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ANALYSIS OF CONICAL DIFFRACTION BY CURVED STRIP GRATINGS BY MEANS OF THE C-METHOD AND THE COMBINED BOUNDARY CONDITIONS METHOD

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Abstract—A rigorous modal theory of conical diffraction from curved strip gratings is presented. In this approach, the C-method with adaptive spatial resolution is used in conjunction with the combined boundary conditions. The method is successfully validated by comparison with a case where the solution can also be obtained in the Cartesian coordinate system.

1. INTRODUCTION

Strip gratings can model such devices as photolithographic masks or frequency selective surfaces either in the optical or in the microwave domain. Likewise, by deposition of a periodic strip at the surface of a dielectric or a metal one obtains a selective surface waveguide

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or even in certain conditions new materials with negative refractive index, the so called metamaterials. This last application has renewed the interest for numerical modeling of strip gratings. The problem of the diffraction of electromagnetic waves by strip gratings has been extensively studied in the past. A possible way to obtain the solution is to express the fields in terms of the Rayleigh expansions above and below the strips and to apply the combined boundary conditions method (C.B.M) introduced in [1]. The key feature of this differential approach is that it combines the continuity equations of the electric and magnetic fields in a unified equation that holds over one full period. The advantage of this method is its simplicity due to the use of Fourier series. Furthermore it offers the numerical possibility to easily mix strip gratings and other gratings provided that they share the same periodicity and they are analysed with any method that uses Fourier expansions. As an example, in a recent paper, Guizal and Granet [2] treated the problem of diffraction from curved strip gratings by using simultaneously the C method and the combined boundary conditions However the main drawback of most Fourier based methods is that they are not able to describe efficiently electromagnetic fields with sharp variations. As a consequence, convergence is achieved with rather large matrices. Here, the tangential component of the field that points toward the axis of periodicity is singular at the edge of the strips! Of course, such problems can be overcome by intricate mathematics but also by the very simple technic of adaptive spatial resolution. By using a non-uniform sampling scheme that places more points around the edge of the strip we have shown that the convergence speed was dramatically improved. [3] Our purpose in the present paper is to further demonstrate the versatility, the effectiveness and the complementarity of C-method, adaptive spatial resolution and combined boundary conditions. As a canonical case we will analyse one dimensional curved gratings illuminated under conical incidence. Such a structure may model open resonators as well as groove shape waveguides.

2. PRESENTATION OF THE PROBLEMS

2.1. Geometry and Excitation

A typical structure is depicted in Fig. 1. It consists of a one dimensional grating, periodic with period d, separating two dielectric homogeneous and isotropic media and over which is deposited an infinitely thin perfectly conducting grating of the same shape and period. The surface of the grating is invariant along the y direction, and described by the function $z = a(x)$. The part of the grating covered by the strip is

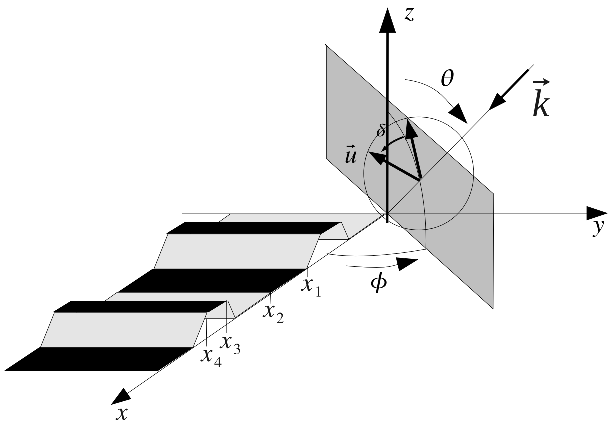

Figure 1. Geometry of the problem.

within the interval $\Omega_1 = [x_1, x_2] \cup [x_3, x_4]$. The grating is illuminated from the upper medium by a monochromatic plane wave that is linearly polarized and has a vacuum wavelength λ and an angular frequency ω. The $\exp(i\omega t)$ time dependance is assumed and will be omitted throughout this paper. The components of the wave vector $\mathbf{k}$ and those of the unit-amplitude electric-field vector $\hat{\mathbf{u}}$ are respectively:

$$\mathbf{k} = \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix} = \begin{bmatrix} -k \sin\theta \cos\phi \\ -k \sin\theta \sin\phi \\ -k \cos\theta \end{bmatrix} \tag{1}$$

with $k = 2\pi/\lambda = \omega\sqrt{\mu_0 \epsilon_0}$, where ϵ_0 and μ_0 are the permittivity and the permeability of vacuum respectively; and:

$$\hat{\mathbf{u}} = \cos\delta \begin{bmatrix} \cos\theta \cos\phi \\ \cos\theta \sin\phi \\ \sin\theta \end{bmatrix} + \sin\delta \begin{bmatrix} -\sin\phi \\ \cos\phi \\ 0 \end{bmatrix} \tag{2}$$

where δ represents the angle between the electric-field vector and the plane of incidence. θ is the inclination angle and ϕ the azimuth angle. The unknowns of the problem are the reflected and transmitted field. The ingredients of the method that we use to solve the present problem have already been presented elsewhere [4, 5] but never combined together in the context of strip gratings. Before going into details, we briefly recall the two main ideas behind our formalism.

3. METHODS

3.1. Coordinate Transformation (C-method)

Any numerical method aimed at solving Maxwell's equations is all the more efficient since it is able to fit the geometry of the problem. One way to do so is to use a coordinate transformation. The C-method that was born in the eighties is precisely one such method. Following, the C-method, we introduce a new coordinate system (u, v, w) deduced from the Cartesian coordinate system by the relations:

$$x = u, \quad y = v, \quad z = w + a(u) \tag{3}$$

hence, the surface profile of the grating becomes the coordinate surface $w = 0$. Then, we have to write Maxwell's equation in this new coordinate system. For that purpose, we use Post's formalism [6] in which Maxwell's equations remain invariant whatever the coordinate system and constitutive relations take into account the geometry through the metric tensor. In a homogeneous and isotropic medium with complex refractive index ν it can be shown that the complex amplitudes E_n and H_n of the covariant components of the electric and the magnetic fields are linked by the following Maxwell's equations:

$$\left\{\begin{array}{l} \xi^{lmn}\partial_m E_n = -i\omega\mu^{lm}H_m \\ \\ \xi^{lmn}\partial_m H_n = i\omega\varepsilon^{lm}E_m \end{array}\right. \tag{4}$$

∂_m denotes the derivation operator with respect to variable x^m. ξ^{lmn} is the Levi-Civita indicator:

$$\xi^{lmn} = \left\{\begin{array}{ll} +1 & \text{if } (lmn) \text{ is an even permutation of } (uvw) \\ -1 & \text{if } (lmn) \text{ is an odd permutation of } (uvw) \\ 0 & \text{otherwise} \end{array}\right. \tag{5}$$

μ^{mn} and ϵ^{mn} are respectively the components of the permeability and the permittivity tensor linked to the contravariant components g^{mn} of the metric tensor by

$$\mu^{mn} = \mu_0\sqrt{g}g^{mn} \tag{6}$$

$$\epsilon^{mn} = \nu^2\epsilon_0\sqrt{g}g^{mn} \tag{7}$$

where g is the determinant of the matrix formed by the covariant components of the metric tensor.

3.2. Combined Boundary Conditions

The boundary conditions at $w = 0$ impose that

- the tangential components of the electric field must be continuous over a whole period.
- the tangential components of the electric field must be null over the strip and
- the tangential components of the magnetic field must be continuous over the complementary of the strips.

The main idea of the combined boundary conditions method is to combine these last two conditions in a single one that is valid over a whole period. The benefit is that one can use a basis defined over the whole period to project the relations deduced from the boundary conditions.

4. FORMULATION OF THE PARAMETRIC C METHOD

4.1. New Coordinate System

The electromagnetic problem reduces in solving a propagation equation and writing boundary conditions. Numerically speaking, we use the method of moments with pseudo-periodic functions as expansion functions and periodic functions as projection functions. In grating theory, many popular methods rely on a similar approach. However, it is well known that Fourier basis are not fitted to represent singular or discontinuous functions because a huge number of coefficients is needed. Unfortunately, in the case of strip gratings some components of the field are singular at the edges of the strip. One way to preserve simplicity of the algorithms without facing slow convergence or even instability is to seek a change of coordinate such that spatial resolution is increased around the singularities. This can be achieved by some more sophisticated function $x(u)$ than the trivial one $x = u$. It follows that we get a parametric representation of the profile [7, 8]. A given point M on the profile previously located by coordinates ($x = u$, $z = a(u)$) is now located by coordinates ($x = x(u)$, $z = a \circ x(u)$). The extremites of strips define transition points where spatial resolution has to be increased. Let us present the coordinate x as a function of u and denote the transition points by x_p in x-space and by u_p in u-space. Between transitions p and $p-1$, we use the function $x_p(u)$ for the mapping between spaces:

$$x_p(u) = x_{p-1}(u) + \frac{x_p - x_{p-1}}{u_p - u_{p-1}} u - \frac{u_p - u_{p-1}}{2\pi} \sin \frac{2\pi(u - u_p)}{u_p - u_{p-1}} \qquad (8)$$

So, now the new coordinate system (u, v, w) is such that

$$x = x_p(u) u \in [u_{p-1}, u_p], \quad y = v, \quad z = w + z(u) \qquad (9)$$

with

$$z(u) = a \circ x(u) \tag{10}$$

Let us denote $\dot{x}$ and $\dot{z}$ the derivative of the functions $x(u)$ and $z(u)$ respectively. We have:

$$\sqrt{g}\,[g^{mn}] = \begin{bmatrix} (\dot{x})^{-1} & 0 & -(\dot{x})^{-1}\dot{z} \\ 0 & \dot{x} & 0 \\ -\dot{z}\,(\dot{x})^{-1} & 0 & \dot{x} + \dot{z}\,(\dot{x})^{-1}\dot{z} \end{bmatrix} \tag{11}$$

The above change of coordinates introduces a metric factor $\dot{x} = \frac{dx}{du}$ along the x direction. At the edges of a strip — $u = u_p$ and $u = u_{p+1}$ — where a singularity of the field occurs, the metric factor is null. Therefore spatial resolution is infinite and an accurate representation of the field at these points is possible.

4.2. TE_w and TM_w Modal Expansions

In this section, assuming an homogeneous medium with a refractive index ν, we derive the eigenvalue equation from which the solution is obtained. First, thanks to the invariability of the problem along the w direction, the components with subscripts u and v can be expressed in terms of the longitudinal components E_w et H_w:

$$\left(k^2\nu^2 + \partial_w^2\right) E_u = \left\{\left(-k^2 + \partial_w\partial_v\right) E_z - ik\mu^{vv}\partial_2 H_z\right\} \tag{12}$$

$$\left(k^2\nu^2 + \partial_w^2\right) H_v = \left\{\left(-ik\partial_w\epsilon^{uw} - ik\epsilon^{uu}\partial_u\right) E_z + \partial_w\partial_v H_z\right\} \tag{13}$$

$$\left(k^2\nu^2 + \partial_w^2\right) H_u = \left\{\left(-k^2\nu^2 + \partial_w\partial_u\right) H_z + ik\epsilon^{vv}\partial_v E_w\right\} \tag{14}$$

$$\left(k^2\nu^2 + \partial_w^2\right) E_v = \left\{\left(ik\partial_w\mu^{uw} - ik\mu^{uu}\partial_u\right) H_w + \partial_w\partial_v E_w\right\} \tag{15}$$

In the above equations, we have renormalized the magnetic field. From now on, H_l, $l \in u, v, w$ designates the covariant components of $Z_0\mathbf{H}$, where Z_0 is the vacuum impedance instead of those of $\mathbf{H}$. Second, provided that the medium is homogeneous it can be shown that the longitudinal components E_w and H_w obey the same scalar wave equation:

$$\partial_n\left(\sqrt{g}g^{n\,m}\partial_m\Phi\right) + k^2\nu^2\sqrt{g}\Phi = 0 \tag{16}$$

Thus, provided that the coefficients of Maxwell's equation are independent of one coordinate, w in the present case, an arbitrary field in a homogeneous source free region can be expressed as a sum of a TE_w field and TM_w field that correspond to $E_w = 0$ and $H_w = 0$ respectively:

$$\Phi = A_{TE}\Phi_{TE} + A_{TM}\Phi_{TM} \tag{17}$$

where Φ is a vector whose components are the functions E_u, H_v, H_u, and E_v. Some additional calculus has to be done to deal with the special case where $k^2\nu^2+\partial_w^2 = 0$. This case that corresponds to normal incidence is such that both E_w and H_w are equal to zero. Hence, the determination of the electromagnetic field amounts to solving Eq. (16) and deducing the TE_w and TM_w vector field according to:

$$\left(k^2\nu^2+\partial_w^2\right)\Phi_{TE_w}=\begin{pmatrix}-ik\mu^{vv}\partial_w\\ \partial_w\partial_v\\ -k^2\nu^2+\partial_w\partial_u\\ ik\partial_w\mu^{uw}-ik\mu^{uu}\partial_u\end{pmatrix}\Phi \quad \text{if} \quad k^2\nu^2+\partial_w^2\neq 0 \tag{18}$$

$$\Phi_{TE_w}=\begin{pmatrix}0\\ 0\\ \dfrac{-i}{k}\dfrac{1}{\mu^{11}}\partial_w\\ 1\end{pmatrix}\Phi \quad \text{if} \quad k^2\nu^2+\partial_w^2=0 \tag{19}$$

$$\left(k^2\nu^2+\partial_w^2\right)\Phi_{TM_w}=\begin{pmatrix}-k^2+\partial_w\partial_v\\ -ik\partial_w\epsilon^{uw}-ik\epsilon^{uu}\partial_u\\ ik\epsilon^{vv}\partial_v\\ \partial_w\partial_v\end{pmatrix}\Phi \quad \text{if} \quad k^2\nu^2+\partial_w^2\neq 0 \tag{20}$$

$$\Phi_{TM_w}=\begin{pmatrix}\dfrac{i}{k}\dfrac{1}{\epsilon^{11}}\partial_w\\ 1\\ 0\\ 0\end{pmatrix}\Phi \quad \text{if} \quad k^2\nu^2+\partial_w^2=0 \tag{21}$$

The propagation equation Eq. (16) can be split into two first order coupled differential equations:

$$\partial_w\begin{bmatrix}\partial_u\sqrt{g}g^{uw}+\sqrt{g}g^{wu}\partial u & \sqrt{g}g^{ww}\\ I & 0\end{bmatrix}\begin{bmatrix}\Phi\\ \partial_w\Phi\end{bmatrix}=\begin{bmatrix}k^2\nu^2+\partial_u^2+\partial_v^2 & 0\\ 0 & I\end{bmatrix}\begin{bmatrix}\Phi\\ \partial_w\Phi\end{bmatrix} \tag{22}$$

In the above matrix operator equation, the coefficients are independent of the w variable. Moreover they depend on v by the ∂_v^2 factor. Therefore Φ can be written as:

$$\Phi(u,v,w)=\exp(-ik\beta_0 v)\exp(-ik\gamma w)\Psi(u) \tag{23}$$

where β_0 is imposed by the incident plane wave: $\beta_0 = \sin\theta\sin\phi$. Eq. (22) is a generalised eigenvector equations whose solutions depend on the boundary conditions in the u direction.

5. NUMERICAL SOLUTION

The numerical solution of Eq. (22) is identical in every respect to that of problems based on C-method approach. We have first to associate a matrix to the matrix operator equation and then numerically search its eigenvalues and eigenvectors. For that purpose the field is expanded into Floquet harmonics:

$$\Psi(u) = \sum_{q=-M}^{q=+M} \Psi_q \exp\left(-ik\alpha_0 u + q\frac{2\pi}{d}\right) \tag{24}$$

where α_0 like β_0 is imposed by the incident plane wave: $\alpha_0 = \sin\theta\cos\phi$ and M is the truncation number. Then Eq. (22) is projected onto exponential functions $\exp(-i\frac{2\pi pu}{d})p \in [-M, M]$. These two steps allow to buid the following generalized matrix eigensystem:

$$\gamma\mathbf{A}\begin{bmatrix}\mathbf{\Psi}\\ \dot{\mathbf{\Psi}}\end{bmatrix} = \mathbf{B}\begin{bmatrix}\mathbf{\Psi}\\ \dot{\mathbf{\Psi}}\end{bmatrix} \tag{25}$$

with

$$\mathbf{A} = \begin{bmatrix}-\alpha\dot{\mathbf{z}}\dot{\mathbf{x}}^{-\mathbf{1}} - \dot{\mathbf{x}}^{-\mathbf{1}}\dot{\mathbf{z}}\alpha & \dot{\mathbf{x}} + \dot{\mathbf{z}}\dot{\mathbf{x}}^{-\mathbf{1}}\dot{\mathbf{z}}\\ \mathbf{I} & \mathbf{0}\end{bmatrix} \tag{26}$$

and

$$\mathbf{B} = \begin{bmatrix}(\nu^2 - \beta_0^2)\dot{\mathbf{x}} - \alpha\dot{\mathbf{x}}^{-\mathbf{1}}\alpha & \mathbf{0}\\ \mathbf{0} & \mathbf{I}\end{bmatrix} \tag{27}$$

where $\dot{\mathbf{x}}$ and $\dot{\mathbf{z}}$ are toeplitz matrices formed by Fourier coefficients of functions $\dot{x}$ and $\dot{z}$ respectively, and $\alpha = \mathrm{diag}(\alpha_{\mathbf{q}})$. It is observed numerically that there are two sets of modes, the number of which are equal: those propagation or decaying in the positive v direction and those propagating or decaying in the opposite direction. We denote these modes by superscripts $+$ and $-$ respectively. In any one medium, the w dependence of an eigenmode is determined by function $\exp(-i\gamma_p w)$. The real eigenmodes have $\Im(\gamma_q) = 0$ and are therefore forward modes if $\Re(\gamma_q) > 0$ or backward modes if $\Re(\gamma_q) < 0$. The complex eigenmode decay from surface $w = 0$ according to $\exp(-\Im(\gamma_q w))$. Thus, they decay forward if $\Im(\gamma_q) < 0$ or backward if $\Im(\gamma_q) > 0$. Furthermore, it has been shown numerically and anlytically, that, as the truncation number increases, the computed real eigenvalues converge to the real Rayleigh eigenvalues $\pm\gamma_q^R$.

$$\lim_{M\to\infty} \pm\gamma_q^M = \pm\gamma_q^R \tag{28}$$

$$\pm\gamma_q^M \approx \pm\gamma_q^R = \pm\sqrt{\nu^2 - \beta_0^2 - \alpha_q^2} \quad \text{and} \quad \nu^2 - \beta_0^2 - \alpha_q^2 > 0 \tag{29}$$

In the above relation, we have added an extra subscript M to indicate the truncation dependance. Indeed, the truncation order M has to be chosen large enough so that the computed real eigenvectors coincide with a great accuracy with their Rayleigh counterpart. In that case, provided that the eigenvalues are not degenerated, up to a multiplicative constant coefficient, the associated computed eigenvectors tend to the corresponding plane waves expressed in the new ccordinate system (u, v, w).

$$\lim_{M\to\infty} \Psi_q^{\pm(M)} = \Psi_q^R \tag{30}$$

$$\begin{aligned}\Psi_q^R &= \exp(\pm ik\gamma_q^R z(u))\exp(-ik\alpha_q x(u))\exp(-ik\beta_0 v)\\ &\quad \text{and} \quad \nu^2-\beta_0^2-\alpha_q^2>0\end{aligned} \tag{31}$$

As Chandezon et al. [4] did, we replace, in the decomposition of Ψ on the eigenvectors of the $\mathbf{B^{-1}A}$ matrix eigenvectors, the computed eigenvectors associated with real eigenvalues by the correponding truncated Rayleigh eigenvector whose components can be easily calculated. Let us consider any real Rayleigh eigen valalue γ_q^R and its associated plane wave Ψ_q^R. We know that this latter is pseudo periodic:

$$\Psi_q^R(u+d) = \exp(-ik\alpha_0 d)\Psi_q^R(u)$$

Hence:

$$\exp(ik\alpha_0 u)\Psi_q^R = \sum \Psi_{mq}^R \exp\left(\frac{-i2\pi mu}{d}\right) \tag{32}$$

with:

$$\Psi_{mq} = \frac{1}{d}\int_0^d \exp(ik\alpha_0 u)\Psi_q^R \exp\left(\frac{i2\pi qu}{d}\right) du \tag{33}$$

5.1. TE_w and TM_w Vectors

In this section we simply give the matrix form of the TE_w and TM_w field vectors

$$\mathbf{\Psi_{TE_w}^{\pm}} = \begin{bmatrix} -\alpha_\mathbf{p}\mathbf{\Psi_q^{\pm}N_q^{\pm}} + \nu^\mathbf{2}\mathbf{\dot{z}\Psi_q^{\pm}D_q^{\pm}} \\ \mathbf{\dot{x}^{-1}}\left(\mathbf{\dot{z}\Psi_q^{\pm}D_q^{\pm}} - \alpha_\mathbf{p}\mathbf{\Psi_q^{\pm}N_q^{\pm}}\right) \\ \beta_0\mathbf{\dot{x}\Psi_q^{\pm}N_q^{\pm}} \\ -\beta_0\mathbf{\Psi_q^{\pm}N_q^{\pm}} \end{bmatrix} \quad \text{if} \quad \nu^2-\gamma_q^{\pm}\neq 0 \tag{34}$$

$$\mathbf{\Psi_{TEw}} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mp\nu\mathbf{\dot{x}\Psi_q^{\pm}} \\ \Psi_q^{\pm} \end{bmatrix} \quad \text{if} \quad \gamma_q^{\pm} = \pm\nu \tag{35}$$

$$\mathbf{\Psi_{TM_w}^{\pm}} = \begin{bmatrix} -\beta_0\mathbf{\dot{x}\Psi_q^{\pm}N_q^{\pm}} \\ -\beta_0\mathbf{\Psi_q^{\pm}N_q^{\pm}} \\ -\alpha_\mathbf{p}\mathbf{\Psi_q^{\pm}N_q^{\pm}} + \nu^\mathbf{2}\mathbf{\dot{z}\Psi_q^{\pm}D_q^{\pm}} \\ -\mathbf{\dot{x}^{-1}}\left(\mathbf{\dot{z}\Psi_q^{\pm}D_q^{\pm}} - \alpha_\mathbf{p}\mathbf{\Psi_q^{\pm}N_q^{\pm}}\right) \end{bmatrix} \quad \text{if} \quad \nu^2-\gamma_q^{\pm}\neq 0 \tag{36}$$

$$\mathbf{\Psi_{TEw}} = \begin{bmatrix} \pm\frac{1}{\nu}\dot{\mathbf{x}}^{-1}\mathbf{\Psi_q^\pm} \\ \Psi_q^\pm \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \quad \text{if} \quad \gamma_q^\pm = \pm\nu \tag{37}$$

with

$$\mathbf{N_q^\pm} = \text{diag}\left(\frac{\gamma_\mathbf{q}^\pm}{\sqrt{(\nu^\mathbf{2} - \gamma_\mathbf{q}^{\pm\mathbf{2}})}}\right) \ \mathbf{D_q^\pm} = \text{diag}\left(\frac{\mathbf{1}}{\sqrt{(\nu^\mathbf{2} - \gamma_\mathbf{q}^{\pm\mathbf{2}})}}\right) \tag{38}$$

5.2. Summary

In matrix form, the field vector writes:

$$\begin{aligned}&\Phi(u,v,w)\\ =&\sum_{q=-M}^{q=M}\exp(-ik\gamma_q^\pm w)\exp(-ik\beta_0 v)\Big(A_{qTE}^\pm\mathbf{\Psi_{qTE}^\pm}+A_{qTM}^\pm\mathbf{\Psi_{qTM}^\pm}\Big)\end{aligned} \tag{39}$$

$$\Phi(u,v,w) = \exp(-i\beta_0 v)\mathbf{P_u^t}\left(\mathbf{\Psi_e}^\pm\mathbf{P_{ew}^\pm} + \mathbf{\Psi_h}^\pm\mathbf{P_{hw}^\pm}\right) \tag{40}$$

where the superscript t is for transposition and where $\mathbf{P_u}$, $\mathbf{P_{ew}}$ and $\mathbf{P_{hw}}$ are column matrices such that:

$$\mathbf{P_u} = \begin{bmatrix} \vdots \\ \exp(-ik\alpha_m u) \\ \vdots \end{bmatrix} \tag{41}$$

$$\mathbf{P_{ew}^\pm} = \begin{bmatrix} \vdots \\ A_{qe}^\pm \exp(-ik\gamma_q^\pm w) \\ \vdots \end{bmatrix} \tag{42}$$

$$\mathbf{P_{hw}^\pm} = \begin{bmatrix} \vdots \\ A_{qh}^\pm \exp(-ik\gamma_q^\pm w) \\ \vdots \end{bmatrix} \tag{43}$$

6. COMBINED BOUNDARY CONDITIONS

Let us consider that the perfectly conducting strips separate two regions labelled by subsripts 1 and 2. Then the boundary conditions write:

$$
\begin{aligned}
E_{1u} &= 0, \quad \forall u \in \Omega_1 && \text{(44a)}\\
E_{2u} &= 0, \quad \forall u \in \Omega_1 && \text{(44b)}\\
E_{1u} &= E_{2u}, \quad \forall u \in \bar{\Omega_1} && \text{(44c)}\\
E_{1v} &= 0, \quad \forall u \in \Omega_1 && \text{(44d)}\\
E_{2v} &= 0, \quad \forall u \in \Omega_1 && \text{(44e)}\\
E_{1v} &= E_{2v}, \quad \forall u \in \bar{\Omega_1} && \text{(44f)}\\
H_{1u} &= H_{2u}, \quad \forall u \in \bar{\Omega_1} && \text{(44g)}\\
H_{1v} &= H_{2v}, \quad \forall u \in \bar{\Omega_1} && \text{(44h)}
\end{aligned}
$$

In order to get relations valid on the interval $[0, d]$, we combine Eq. (44a) , Eq. (44b) and Eq. (44c): Taking into account that each component of the field is a linear combination of a TE_w and TM_w field, we get:

$$
\begin{aligned}
E_{1ue} + E_{1uh} &= E_{2ue} + E_{2uh}, \quad \forall x \in [0, d] && (45)\\
E_{1ve} + E_{1vh} &= E_{2ve} + E_{2vh}, \quad \forall x \in [0, d] && (46)
\end{aligned}
$$

Eq. (45) and Eq. (44c) can also be combined in a single equation that holds for every u in $[0, d]$

$$\chi(u)\begin{bmatrix} E_{2ue} + E_{2uh} \\ E_{2ve} + E_{2vh} \end{bmatrix} + \bar{\chi}(u)\begin{bmatrix} H_{1ve}+H_{1vh}-H_{2ve}-H_{2vh} \\ H_{1ue}+H_{1uh}-H_{2ve}-H_{2vh} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (47)$$

Equations are projected on the periodic functions $\exp(i\frac{2\pi u}{d})$ which allows to obtain the **S** matrix associated to the strip grating problem. χ and $\bar{\chi}$ are the toeplitz matrix formed by the fourier coefficient of the characteristic function $\chi(u)$ and $\bar{\chi(u)}$. The **S** matrix is defined by:

$$\begin{bmatrix} \mathbf{A_{1e}^+} \\ \mathbf{A_{1h}^+} \\ \mathbf{A_{2e}^-} \\ \mathbf{A_{2h}^-} \end{bmatrix} = \mathbf{S} \begin{bmatrix} \mathbf{A_{1e}^-} \\ \mathbf{A_{1h}^-} \\ \mathbf{A_{2e}^+} \\ \mathbf{A_{2h}^+} \end{bmatrix} \mathbf{S} = \mathbf{L^{-1}R} \quad (48)$$

$$\mathbf{L} = \begin{bmatrix} \mathbf{E_{1ue}^+} & \mathbf{E_{1uh}^+} & \mathbf{E_{2ue}^-} & \mathbf{E_{2uh}^-} \\ \bar{\chi}\mathbf{H_{1ve}^+}+ & \bar{\chi}\mathbf{H_{1vh}^+} & \bar{\chi}\mathbf{H_{2ve}^-}+\chi\mathbf{E_{2ue}^-} & \bar{\chi}\mathbf{H_{2vh}^-}+\chi\mathbf{E_{2uh}^-} \\ \mathbf{E_{1ve}^+} & \mathbf{E_{1vh}^+} & \mathbf{E_{2ve}^-} & \mathbf{E_{2vh}^-} \\ \bar{\chi}\mathbf{H_{1ue}^+} & \bar{\chi}\mathbf{H_{1uh}^+} & \bar{\chi}\mathbf{H_{2ue}^-}+\chi\mathbf{E_{2ve}^-} & \bar{\chi}\mathbf{H_{2uh}^-}+\chi\mathbf{E_{2vh}^-} \end{bmatrix} \quad (49)$$

$$\mathbf{R} = \begin{bmatrix} \mathbf{E_{1ue}^-} & \mathbf{E_{1uh}^-} & \mathbf{E_{2ue}^+} & \mathbf{E_{2uh}^+} \\ \bar{\chi}\mathbf{H_{1ve}^-} & \bar{\chi}\mathbf{H_{1vh}^-} & \bar{\chi}\mathbf{H_{2ve}^+}+\chi\mathbf{E_{2ue}^+} & \bar{\chi}\mathbf{H_{2vh}^+}++\chi\mathbf{E_{2uh}^+} \\ \mathbf{E_{1ve}^-} & \mathbf{E_{1vh}^-} & \mathbf{E_{2ve}^+} & \mathbf{E_{2vh}^+} \\ \bar{\chi}\mathbf{H_{1ue}^-} & \bar{\chi}\mathbf{H_{1uh}^-} & \bar{\chi}\mathbf{H_{2ue}^+}+\chi\mathbf{E_{2ve}^+} & \bar{\chi}\mathbf{H_{2uh}^+}+\chi\mathbf{E_{2ve}^+} \end{bmatrix} \quad (50)$$

7. NUMERICAL RESULTS

In order to check our method, we consider a trapezoidal profile with flat parts at top and bottom that separates two identical media and thus only represents a fictitious interface see Fig. 2. Here we consider that the two media are vacuum. By putting two strips on the flat parts we obtain a strip grating, the diffraction response of which may be calculated following two different approaches: method A: we consider it as a stack of planar strip gratings located at $h = 0$ and $h = z$ respectively and we stay in Cartesian coordinates; method B: we define a new coordinate system that matches the fictitious interface. We have chosen the following parameters $\theta = 28$, $\phi = 90$, $\lambda/d = 0.7$.

In Table 1 we give the reflected and transmitted efficiencies in order q obtained with both methods. err designates the defect in energy balance. We see the agreement is excellent. Moreover, we have checked that results obtained with method A and method B converge to each other as could be expected when the truncation number is

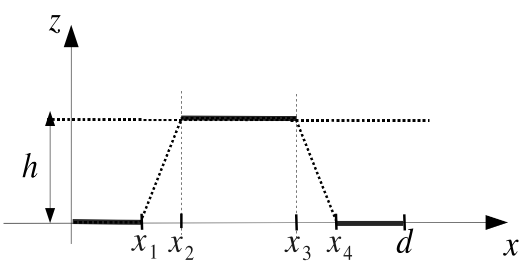

Figure 2. Geometry of the test case. $x_1 = d/6$, $x_2 = 2d/6$, $x_3 = 2d/3$, $x_4 = 5d/6$, $h/d = 1/3.46$. The grating is suspended in vacuum.

Table 1. Zeroth order and minus one order reflected efficiencies R_0, R_{-1} and transmitted efficiencies T_0, T_{-1}. calculated with two different methods for the grating of Fig. 2. $\lambda/d = 0.7$, $\theta = 28$, $\phi = 90$.

	A		B	
	TM_w	TE_w	TM_w	TE_w
R_0	0.1189	0.0856	0.1188	0.0854
R_{-1}	0.2724	0.2447	0.2724	0.2417
T_0	0.3067	0.2806	0.3052	0.2849
T_{-1}	0.0148	0.0721	0.0148	0.0727
err	4.5 (-7)	5.6(-6)	1.6(-3)	2.7(-5)

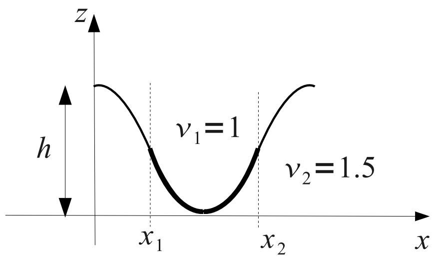

Figure 3. Geometry of a sinusoidal grating with strips. $x_1 = 0.2$, $x_2 = 0.8$, $d = 1$, $h = 0.2762$, $\nu_1 = 1$, $\nu_2 = 1.5$.

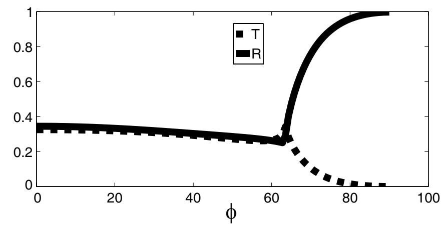

Figure 4. Zeroth-order reflectance (full line) and zeroth-order transmittance (dashed line) versus azimuth angle for the grating of Fig. 3 $\lambda = 1.1055$, $\theta = 20$.

increased. It should be emphasized that the above example is only given for validation purpose. Indeed,the solution of this particular case does not require the use of curvilinear coordinates. Besides, it can be observed that defect in energy balance is larger with method B than with method A. As a typical case of application of our method, we consider a sinusoidal grating covered by strips as shown in Fig. 3. Fig. 4 shows the zeroth-order reflectance and the zeroth-order transmittance versus the azimuth angle under TM_w polarisation.

8. CONCLUSION

We have combined the parametric C-method and the CBMC in the case of conical incidence. This approach allows the study of curved strip gratings of various shapes with low computational effort. Potential applications of the present extension include analysis of waveguide of various cross section. It should also be noted that the CBMC may be easily combined with any method that uses modal expansions. In our future work we will develop the theory of C-method and CBCM as applied to the analysis of two dimensional curved strip grating.

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