0.07 s and 1.58 s, respectively.

Details of each step of ISM/MoM are described as follow

Step 1 Define the design parameter and initial auxiliary parameter x_a, set $i = 0$.

Step 2 Optimize the design parameter $x_c^{(i)}$ of the coarse model with $x_a^{(i)}$ being fixed.



(a) Coarse model (b) Fine model

Figure 1. MoM mesh models of dipole array.

Step 3 Set $x_f^{(i+1)} = x_c^{(i)}$.

Step 4 Solve the fine model at $x_f^{(i+1)}$ by fine-mesh MoM solver.

Step 5 Judge whether the termination criteria is satisfied (e.g., response meets specifications).

Step 6 Calibrate the coarse model by extracting the auxiliary parameter $x_a^{(i)}$ (PE).

Step 7 Set $i = i + 1$, go to Step 2.

3. EXAMPLE DESIGNS

In order to illustrate the implicit space mapping, the application of the ISM to linear array antenna design problems is taken into consideration. The optimization ranges of the excitation magnitude and phase of each element are from 0 to 1 and from 0° to 180°, respectively. The following fitness function can be used in the ISM and the PSO

$$\begin{aligned} \mathit{fitness} \;=\; & \alpha \times \mathit{sum}(F_d(\theta) - F(\theta)) + \beta \times \mathit{std}(F_d(\theta) - F(\theta)) \\ & + \gamma \times (\max(F_{dp}(\theta)) - \max(F_p(\theta))) \end{aligned} \tag{8}$$

where $F_d(\theta)$ is the desired pattern, $F_(\theta)$ is the calculated pattern, $F_{dp}(\theta)$ and $F_p(\theta)$ are the peaks of both patterns. The coefficient α, β and γ adjust the relative weights of these three objectives. All these patterns were calculated by MOM.

3.1. Null Controlled Pattern Design

The array is a 16-element asymmetric linear array, and the desired antenna pattern is marked in Fig. 2 by the dotted lines. The excitation magnitudes of array elements have been optimized and the corresponding array factor has nulls in specified directions. Two nulls are desired to exist between 44° and 53° and between 128° and 137°, and their magnitudes should be lower than −40 dB.

The excitation phase is selected as the auxiliary parameter in this example. After 3 iterations, an optimal null control pattern is obtained and presented in Fig. 2, and it shows the contrast of the optimal pattern between the ISM and the PSO. The desired pattern is obtained by using 80 particles and running around 300 iterations through PSO. Thus, it calculates the fine model 24000 times ($D \times N_{PSO} = 80 \times 300 = 24000$, D is the number of the particles, N_{PSO} is the iteration times of the PSO). But only 3 iterations are required to achieve the same goal by ISM, so it calculates the fine model only 3 times, and calculates the coarse model 168000 times ($N_{PSO} \times D \times (N_{PSO} + 1) + N_{PE} \times N_{ISM} \times D = 300 \times 80 \times 4 + 300 \times 3 \times 80 = 168000$, D is the number of the particles, and N_{PSO}, N_{PE} and N_{ISM} are the iteration times of the PSO, PE and ISM, respectively.). The cost time of calculation of the fine model is about 22.5 times more than that of the coarse model, so it is equivalent to calculate the fine model 7467 times. The cost time of the PSO is

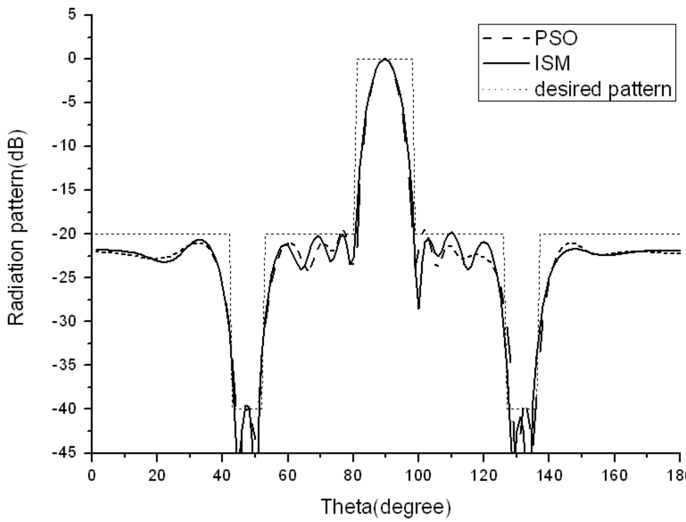


Figure 2. Null controlled pattern of an optimized 16-element asymmetric linear array.

Table 1. Design and auxiliary parameter.

Iteration	Magnitude (design parameter $x_f^{(i)}$)
1	0.040, 0.406, 0.203, 0.824, 0.125, 0.977, 0.137, 1.000, 0.411, 0.998, 0.914, 0.690, 0.433, 0.398, 0.487, 0.375
2	0.001, 0.375, 0.490, 0.457, 0.503, 0.869, 0.834, 0.989, 0.195, 1.000, 0.166, 0.875, 0.198, 0.533, 0.115, 0.046
3	0.105, 0.655, 0.164, 0.921, 0.182, 1.000, 0.225, 1.000, 0.913, 0.822, 0.544, 0.453, 0.511, 0.420, 0, 0.109
Iteration	Phase (auxiliary parameter $x_a^{(i)}$) (degree)
1	180, 180, 180, 180, 180, 180, 180, 180, 180, 180, 180, 180, 180, 180, 180, 180
2	35.44, 63.92, 65.49, 64.11, 62.86, 63.32, 62.53, 63.89, 64.70, 64.48, 66.08, 64.01, 65.63, 64.47, 65.89, 65.02
3	16.25, 38.59, 38.88, 38.32, 38.37, 38.60, 38.25, 38.63, 37.82, 38.56, 39.12, 39.32, 34.44, 40.40, 39.75, 88.22

10.6 hours and the cost time of the ISM is 3.2 hours. Design and auxiliary parameter from the 1th to the 3th iteration are shown in Table 1. It shows the variation of the design parameter and the auxiliary parameter in each iteration.

3.2. Sector Beam Pattern Design

Given a 20-element symmetric spaced linear array, the desired antenna pattern is marked in Fig. 3 by the dotted lines. We optimize the excitation phases of the array elements, and select the excitation magnitudes as the auxiliary parameter.

The requirements for the sector beam pattern are shown in Fig. 3 using dotted lines. To define the sector beam, the ripples should be smaller than 0.5 dB, the sidelobe are all below -25 dB between $0°$ and $70°$ and between $110°$ and $180°$.

When the optimization process has been executed for 3 iterations, an optimal sector beam pattern is obtained and presented in Fig. 3.

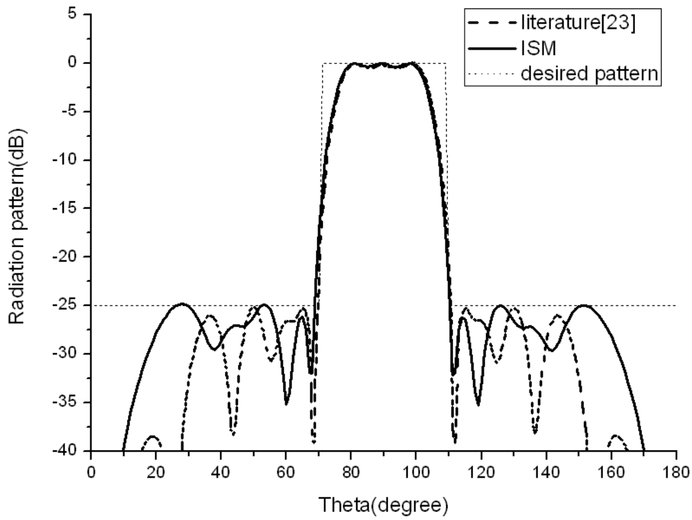


Figure 3. Sector beam pattern of an optimized 20-element symmetric linear array.

Table 2. Design and auxiliary parameter.

Iteration	Phase (design parameter $x_f^{(i)}$) (degree)
1	115.36, 127.44, 180.00, 66.49, 31.34, 71.83, 84.47, 161.70, 177.93, 178.47
2	167.99, 0, 175.90, 30.49, 165.25, 92.32, 45.49, 21.13, 12.34, 10.28
3	62.88, 112.42, 180, 120.53, 165.63, 122.74, 114.73, 33.49, 14.84, 13.06
Iteration	Magnitude (auxiliary parameter $x_a^{(i)}$)
1	0, 0, 0, 0.181, 0.297, 0.302, 0.279, 0.430, 0.734, 1.000
2	0.021, 0.067, 0.036, 0.058, 0.184, 0.150, 0.050, 0.491, 0.812, 1.000
3	0, 0, 0, 0.149, 0.271, 0.280, 0.277, 0.436, 0.740, 1.000

The same sector beam problem was optimized in [23] using Taguchi's method. The desired pattern was obtained by using 82 particles and running around 60 iterations. Thus, it calculated the fine model 4920 times. But only 3 iterations are required to achieve the same goal by ISM using 70 particles, so it calculates the fine model only 3 times, and calculates the coarse model 58800 times, which is equivalent to calculate the fine model 2613 times. Design and auxiliary parameter from the 1th to the 3th iteration are shown in Table 2. The original value of the auxiliary parameter is obtained by optimizing both excitation magnitudes and phases of the coarse model. Then the optimized magnitude is selected as the original value of auxiliary parameter.

3.3. Cosecant Squared Pattern Design

To further demonstrate the validity of ISM, a relatively complex case, a cosecant squared pattern design, is attempted here. In this design, the desired antenna pattern is marked in Fig. 4 by the dotted lines. The phases of array elements have been optimized, and the excitation magnitudes as the auxiliary parameter have been selected. The original value of the auxiliary parameter is obtained the same as the example above.

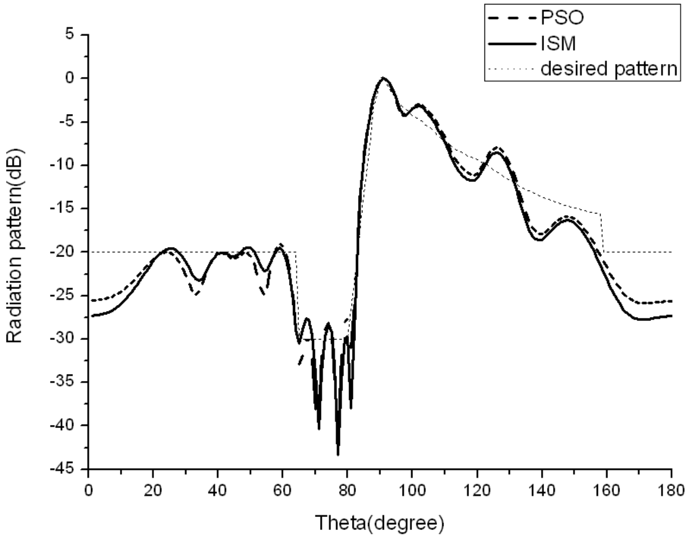


Figure 4. Cosecant squared pattern of an optimized 16-element asymmetric linear array.

The desired antenna pattern is marked in Fig. 4 by the dashed lines, in which a null is desired to exist between 65° and 80°. An optimal cosecant squared pattern is obtained and presented in Fig. 4 through 4 iterations, and it shows the contrast of the optimal pattern between the ISM and the PSO. The desired pattern is obtained by using 100 particles and running around 500 iterations through PSO. Thus, it calculates the fine model 50000 times. But only 4 iterations are required to achieve the same goal by ISM, so it calculates the fine

Table 3. Design and auxiliary parameter.

Iteration	Phase (design parameter $x_f^{(i)}$) (degree)
1	0.04, 30.78, 60.59, 53.44, 68.38, 106.28, 179.61, 28.20, 133.51, 57.50, 91.69, 80.90, 126.30, 45.58, 107.88, 103.32
2	1.57, 69.29, 77.88, 128.19, 84.88, 147.82, 174.13, 180.00, 173.69, 119.79, 164.46, 161.78, 164.56, 174.25, 95.92, 164.80
3	3.21, 35.73, 63.36, 69.04, 71.77, 103.90, 174.31, 86.44, 177.23, 60.61, 110.06, 97.70, 144.15, 118.73, 180.00, 115.73
4	2.08, 28.88, 61.65, 56.04, 73.14, 109.60, 180.00, 40.40, 145.37, 77.19, 97.23, 87.13, 124.00, 76.94, 87.57, 116.55
Iteration	Magnitude (auxiliary parameter $x_a^{(i)}$)
1	0.351, 0.399, 0.439, 0.541, 0.836, 1.000, 0.809, 0.395, 0.169, 0.293, 0.296, 0.253, 0.215, 0.099, 0.088, 0.225
2	0.452, 0.508, 0.549, 0.617, 0.848, 1.000, 0.730, 0.390, 0.133, 0.337, 0.305, 0.236, 0.201, 0.099, 0.078, 0.199
3	0.284, 0.385, 0.410, 0.545, 0.769, 1.000, 0.816, 0.402, 0.184, 0.325, 0.382, 0.326, 0.294, 0.165, 0.034, 0.203
4	0.306, 0.406, 0.443, 0.561, 0.865, 1.000, 0.812, 0.368, 0.175, 0.232, 0.251, 0.194, 0.178, 0.037, 0.078, 0.176

model only 4 times, and calculates the coarse model 450000 times, which is equivalent to calculate the fine model 20000 times. The cost time of the PSO is 21.9 hours and the cost time of the ISM is 8.8 hours. Design and auxiliary parameter from the 1st to the 4th iteration are shown in Table 3.

4. CONCLUSION

An efficient method for the integration of ISM optimization with MoM for array synthesis has been presented. ISM was applied successfully in the array synthesis. The optimization kernel was constructed using ISM and MoM was integrated into the optimizer for fitness evaluations. For different optimization problems, the phase or the amplitude is used as the auxiliary parameter to achieve approximation between the coarse model and the fine model. Optimized results show that the desired array factors, a null controlled pattern, a sector beam pattern and a cosecant squared pattern are successfully obtained. It is found that implicit space mapping is a good candidate for optimizing various electromagnetics applications. ISM algorithm allows improved optimization efficiency, higher simulation accuracy, and less computational time.

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