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COMBINED ESPRIT-ROOTMUSIC FOR DOA-DOD ESTIMATION IN POLARIMETRIC BISTATIC MIMO RADAR

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Abstract—In this paper, we investigate the exploitation of the polarimetric diversity signal properties in a bistatic polarimetric MIMO radar to improve the performance of joint estimation of direction of arrival (DOA) and direction of departure (DOD) of targets using combined ESPRIT-RootMUSIC technique. Numerical simulations are carried out to illustrate the performance of the proposed approach.

1. INTRODUCTION

The Multi-Input Multi-Output (MIMO) radar uses multiple antennas to transmit simultaneously several linearly orthogonal waveforms. It also uses multiple antennas to receive the reflected signals [1].

A bistatic MIMO radar scheme has been considered in [2], where a two-dimensional (2-D) spatial spectrum estimation technique based on the Capon method is developed. The proposed method needs an exhaustive search through all the 2-D space to find the DOA and DOD of the targets.

Exploiting the same MIMO radar configuration, Jin et al. [3] have proposed a joint DOA and DOD estimation approach based on the ESPRIT technique. A closed form of the target DOA and DOD is obtained and automatically paired. However, the number of targets which can be localized by this method is smaller than the number of receivers.

With the same architecture of the bistatic MIMO radar, another ESPRIT-based method is proposed in [5] by exploiting the invariance

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property of both the transmitting array and receiving array to estimate the DOA and DOD of targets. The pairing is not obtained automatically by this algorithm, but an additional pairing operation is given.

In [7], we have proposed an approach based on the combination of both the ESPRIT and Root-MUSIC methods to jointly estimate DOA-DOD of targets for a bistatic MIMO radar. This approach allows an automatic pairing between the target's DOA and DOD.

Polarization diversity have been proven to be useful in array signal processing and various types of radar systems [8, 9, 11, 12].

Recently, by exploiting the invariance property of both the transmitting array and the receiving array, a method based on the ESPRIT technique is proposed in [10], to jointly estimate the parameter DOA-DOD-polarization of target for a polarimetric bistatic MIMO radar, but a pairing operation is needed.

In this work, we propose an extension of the technique developed in [7] by exploiting the polarization diversity in reception antennas. Besides the polarisation diversity, this approach allows an automatic pairing between the target's DOA and DOD. Simulation results are provided to illustrate the performance of the proposed approach.

2. SIGNAL MODEL

The considered system is a bistatic polarimetric MIMO radar system with M closely spaced transmit antennas and N closely spaced receive antennas. Both of them are uniform linear arrays (ULA) and all the elements are omnidirectional. Δ_t and Δ_r are the inter-element spacing at the transmitter and the receiver, respectively. It is assumed that the Doppler frequencies have almost no effect on the orthogonality of the signals. Therefore, the variation of the phase within pulses caused by Doppler frequency can be ignored. The targets range is assumed to be much larger than the aperture of transmit array and receive array. Each element of the transmitter transmits orthogonal waveforms $\mathbf{r}_m$, $m = 1 \ldots M$. These signals are reflected by a target assumed at position $(\theta_\mathrm{r}, \theta_\mathrm{t})$ with θ_t and θ_r denoting DOD and DOA, respectively. For P targets located at the same range bin, the output of the entire matched filters at the receiver can be written as [1–3]

$$\mathbf{z}\left(t\right) = \mathbf{C}\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

with the pth column of the mixing matrix $\mathbf{C}$ given by

$$\mathbf{c}\left(\theta_r^{(p)}, \theta_t^{(p)}\right) = \mathbf{b}\left(\theta_t^{(p)}\right) \otimes \mathbf{a}\left(\theta_r^{(p)}\right)\Big|_{p=1,\ldots,P} \tag{2}$$

where

$$\mathbf{s}(t)=\begin{bmatrix} s_1(t) \\ \vdots \\ s_P(t) \end{bmatrix}=\begin{bmatrix} \alpha_1 e^{j2\pi f_{d1}t} \\ \vdots \\ \alpha_P e^{j2\pi f_{dP}t} \end{bmatrix} \tag{3}$$

$\mathbf{a}(\theta_r)=\left[1\ e^{j2\pi\frac{\triangle_r \sin(\theta_r)}{\lambda}}\ \ldots\ e^{j2\pi(N-1)\frac{\triangle_r \sin(\theta_r)}{\lambda}}\right]^T \in C^{N\times 1}$ is the receiver steering vector; $\mathbf{b}(\theta_t)=\left[1\ e^{j2\pi\frac{\triangle_t \sin(\theta_t)}{\lambda}}\ \ldots\ e^{j2\pi(M-1)\frac{\triangle_t \sin(\theta_t)}{\lambda}}\right]^T \in C^{M\times 1}$ is the transmitter steering vector; $\otimes$ is the Kronecker operator; α is the reflection coefficient depending on the target radar cross section; $e^{j2\pi f_d t}$ is due to the target Doppler shift; $\mathbf{n}(t)$ is the noise vector, whose elements are assumed to be independent, zero mean and complex Gaussian distributed.

2.1. Polarimetric Model

In order to exploit the polarization diversity in reception, we assume that the receiver is composed of N dual polarization antennas. For a transverse electromagnetic (TEM) wave incident on the array, the simplified horizontal and vertical polarization components of the target reflected signal can be expressed as [8, 9, 12]

$$\mathbf{z}_{po}(t)=\begin{bmatrix} \mathbf{C}^{(v)} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}^{(h)} \end{bmatrix}\begin{bmatrix} \mathbf{s}^{(v)}(t) \\ \mathbf{s}^{(h)}(t) \end{bmatrix}+\begin{bmatrix} \mathbf{n}^{(v)}(t) \\ \mathbf{n}^{(h)}(t) \end{bmatrix} \tag{4}$$

where v and h denote the vertical and horizontal polarizations, respectively, and $\mathbf{s}^{(v)}=[s_1(t)\cos(\gamma_1),\ldots,s_P(t)\cos(\gamma_P)]^T$, $\mathbf{s}^{(h)}=\left[s_1(t)\sin(\gamma_1)e^{j\xi_1},\ldots,s_P(t)\sin(\gamma_P)e^{j\xi_P}\right]^T$ are the targets signal vectors for polarizations v and h, respectively, the noise terms are assumed to be uncorrelated and complex Gaussian distributed with polarization angle $\gamma\in[0,\pi/2]$ and polarization phase difference $\xi\in[-\pi,\pi]$.

Assuming that the mixing matrix is polarization-independent, i.e., $\mathbf{C}^{(v)}(\theta_r,\theta_t)=\mathbf{C}^{(h)}(\theta_r,\theta_t)=\mathbf{C}(\theta_r,\theta_t)$, the extended signal model can be written as

$$\mathbf{z}_{po}(t)=\mathbf{F}\mathbf{s}(t)+\begin{bmatrix} \mathbf{n}^{(v)}(t) \\ \mathbf{n}^{(h)}(t) \end{bmatrix} \tag{5}$$

where $\mathbf{F}=\begin{bmatrix} \mathbf{C} & \mathbf{0} \\ \mathbf{0} & \mathbf{C} \end{bmatrix}\begin{bmatrix} \mathbf{G}^{(v)} \\ \mathbf{G}^{(h)} \end{bmatrix}$, and $\mathbf{G}^{(v)}$ and $\mathbf{G}^{(h)}$ are diagonal matrices with diagonal vectors $\mathbf{g}^{(v)}$ and $\mathbf{g}^{(h)}$ given by

$$\begin{aligned} \mathbf{g}^{(v)} &= [\ \cos(\gamma_1),\ \ldots\ ,\cos(\gamma_P)\ ] \\ \mathbf{g}^{(h)} &= [\ \sin(\gamma_1)e^{j\xi_1},\ \ldots\ ,\sin(\gamma_P)e^{j\xi_P}\ ] \end{aligned} \tag{6}$$

Then, the covariance matrix of the extended observation vector $\mathbf{z}_{po}$ can be expressed as

$$\mathbf{R}_p = \mathbf{F}\boldsymbol{\Gamma}_{\mathbf{s}}\mathbf{F}^H + \sigma^2 \mathbf{I}_{2MN} \tag{7}$$

where $\boldsymbol{\Gamma}_{\mathbf{s}}$ is the covariance matrix of $\mathbf{s}(t)$ and σ^2 is the noise power.

The directional vector for a single target $\mathbf{f}$ is given by

$$\mathbf{f}(\theta_r, \theta_t) = \begin{bmatrix} \mathbf{b}(\theta_t) \otimes \mathbf{a}(\theta_r)\cos(\gamma) \\ \mathbf{b}(\theta_t) \otimes \mathbf{a}(\theta_r)\sin(\gamma)e^{j\xi} \end{bmatrix} \tag{8}$$

3. POLARIMETRIC COMBINED ESPRIT-ROOTMUSIC

The MUSIC technique is based on the orthogonality between the noise subspace and the signal subspace. These subspaces can be obtained by the eigenvalues decomposition of the covariance matrix $\mathbf{R}_p \in \mathcal{C}^{2MN \times 2MN}$

$$\mathbf{R}_p = \mathbf{U}_{sp}\Gamma_{sp}\mathbf{U}_{sp}^H + \sigma^2 \mathbf{U}_{np}\mathbf{U}_{np}^H \tag{9}$$

The noise subspace $\mathbf{U}_{np}$ is orthogonal to the actual target directional vectors, i.e.,

$$\mathbf{f}(\theta_r, \theta_t)^H \mathbf{U}_{np}\mathbf{U}_{np}^H \mathbf{f}(\theta_r, \theta_t) = 0, \qquad \left.\begin{matrix} \theta_r = \theta_r^{(p)} \\ \theta_t = \theta_t^{(p)} \end{matrix}\right|_{p=1,\ldots,P} \tag{10}$$

where $\mathbf{f}$ can be written as

$$\begin{aligned} \mathbf{f}(\theta_r, \theta_t) &= \mathbf{B}(\theta_t) \begin{bmatrix} \mathbf{a}(\theta_r)\cos(\gamma) \\ \mathbf{a}(\theta_r)\sin(\gamma)e^{j\xi} \end{bmatrix} \\ \mathbf{B}(\theta_t) &= \begin{bmatrix} [\mathbf{b}(\theta_t) \otimes \mathbf{I}_N] & \mathbf{0} \\ \mathbf{0} & [\mathbf{b}(\theta_t) \otimes \mathbf{I}_N] \end{bmatrix} \end{aligned}$$

This expression allows to estimate separately the directions. Therefore, the proposed approach can be subdivided in two steps. In the first step, we determine only the DOD by the ESPRIT method and in the second step, for each estimated DOD, the estimated direction matrix $\mathbf{B}(\hat{\theta}_t)$ is substituted in (10) and the RootMUSIC technique is applied to estimate the DOA.

According to the principle of ESPRIT technique [5], to exploit the rotational invariance, in the transmitter array, we define the submatrices $\mathbf{C}_{t1} \in C^{(MN-N)\times P}$ and $\mathbf{C}_{t2} \in C^{(MN-N)\times P}$, related to the DOD, as

$$\begin{aligned} \mathbf{C}_{t1} &= \left[\mathbf{b}_1\left(\theta_t^{(1)}\right) \otimes \mathbf{a}\left(\theta_r^{(1)}\right), \ldots, \mathbf{b}_1\left(\theta_t^{(P)}\right) \otimes \mathbf{a}\left(\theta_r^{(P)}\right)\right] \\ \mathbf{C}_{t2} &= \left[\mathbf{b}_2\left(\theta_t^{(1)}\right) \otimes \mathbf{a}\left(\theta_r^{(1)}\right), \ldots, \mathbf{b}_2\left(\theta_t^{(P)}\right) \otimes \mathbf{a}\left(\theta_r^{(P)}\right)\right] \end{aligned} \tag{11}$$

with $\mathbf{b}_1(\theta_t^{(p)})$ and $\mathbf{b}_2(\theta_t^{(p)})$ are the first and the last $M-1$ elements of $\mathbf{b}(\theta_t^{(p)})$, respectively.

Then, it is easy to show

$$\mathbf{F}_{t2} = \mathbf{F}_{t1}\mathbf{\Phi}_t \tag{12}$$

where $\mathbf{F}_{t1(2)} = \begin{bmatrix} \mathbf{C}_{t1(2)} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_{t1(2)} \end{bmatrix} \begin{bmatrix} \mathbf{G}^{(v)} \\ \mathbf{G}^{(h)} \end{bmatrix}$ and $\mathbf{\Phi}_t$ is a diagonal matrix with its main diagonal elements given by $\rho_t^{(p)} = e^{j2\pi\frac{\triangle_t \sin(\theta_t^{(p)})}{\lambda}}$ for $p = 1, \ldots, P$.

The vectors of $\mathbf{U}_{sp}$ span the signal subspace, then there exists a non singular transformation matrix $\mathbf{T}$ satisfying

$$\mathbf{U}_{sp} = \mathbf{F}\mathbf{T} \tag{13}$$

Let $\mathbf{U}_{sp_{t1}}$ and $\mathbf{U}_{sp_{t2}}$ be the submatrices formed from $\mathbf{U}_{sp}$ in the same way as that $\mathbf{F}_{t1}$ and $\mathbf{F}_{t2}$ are formed from $\mathbf{F}$, we can write

$$\mathbf{U}_{sp_{t1}} = \mathbf{F}_{t1}\mathbf{T}, \quad \mathbf{U}_{sp_{t2}} = \mathbf{F}_{t2}\mathbf{T} \tag{14}$$

Then, the diagonal elements of $\mathbf{\Phi}_t$ are the eigenvalues $\omega_{p=1,\ldots,P}$ of the following matrix $\mathbf{\Omega}_t = \mathbf{T}^{-1}\mathbf{\Phi}_t\mathbf{T}$ which satisfies $\mathbf{U}_{sp_{t2}} = \mathbf{U}_{sp_{t1}}\mathbf{\Omega}_t$, therefore, the DOD can be estimated by

$$\hat{\theta}_t^{(p)} = \arcsin\left(\frac{\lambda}{2\pi\Delta_t}\arg\left(\omega_p\right)\right) \tag{15}$$

Therefore, for each estimated angle $\hat{\theta}_t^{(p)}$, if θ_r is the corresponding direction of an actual target, then the directional vector is orthogonal to the noise subspace which allows to rewrite the expression (10) as

$$\dot{\mathbf{g}}^H\mathbf{A}^T(1/z_r)\begin{bmatrix} \mathbf{\Pi}_{\text{n}11} & \mathbf{\Pi}_{\text{n}12} \\ \mathbf{\Pi}_{\text{n}21} & \mathbf{\Pi}_{\text{n}22} \end{bmatrix}\mathbf{A}(z_r)\dot{\mathbf{g}} = 0 \tag{16}$$

where $\mathbf{A}(z_r) = \begin{bmatrix} \mathbf{a}(z_r) & \mathbf{0} \\ \mathbf{0} & \mathbf{a}(z_r) \end{bmatrix}$, $z_r = e^{j2\pi\frac{\triangle_r \sin(\theta_r)}{\lambda}}$, $\dot{\mathbf{g}} = \begin{bmatrix} \cos(\gamma) \\ \sin(\gamma)e^{j\xi} \end{bmatrix}$, and $\begin{bmatrix} \mathbf{\Pi}_{\text{n}11} & \mathbf{\Pi}_{\text{n}12} \\ \mathbf{\Pi}_{\text{n}21} & \mathbf{\Pi}_{\text{n}22} \end{bmatrix} = \mathbf{B}(\hat{\theta}_t)^H\mathbf{U}_{np}\mathbf{U}_{np}^H\mathbf{B}(\hat{\theta}_t)$.

The set (16) can be solved by finding the roots of the following polynomial

$$det\left(\mathbf{A}^T(1/z_r)\begin{bmatrix} \mathbf{\Pi}_{\text{n}11} & \mathbf{\Pi}_{\text{n}12} \\ \mathbf{\Pi}_{\text{n}21} & \mathbf{\Pi}_{\text{n}22} \end{bmatrix}\mathbf{A}(z_r)\right) = 0 \tag{17}$$

which can be expressed by

$$\sum_{k=1}^{4N-3} q_k z^{k-1-2(N-1)} = 0 \tag{18}$$

The parameters q_k are calculated as

$$q_k = \sum_{i=\max[1,k-2N+2]}^{\min[2N-1,k]} \mathbf{Q}(i, k-i+1) \tag{19}$$

where $\mathbf{Q} = \mathbf{v}_{11}\mathbf{v}_{22}^T - \mathbf{v}_{12}\mathbf{v}_{21}^T$, and the elements of $\mathbf{v}_{ij}$ are given by

$$\mathbf{v}_{ij}(k) = \sum_{l=\max[1,1-k]}^{\min[N,N-k]} [\mathbf{\Pi}_{nij}]_{l,k+l} \Bigg|_{k=-(N-1),\ldots,(N-1)} \tag{20}$$

Therefore, the estimation of the angle θ_r which minimizes the projection of the directional vector on the noise subspace, is equivalent to finding the roots of the polynomial (18) for each given $\hat{\theta}_t$.

For each estimated DOD, the roots $\hat{z}_r$ inside and closest to the unitary circle of the polynomial (18) allow to estimate the corresponding DOA angle.

$$\hat{\theta}_r^{(p)} = \arcsin\left(\frac{\lambda}{2\pi\Delta_r} \arg\left(\hat{z}_r\right)\right) \tag{21}$$

We note that, for this proposed method the pairing is automatically obtained between the DOD and DOA angles.

This approach exploiting the polarimetric diversity in bistatic polarimetric MIMO radar is able to handle more targets than the classical techniques. The maximum number of targets can be localized by this approach is $2N(M-1)$.

4. SIMULATION RESULTS

In this section, we investigate the performance of the proposed approach. For all the simulations, the parameters of targets $((\alpha, f_d, \gamma, \xi)$ are randomly generated for each target.

Firstly, we consider a bistatic polarimetric MIMO radar composed of $M = 3$ transmitter elements and $N = 2$ elements in receiver array spaced by a half wavelength. The data length is $L = 500$ samples and the number of trials is $K = 200$. For $SNR = 20\,\text{dB}$, Figure 1 shows that the proposed approach is able to localize up to $P = 8$ targets.

Secondly, we set $M = 3$, $N = 4$, $L = 256$ and $K = 200$ for two targets $(P = 2)$ located at the angles $(\theta_r, \theta_t) = (35°, 60°)$ $(\theta_r, \theta_t) = (80°, 15°)$, Figures 2 and 3 show the Root Mean Square Error (RMSE) versus SNR for DOA estimation and DOD estimation, respectively, compared with the Cramer-Rao Bound (CRB).

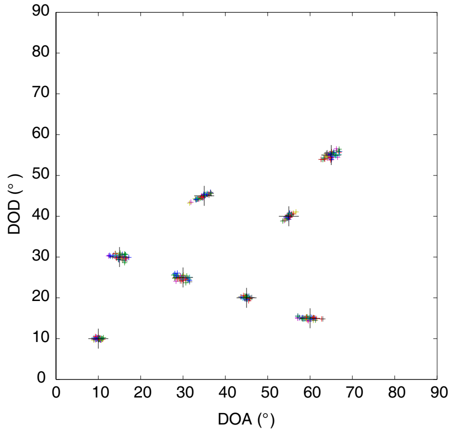

Figure 1. Joint DOD-DOA estimation of $P = 8$ targets by the proposed approach.

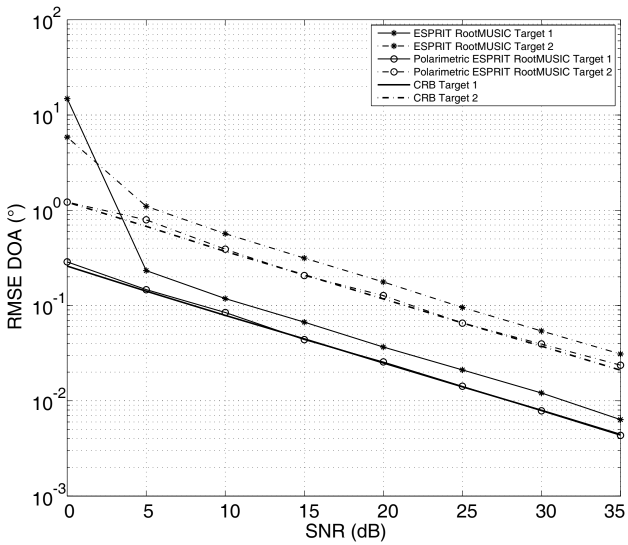

Figure 2. RMSE for DOA estimation versus SNR.

These figures present a comparison between the proposed Polarimetric ESPRIT-RootMUSIC approach and the ESPRIT-RootMUSIC developed in [7]. It can be seen that the curve corresponding to the Polarimetric ESPRIT-RootMUSIC approach is more closer to the CRB curve than the curve related to the ESPRIT-RootMUSIC method.

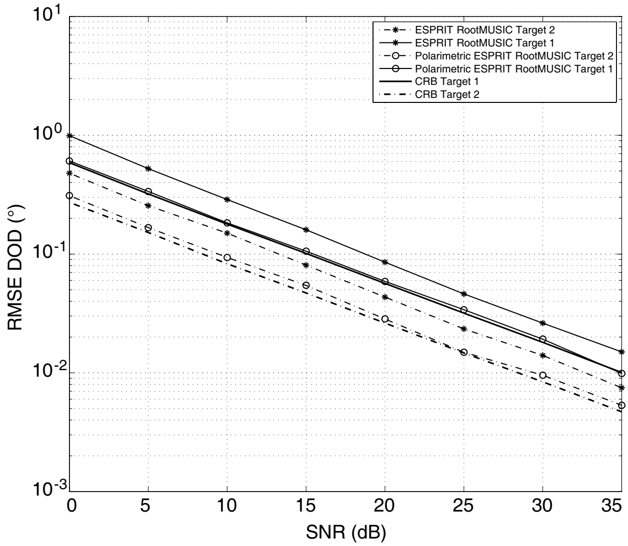

Figure 3. RMSE for DOD estimation versus SNR.

Therefore, the exploitation of the polarization diversity in reception improves considerably the estimation performance because the observations space is extended to $2MN \times 2MN$ from $MN \times MN$ which increases also the number of localizable targets and the resolution power.

5. CONCLUSION

We have exploited the additional degrees-of-freedom of the polarization diversity to improve the performances of the combined ESPRIT-RootMUSIC proposed in [7] for joint DOA-DOD estimation in a bistatic MIMO radar. The exploitation of the polarimetric diversity allows to handle more targets than the classical algorithms and improve the localization performances.

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