Progress In Electromagnetics Research B, Vol. 30, 239–253, 2011

COMPARISON BETWEEN TWO PHASE-RETRIEVAL METHODS FOR ELECTROMAGNETIC SOURCE MODELING

M. Johansson*, A. Fhager, H.-S. Lui, and M. Persson

Department of Signals and Systems, Chalmers University of Technology, SE 41296, Gothenburg, Sweden

Abstract—Phase-retrieval from measured phaseless field data is of interest for various applications including electromagnetic dosimetry, electromagnetic compatibility investigations, near-field to far-field transformations and antenna diagnostics. In this study two phase-retrieval methods, namely the adjoint field method and the phase angle gradient method, are compared using 3D numerical test cases. The methods were previously presented by us, but the adjoint field method was at that time only implemented in 2D. In this study the adjoint field method has been extended to 3D, which makes it possible to test the method for more realistic test cases and to compare it with the phase angle gradient method. The results show that the phase angle gradient method is able to retrieve the phase with better accuracy than the adjoint field method. Moreover it gives results that agree well with correct phase. The phase angle gradient method was also tested with measured magnetic field. The obtained phase angles on a measurement plane in front of the source gave calculated field amplitudes that agree well with measured field.

1. INTRODUCTION

Phase-retrieval from measured phaseless field data is of interest for numerical modeling of field distributions from electromagnetic sources. There are several applications where such modeling is useful. Some examples are near-field to far-field transformations [1], antenna diagnostics [2] and electromagnetic dosimetry [3]. Phase-retrieval methods are interesting, since measurements of both amplitude and phase generally are more complicated and require more expensive equipment than phaseless measurements.

Received 5 February 2011, Accepted 9 May 2011, Scheduled 13 May 2011

* Corresponding author: Markus Johansson (markusjo@chalmers.se).

In dosimetry studies it is important to know the field distributions from electromagnetic sources, in order to determine whether exposure safety guidelines are complied with. To avoid complicated and time consuming modeling of all the details of an electromagnetic source, the field in front of the source can be measured instead. If both the amplitude and the phase of the field on a surface that encloses an electromagnetic source in free space are known, the field outside the surface can be calculated. Knowledge of the field on a planar surface between the source and the area of interest is enough to make it possible to calculate the field with a high accuracy, provided that the planar surface is large enough. In electromagnetic dosimetry, however, usually only the amplitude values of the field are measured. Therefore a method that can retrieve the phase from measured phaseless field data on a set of parallel planes close to the source is needed. A typical setup for a phase-retrieval problem based on amplitude-only measurements is shown in Figure 1. The field amplitudes on a set of plane surfaces in front of an electromagnetic source are measured and the phase information of the field is reconstructed, based on the amplitude-only data.

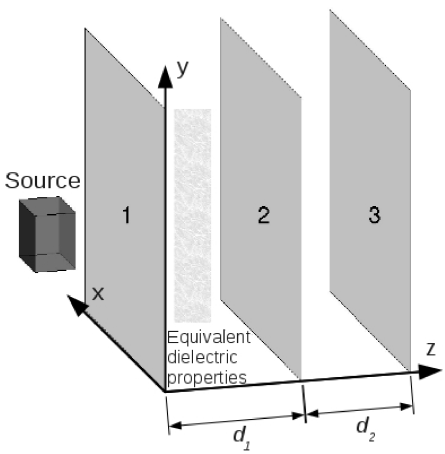

Figure 1. Field amplitudes can be measured on a set of planes in front of an electromagnetic source. The adjoint field method involves finding equivalent dielectric properties between plane 1 and plane 2. These properties should together with measured amplitudes and the field in all points set to be in phase, on plane 1, give correct field on the other planes.

Phase-retrieval methods of different kinds can be found in the scientific literature [1, 2, 4–18]. Some of them [1, 2, 4–9] are gradient-based and try to minimize a functional. As an example, phase-retrieval for complex geometry antenna and field acquisition domains is discussed in the recent work by Álvarez et al. [9]. Other methods [10–14] instead aim to recreate the phase angles in an iterative fashion, by propagating field estimates back and forth between different surfaces. Algorithms that combine more than one method, have also been presented [15, 16].

In [3], we have introduced two phase retrieval methods, the adjoint field method and the phase angle gradient method (PAGM). Results for both methods were given. It was however not clear from the results which method was the best. The reason was that results for 3D test cases were presented for the PAGM, while only 2D results were shown for the adjoint field method, since only a 2D implementation was available.

Later the performance of the PAGM was evaluated in [19]. The PAGM was found to perform well for source reconstruction problems with different electromagnetic sources, as well as for different distances between and sizes of the measurement planes. Moreover it was demonstrated that the PAGM can accurately retrieve the phase information for test cases with dimensions of the measurement planes and plane separations much less than a wavelength. This is of large interest, since other phase-retrieval methods, see for example [1, 2, 4, 12, 15], normally use distances between the measurement planes that are larger than a wavelength, which is further discussed in [19]. It is common that measured field amplitudes on only two measurement planes are used in phase-retrieval methods. This is for example done in [1] where the complex field is searched for and a functional is formulated so that the inverse problem becomes quadratic. In the PAGM on the other hand, only the phase angles are searched for and the field amplitudes, which are known from measurements, remain to be the same in all iterations of the phase-retrieval process. The correct phase angles that together with measured field amplitudes on one measurement plane give correct calculated field amplitudes on two other planes, are searched for. As a result the nonlinearity is different.

The adjoint field method and the PAGM are fundamentally different and it is of large interest to try to determine which method performs better. In this study distances between the measurement planes that are smaller than a wavelength are used and it is therefore suitable to compare the 3D implementation of the adjoint field method with the PAGM, which performs well for such small distances between the planes. Both methods are using measured field amplitudes on 3 measurement planes, as illustrated in Figure 1. In the adjoint field method, the measured amplitudes on plane 1 are used as a source, with the field in all points on the plane set to be in phase. Furthermore equivalent dielectric properties between measurement planes 1 and 2 are searched for, see Figure 1. These dielectric properties together with the source on plane 1 should give correct field, amplitude as well as phase, on the other planes. A gradient-based optimization algorithm, that minimizes the difference between calculated and measured field, is used to search for the equivalent dielectric properties. The PAGM on the other hand is a gradient-based method that searches for the correct phase angles directly. It searches for phase angles on plane 1, that minimize the difference between calculated and measured field amplitudes on the other planes.

In this study a 3D version of the adjoint field method is presented and compared with the PAGM. The rest of the paper is organized as follows. In Section 2, the details about the two phase-retrieval methods are covered. Numerical results of phase-retrieval for the two methods are given in Section 3. Results for measured field are presented in Section 4 and conclusions are presented in Section 5.

2. METHODS

2.1. The Adjoint Field Method

The purpose of the adjoint field method is to retrieve the phase of the field in front of an electromagnetic source using measured field amplitudes, see Figure 1. Measured field amplitudes on 3 measurement planes in front of the source are used. The measured amplitudes on plane 1 are used as a source, with the field in all points on the plane set to be in phase. Moreover equivalent dielectric properties between measurement plane 1 and 2 are searched for, see Figure 1. These properties should together with the source on plane 1 give numerically calculated correct field, amplitude as well as phase, on the other planes. A gradient-based optimization algorithm, that minimizes the difference between calculated and measured field, is used to search for the equivalent dielectric properties. The finite-difference time-domain (FDTD) method is used for the field calculations in the algorithm.

Versions of this type of optimization algorithm is used in tomography and can be found in [20–23]. In [3] a 2D version of the adjoint field method, which used a 2D FDTD code, was presented. To make it possible to use the adjoint field method in 3D as well as to do tomography in 3D, a code including a 3D FDTD solver was later implemented. In the original [22, 23] optimization algorithm the functional to be minimized is

$$F(\epsilon, \sigma) = \int_0^T \sum_{m=1}^{M} \sum_{n=1}^{N} |\bar{E}_m(\epsilon, \sigma, \bar{R}_n, t) - \bar{E}_m^{meas}(\bar{R}_n, t)|^2 dt, \qquad (1)$$

where $\bar{E}_m(\epsilon, \sigma, \bar{R}_n, t)$ is the calculated field and $\bar{E}_m^{meas}(\bar{R}_n, t)$ the measured field on planes 2 and 3. M is the number of sources and N is the number of measurement points on planes 2 and 3. The minimization is done with a conjugate-gradient algorithm. The gradients can be written as

$$G_\epsilon(\bar{x}) = 2 \sum_{m=1}^{M} \int_0^T \tilde{\bar{E}}_m(\epsilon, \sigma, \bar{x}, t) \cdot \partial_t \bar{E}_m(\epsilon, \sigma, \bar{x}, t) dt \qquad (2)$$

$$G_{\sigma/\langle\sigma\rangle}(\bar{x}) = 2\langle\sigma\rangle \sum_{m=1}^{M} \int_0^T \tilde{\bar{E}}_m(\epsilon, \sigma, \bar{x}, t) \cdot \bar{E}_m(\epsilon, \sigma, \bar{x}, t) dt, \qquad (3)$$

where $\bar{E}_m(\epsilon, \sigma, \bar{x}, t)$ is the numerically computed E-field in the area where equivalent dielectric properties are searched for and $\tilde{\bar{E}}_m(\epsilon, \sigma, \bar{x}, t)$ is the solution to the adjoint problem with the residual between the computed field and $\bar{E}_m^{meas}(\bar{R}_n, t)$ as sources. $\langle\sigma\rangle$ is a parameter compensating for the different scaling of the gradients.

We have here modified the algorithm slightly so that the phase of $\bar{E}_m^{meas}(\bar{R}_n, t)$ is set to be equal to that of $\bar{E}_m(\epsilon, \sigma, \bar{R}_n, t)$ at each iteration step of the procedure. The amplitudes of $\bar{E}_m^{meas}(\bar{R}_n, t)$ are obtained from the measurements. The calculation of $\langle\sigma\rangle$ is performed using the same principle as in [21, 23] and is based entirely on knowledge of the amplitude spectrum of the received signals. Thus having no information about the phase does not put any restriction on the calculation.

2.2. The Phase Angle Gradient Method

The aim of the PAGM is to retrieve the phase of the field from an electromagnetic source, using measured field amplitudes on a set of planes in front of the source, see Figure 1. Unlike the adjoint field method, the PAGM searches for the correct phase on plane 1 directly. The PAGM is gradient-based and searches for phase angles on plane 1, that minimize the difference between calculated and measured field amplitudes on planes 2 and 3.

According to the field equivalence principle, the source in Figure 1 can be replaced by an equivalent magnetic surface current density on plane 1

$$\bar{M}_s = -2\hat{n} \times \bar{E}_{p1}, \qquad (4)$$

where $\hat{n}$ is a unit vector perpendicular to plane 1 pointing towards the other planes and $\bar{E}_{p1}$ is the electric field on plane 1.

One can divide plane 1 into a square grid with the grid cell area ΔS. If the point in the middle of the square number p is represented by $\bar{r}'_p$, the Cartesian components of the electric field $\bar{E}$ in point $\bar{r}$ can be calculated [3] with the expressions

$$E_x(\bar{r}) = -\frac{\Delta S}{2\pi}\sum_p E_x(\bar{r}'_p)\frac{\partial G(\bar{r},\bar{r}'_p)}{\partial z} \quad (5)$$

$$E_y(\bar{r}) = -\frac{\Delta S}{2\pi}\sum_p E_y(\bar{r}'_p)\frac{\partial G(\bar{r},\bar{r}'_p)}{\partial z} \quad (6)$$

$$E_z(\bar{r}) = \frac{\Delta S}{2\pi}\sum_p \left(E_x(\bar{r}'_p)\frac{\partial G(\bar{r},\bar{r}'_p)}{\partial x} + E_y(\bar{r}'_p)\frac{\partial G(\bar{r},\bar{r}'_p)}{\partial y}\right). \quad (7)$$

The system of coordinates is here chosen such that plane 1 is part of the $z = 0$-plane and $G(\bar{r},\bar{r}')$ is the Green's function

$$G(\bar{r},\bar{r}') = \frac{e^{-jk|\bar{r}-\bar{r}'|}}{|\bar{r}-\bar{r}'|}, \quad (8)$$

where k is the wavenumber. If plane 1 is chosen large enough and ΔS small enough, the Equations (5), (6) and (7) give a good approximation for the electric field on the planes 2 and 3. Since the field amplitudes are known on plane 1, the field on the other planes can be regarded as a function of the unknown phase angles of the tangential components of $\bar{E}$ on plane 1.

After the phase angles on plane 1 have been initiated, the resulting field estimates on the planes 2 and 3 can be calculated. To find the correct phase, the initial angles are altered in small steps, so that the field amplitudes $|E_i|_n$, where n is a computational grid point on plane 2 or 3, converge to the measured values $|E_i^m|_n$. A functional J of the phase can be defined as

$$J \equiv \frac{1}{2}\sum_n \Big((|E_x|_n - |E_x^m|_n)^2 + \big(|E_y|_n - |E_y^m|_n\big)^2 + (|E_z|_n - |E_z^m|_n)^2\Big). \quad (9)$$

The phase angles are changed in the opposite direction of the phase angle gradients of J, so that J is minimized.

3. RESULTS FOR NUMERICAL TEST CASES

To test the 3D version of the adjoint field method and compare it with the PAGM, different numerical test cases were used. For simplicity one Cartesian field component was considered in each test case. That is measured amplitudes for only one field component was used and equivalent dielectric properties that would give correct amplitude and phase for that field component was searched for.

3.1. Field from Chosen Dielectric Properties

Although the adjoint field method is meant to be used for retrieving the phase from measured amplitudes in front of a source, a suitable way to test the implementation of the adjoint field method is to use a test case with chosen dielectric properties between the planes 1 and 2, see Figure 1. Therefore a test case was used with $\epsilon_r = 2$ in part of the volume between the planes.

The amplitudes in 74×74 field points on plane 1 were set to 1 V/m for E_y, a field component parallel to one of the edges of the plane, and to zero for the other component tangential to the plane, E_x. The phase for E_x and E_y was also set to zero on the plane. The value of ϵ_r was set to 2 in a box with the height 96 cm in the y-direction, in part of the space between planes 1 and 2. In the rest of the computational volume ϵ_r was set to 1. The conductivity σ was set to zero in the whole computational volume. A cross section of the dielectric box and the rest of the domain for reconstruction of the equivalent dielectric properties, is shown in Figure 2(b).

The chosen field on plane 1 was used as a source with the frequency 250 MHz. FDTD was used to calculate the field that the source together with the chosen dielectric properties gave on the other planes. The number of measurement points was 74×74 on both planes 2 and 3. The distance between the points on each of the three planes in x and in y-direction was 1.5 cm. From plane 1 to plane 2 and from plane 2 to plane 3 the distances were $d_1 = 15.5$ cm and $d_2 = 5$ cm respectively.

The field amplitudes of E_y on the planes were used for retrieving the phase of the field and the dielectric properties between planes 1 and 2. In Figure 3 the retrieved phase for E_y on plane 2 and the difference between the retrieved and the correct phase are shown. It can be seen that the error for the phase is small, less than 0.04 rad. Figure 2 shows cross sections of the retrieved ϵ_r and the correct ϵ_r. It can be noticed that the retrieved ϵ_r for the dielectric box is somewhat lower than the correct value. The correct ϵ_r is 2 and the retrieved ϵ_r has a maximum value of around 1.3. The retrieved object is, however, also larger than the correct object in z-direction. Therefore it appears reasonable that the resulting field on planes 2 and 3 can be approximately the same with retrieved ϵ_r as with correct ϵ_r. More sources and receivers placed on different sides of the dielectric box could probably improve the reconstruction of ϵ_r.

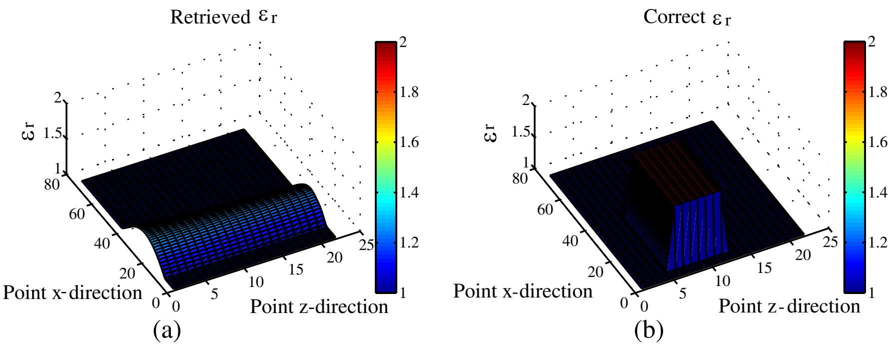

Figure 2. (a) The retrieved ϵ_r. (b) The correct ϵ_r. The distances between the points in x- and z-direction were 1.5 cm and 0.5 cm respectively.

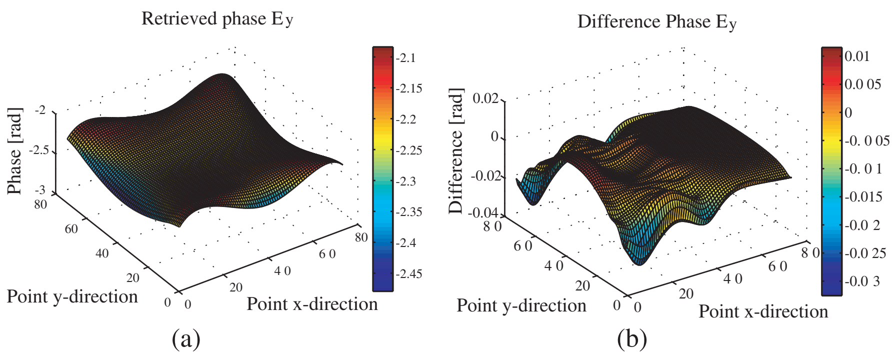

Figure 3. (a) The retrieved phase for E_y. (b) Difference between the retrieved and the correct phase.

3.2. Field from Infinitesimal Dipole

The adjoint field method was also tested with 500 MHz field from an infinitesimal dipole. Field values calculated with an analytical formula for a y-directed infinitesimal dipole were utilized to test the method. As before the y-direction was parallel to one of the edges of the planes and the x-direction was tangential to the planes but perpendicular to the y-direction. Calculated field amplitudes of E_x for 3 parallel planes in front of the source, see Figure 1, were used to calculate the phase angles. The infinitesimal dipole source was centered and the right-angled distance between the center of the plane closest to the source, plane 1, and the infinitesimal dipole was 5.75 cm. On each of the three

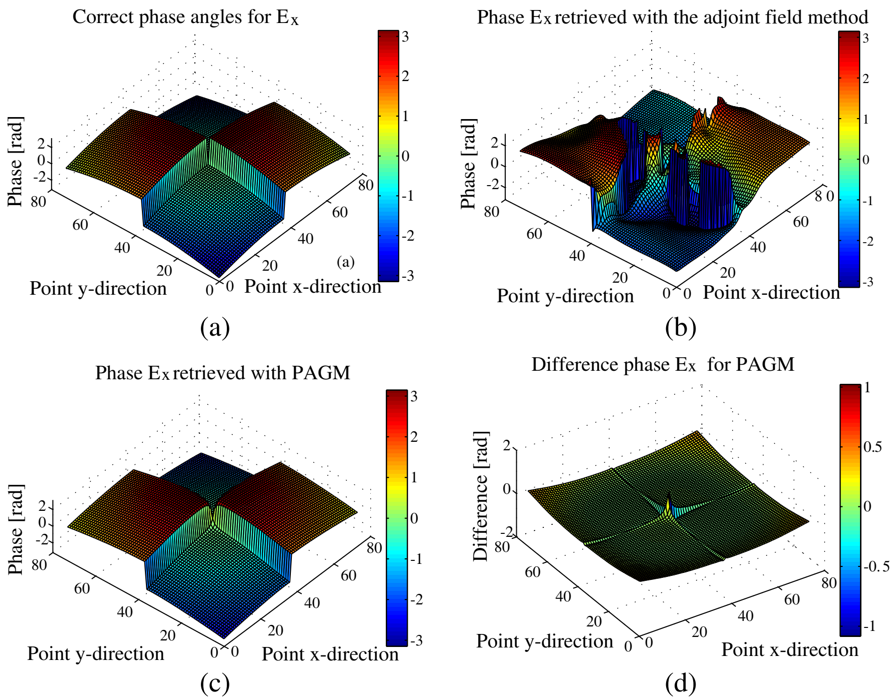

Figure 4. Results for field from infinitesimal dipole. (a) Correct phase angles for E_x. (b) Phase angles for E_x retrieved with the adjoint field method. (c) Phase angles for E_x retrieved with the PAGM. (d) Difference between retrieved and correct E_x phase for the PAGM.

planes 74×74 measurement points were used. The distance between the points on each plane in x and in y-direction was 1 cm. From plane 1 to plane 2 and from plane 2 to plane 3 the distances were $d_1 = 15.5\,\text{cm}$ and $d_2 = 5\,\text{cm}$ respectively.

To achieve the best result, the adjoint field method was run 7 times for the test case. E_y on plane 1 was set to zero for all runs. In the first run, the phase for E_x was set to zero on plane 1 as usual and then on each of the later runs the phase of E_x was set to the retrieved phase on plane 2 from the previous run. This was done since the phase on plane 2 for the test case can be expected to be a better approximation of the phase on plane 1 than the phase zero on the whole plane. After 7 runs not much more improvement was observed between consecutive runs and no more run was performed. The correct and retrieved phase for E_x on plane 2 are shown in Figures 4(a) and (b).

The PAGM was also tested with the field from the infinitesimal dipole. Both from plane 1 to plane 2 and from plane 2 to plane 3 the distance was 7.75 cm. Except for that the same test case as before was used. For the adjoint field method a sufficiently large space for the equivalent dielectric properties is required, while for the PAGM a shorter distance between the planes was preferred to reduce the truncation errors due to the finite size of the planes. Since plane 3 for the PAGM case coincide with plane 2 for the adjoint field method case, the results for the two methods can be conveniently compared. The placement of the planes is illustrated by Figure 5. To make the comparison fair the PAGM was used only for E_x and not in the normal way with all field components. That is with the help of amplitudes for only E_x, only the phase for E_x was retrieved. The retrieved phase on plane 3 that the PAGM gave and the difference between retrieved and correct phase are shown in Figures 4 (c) and (d). It can be seen that there is an excellent agreement with the correct phase for the PAGM. The peak with somewhat larger phase errors for E_x in the middle of the plane is unimportant, since the amplitude for E_x is small near the middle of the plane. Although there are similarities between the correct phase and the phase that was retrieved by the adjoint field method, it can be seen in Figure 4 that the PAGM gave much better result. To facilitate the comparison between the two phase-retrieval methods the results for the dipole test case are also shown in an alternative way in Figure 6. It can clearly be seen in the Figure that the PAGM is the method that performs best.

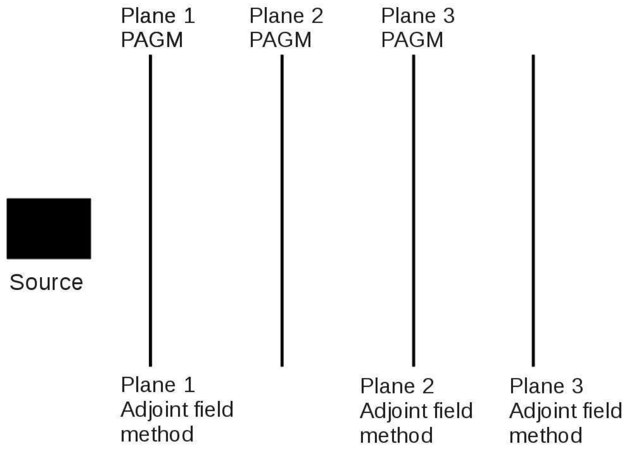

Figure 5. Placement of the measurement planes for the PAGM and the adjoint field method. Plane 3 for the PAGM case coincide with plane 2 for the adjoint field method case.

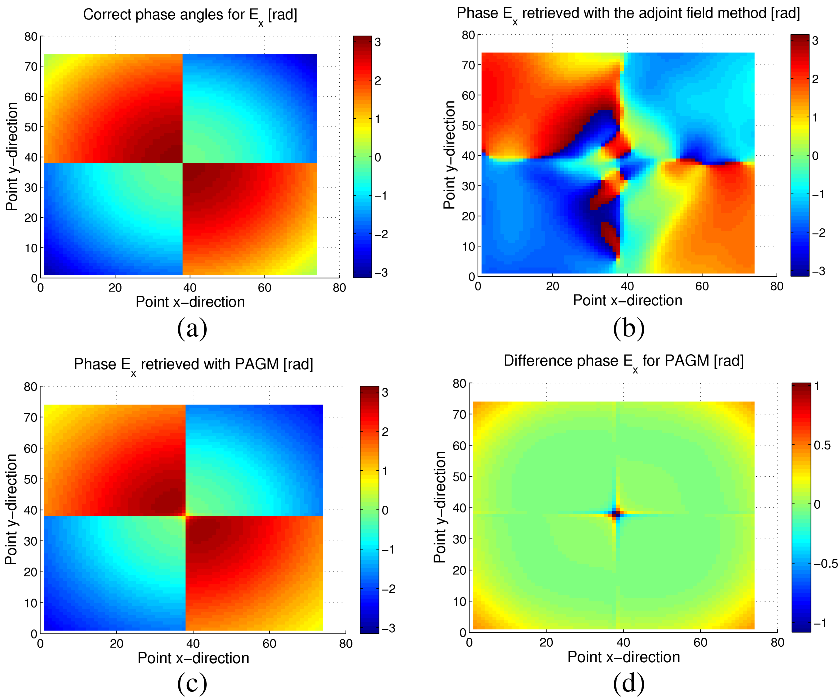

Figure 6. Results for field from infinitesimal dipole. (a) Correct phase angles for E_x. (b) Phase angles for E_x retrieved with the adjoint field method. (c) Phase angles for E_x retrieved with the PAGM. (d) Difference between retrieved and correct E_x phase for the PAGM.

4. RESULTS FOR MEASURED FIELD

The PAGM, which was the method that gave best results for the numerically calculated field in the previous section, was also tested for measured field. A test case with measured 50 Hz magnetic field was used. The use of the FDTD solver in our implementation of the adjoint field method is limited by the large number of time steps that is required for such a low frequency, so the test case was not run for the adjoint field method. The PAGM was originally developed for calculation of the phase angles of the electric field from measured electric field amplitudes, but because of the symmetry of Maxwell's equations it can also be used to calculate the phase angles of the magnetic flux density $\bar{B}$ from measured amplitudes of $\bar{B}$.

Measurements of the 50 Hz field component of $\bar{B}$ in front of a transformer were performed. A measurement probe was moved with

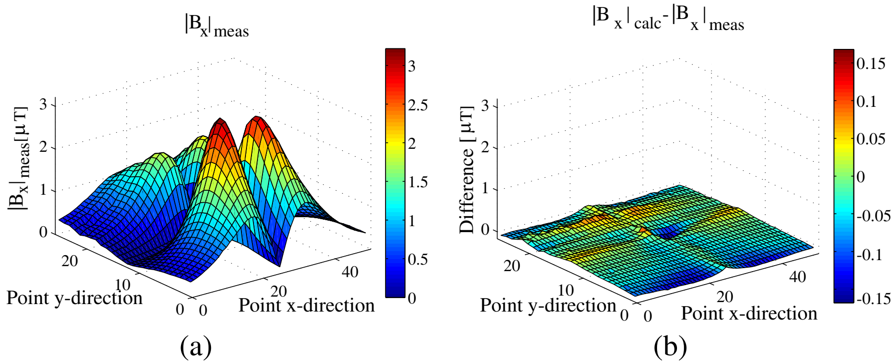

Figure 7. Measured field amplitudes and difference between calculated and measured amplitudes, for B_x on plane 3.

the help of a robot between different points where field amplitudes were measured. Measured field from 3 parallel planes were used to calculate the phase. Field amplitudes from 54×30 points on the plane closest to the transformer, plane 1, and from 50×26 points on each of the planes 2 and 3, were used. The distance between the points in horizontal as well as in vertical direction was 3 cm, on each of the planes. The distances from measurement plane 1 to plane 2 and from plane 2 to plane 3 were 5 cm and 2.5 cm respectively.

If the phase angles, given by the PAGM, are similar to the correct ones, they should give calculated field amplitudes that are similar to the measured amplitudes. Therefore, the phase angles obtained with the PAGM on plane 1 were used to calculate the field amplitudes on the other planes. In Figure 7 measured field amplitudes and the differences between calculated and measured amplitudes can be seen. Results on plane 3 for one of the field components tangential to the planes, B_x, are shown. It can be noted that the calculated field amplitudes that the obtained phase gave agree well with the correct field. The largest difference between calculated and measured amplitudes divided by the largest measured amplitude was 5.22%, for B_x on plane 3.

5. CONCLUSION

Phase-retrieval from measured phaseless field data is useful for modeling of the field distributions from electromagnetic sources in situations where the sources are complex or time consuming to make accurate models of. It has the important advantage that it can be used for many different sources without a priori knowledge of the details. Applications where this kind of modeling is of interest include electromagnetic dosimetry, electromagnetic compatibility investigations, near-field to far-field transformations and antenna diagnostics.

In this study the two phase-retrieval methods, the adjoint field method and the PAGM, have been compared. The 3D version of the adjoint field method presented here gave phase angles similar to the correct ones for the test case with known dielectric properties. The error was less than 0.04 rad. The difference between retrieved and correct ϵ_r is reasonable since only one source was used. The correct ϵ_r for the dielectric box is 2 and the retrieved ϵ_r has a maximum value of around 1.3. The retrieved object is, however, also larger than the correct object in z-direction. Therefore it appears reasonable that the resulting field on planes 2 and 3 can be approximately the same with retrieved ϵ_r as with correct ϵ_r. If the main aim would be retrieval of the dielectric properties and not phase-retrieval, more sources and receivers on different sides of the object could be used, which probably could improve the result for the reconstruction of the dielectric properties significantly. It seems reasonable to assume that for such a configuration dielectric properties closer to the correct ones would be required to produce a field close to the correct field in all receiver positions, for each of the sources. Although there are similarities between the correct phase and the phase that was retrieved by the adjoint field method, for the test case with field from an infinitesimal dipole, the PAGM gave much better result. The main conclusion from this study is therefore that the PAGM performed best and gave retrieved phase angles that are in excellent agreement with the correct phase. Moreover, the phase angles that were obtained with the PAGM gave calculated field amplitudes that agree well with measured field for the test case with measured magnetic field.

ACKNOWLEDGMENT

The computations were performed on C3SE computing resources. The authors would also like to thank The Swedish Labour Market Insurance, AFA for their financial support to this project.

REFERENCES

1. Isernia, T., G. Leone, and R. Pierri, "Radiation pattern evaluation from near-field intensities on planes," *IEEE Transactions on Antennas and Propagation*, Vol. 44, No. 5, 701–710, 1996.
2. Las-Heras, F. and T. K. Sarkar, "A direct optimization approach for source reconstruction and NF-FF transformation

using amplitude-only data," *IEEE Transactions on Antennas and Propagation*, Vol. 50, No. 4, 500–510, 2002.

3. Johansson, M., L. E. Nord, R. Kopecký, A. Fhager, and M. Persson, "Computational methods for modeling of complex sources," *COMPEL: The International Journal for Computation and Mathematics in Electrical and Electronic Engineering*, Vol. 27, No. 1, 133–143, 2008.
4. Isernia, T., G. Leone, and R. Pierri, "Results for a truncated phaseless near field technique," *Electronics Letters*, Vol. 29, No. 5, 505–506, 1993.
5. Bucci, O. M., G. D'Elia, G. Leone, and R. Pierri, "Far-field pattern determination from the near-field amplitude on two surfaces," *IEEE Transactions on Antennas and Propagation*, Vol. 38, No. 11, 1772–1779, 1990.
6. Bucci, O. M., G. D'Elia, and M. D. Migliore, "An effective near-field far-field transformation technique from truncated and inaccurate amplitude-only data," *IEEE Transactions on Antennas and Propagation*, Vol. 47, No. 9, 1377–1385, 1999.
7. Sarkar, T. K. and F. Las-Heras, "A direct optimization approach for source reconstruction NF-FF using amplitude only data," *IEEE International Symposium on Electromagnetic Compatibility*, Vol. 1, 399–402, 2001.
8. Soldovieri, F., A. Liseno, G. D'Elia, and R. Pierri, "Global convergence of phase retrieval by quadratic approach," *IEEE Transactions on Antennas and Propagation*, Vol. 53, No. 10, 3135–3141, 2005.
9. Álvarez, Y., F. Las-Heras, and M. R. Pino, "The sources reconstruction method for amplitude-only field measurements," *IEEE Transactions on Antennas and Propagation*, Vol. 58, No. 8, 2776–2781, 2010.
10. Gerchberg, R. W. and W. O. Saxton, "A practical algorithm for the determination of phase from image and diffraction plane pictures," *Optik*, Vol. 35, No. 2, 237–246, 1972.
11. Tennant, A., G. Junkin, and A. P. Anderson, "Antenna far-field predictions from two phaseless cylindrical near-field measurements," *Electronics Letters*, Vol. 28, No. 23, 2120–2122, 1992.
12. Yaccarino, R. G. and Y. Rahmat-Samii, "Phaseless bi-polar planar near-field measurements and diagnostics of array antennas," *IEEE Transactions on Antennas and Propagation*, Vol. 47, No. 3, 574–583, 1999.

13. Fridén, J., M. Siegbahn, B. Thors, and L. Hamberg, "Quick SAR assessment using dual-plane amplitude-only measurement," *Proc. 'EuCAP 2006'*, Nice, France, 2006.
14. Ribiere-Tharaud, N., M. Lambert, and P. Levesque, "Wideband validation of a phase retrieval process applied to infrared planar near-field measurements," *Progress In Electromagnetics Research B*, Vol. 23, 39–54, 2010.
15. Razavi, S. F. and Y. Rahmat-Samii, "A new look at phaseless planar near-field measurements: limitations, simulations, measurements, and a hybrid solution," *IEEE Antennas and Propagation Magazine*, Vol. 49, No. 2, 170–178, 2007.
16. Fridén, J. and H. Isaksson, "Robust phase-retrieval for quick SAR assessment using dual plane amplitude-only data," *EMB 07 Computational Electromagnetics Methods and Applications Conference proceedings*, Lund, Sweden, 2007.
17. Fienup, J. R., "Phase retrieval algorithms: a comparison," *Applied Optics*, Vol. 21, No. 15, 2758–2769, 1982.
18. Roczniak, A., E. M. Petriu, and G. I. Costache, "3-D electromagnetic field modeling based on near field measurements," *Quality Measurements: The Indispensable Bridge between Theory and Reality (No Measurements? No Science!) Joint Conference — 1996: IEEE Instrumentation and Measurement Technology Conference and IMEKO Technical Committee 7. Conference Proceedings (Cat. No. 96CH35936)*, Vol. 2, 1124–1127, 1996.
19. Johansson, M., H.-S. Lui, and M. Persson, "Performance evaluation of phase-angle gradient method for phase retrieval based on low-frequency amplitude-only near-field data," *Progress In Electromagnetics Research B*, Vol. 25, 113–130, 2010.
20. Takenaka, T., T. Tanaka, H. Harada, and S. He, "FDTD approach to time-domain inverse scattering problem for stratified lossy media," *Microwave and Optical Technology Letters*, Vol. 16, No. 5, 292–296, 1997.
21. Gustafsson. M. and S. He, "An optimization approach to two-dimensional time domain electromagnetic inverse problems," *Radio Science*, Vol. 35, No. 2, 525–536, 2000.
22. Fhager, A. and M. Persson, "Comparison of two image reconstruction algorithms for microwave tomography," *Radio Science*, Vol. 40, No. RS3017, 2005.
23. Fhager, A., S. K. Padhi, and J. Howard, "3D image reconstruction in microwave tomography using an efficient FDTD model," *IEEE Antennas and Wireless Propagation Letters*, Vol. 8, 1353–1356, 2009.