Progress In Electromagnetics Research Letters, Vol. 21, 177–186, 2011

NONRECIPROCAL PROPERTIES OF SURFACE PLASMON-POLARITONS AT THE INTERFACE BETWEEN TWO MAGNETIZED MEDIA: EXACT ANALYTICAL SOLUTIONS

V. Dmitriev and A. O. Silva

Electrical Engineering Department
Federal University of Para, Belem, Para, Brazil

Abstract—In this paper, we investigate surface plasmon-polariton propagation at the interface of metal and magnetooptic dielectric semi-infinite layers where both layers are magnetized. Using magnetic group theory, we calculate scattering matrices for the waveguide sections with three orientations of dc magnetic field. Solving analytically the wave equation for the Voigt configuration, we obtain exact dispersion relation for this waveguide. Numerical examples show that the nonreciprocal phase shift between forward and backward waves can be increased significantly as compared with the case where only the magnetooptic dielectric layer is magnetized.

1. INTRODUCTION

The diffraction limit of light can be overcome using surface-plasmon-polaritons (SPPs). SPPs are electromagnetic waves propagating along a metal-dielectric interface due to resonant interaction between photons and free electrons in the metal surface. The wavelength of an SPP could be far much less than the wavelength of light. As a consequence, the plasmonic waves can confine and enhance electromagnetic fields at nanometric scale. These attractive characteristics can serve for implementation of a new generation of optical components [1].

For development of all-optical networks, it is necessary to have components for control and manipulation of SPPs. Magnetooptic (MO) effects can serve for this purpose. One of the main problems in implementation of MO control and nonreciprocal components in optic

Received 19 January 2011, Accepted 7 March 2011, Scheduled 14 March 2011
Corresponding author: Anderson Oliveira Silva (aosilva@ufpa.br).

region is a small value of the Voigt parameter for existing magnetooptic materials. This parameter defining nonreciprocal properties of media is calculated as ratio of nondiagonal element of the permittivity tensor and diagonal one. With small value of the Voigt parameter, a possible way to increase nonreciprocal effects is to increase the volume of the nonreciprocal material interacting with electromagnetic waves.

A promising structure which can serve as a nonreciprocal waveguide is the Voigt configuration with magnetization in-plane of the metal-MO dielectric interface and perpendicular to the wave vector of SPP. SPP propagation in such structures was discussed in various papers (see, for example, [2, 3] and references in them) but in most of them magnetic properties of the metal media were not taken into account. However, magnetization of plasma at the metallic surface may lead also to significant MO activity.

In this paper we investigate a basic structure consisting of two semi-infinite layers where both a MO dielectric layer and a metal are magnetized, because, firstly, in practice magnetization of the MO material will lead to magnetization of the metal as well because the MO material and the metal are juxtaposed. Secondly, and most important, the analysis of properties of SPPs at MO material-metal interface, taking into account the effects of magnetization of the plasma at the metal surface, can give a more realistic evaluation of nonreciprocal properties of these structures.

Notice that our analysis is fulfilled using exact analytical methods which give a better physical insight into the problems. To the best of our knowledge, any analytical solution for SPPs propagating along two semi-infinite magnetized layers described by nonsymmetrical tensor permittivities has not been reported so far.

A general symmetry analysis of the waveguide with three orientations of dc magnetic fields presented below allows one to discuss a realizability of nonreciprocal effects without using Maxwell equations. We also solve the wave equation for the Voigt geometry and obtain the exact analytical dispersion relation for such waveguide. Finally, numerical examples illustrating the theory are given.

2. THEORY

2.1. Symmetry Analysis Using Magnetic Groups

The geometries of the problem for three orientations of dc magnetic field $\mathbf{B}_0$ are shown in Figure 1. The axis z is oriented normally to the interface xy of two semi-infinite regions and every region is described by a nonsymmetrical permittivity tensor. The direction of wave propagation is along the y-axis.

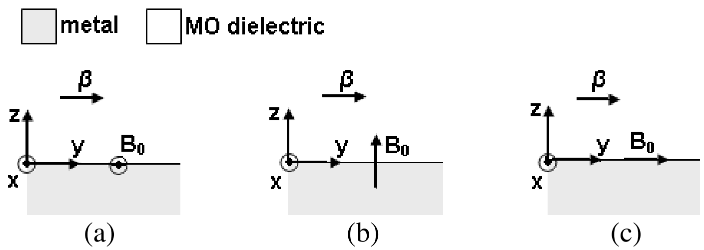

Figure 1. Interface between MO medium and metal: (a) Voigt geometry, (b) transversal magnetization, (c) Faraday geometry. β is the wave vector of SPP.

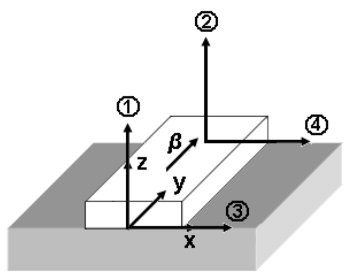

Figure 2. Four-port circuit theory model of SPP waveguides.

The structures are uniform in the x-direction. It should be noted that the following symmetry analysis is valid also for waveguides with a limited width $2a$ in the x-direction if the boundary conditions on the planes $x = \pm a$ are the same. The waveguides can also be finite in the z-direction but in this case there is no restriction on the symmetry of the boundary condition in the plane $z =$ const. An example of such structure with the port numbering is shown in Figure 2.

Depending on magnetization orientation, different eigenmodes can exist in the waveguides under investigation. The following symmetry analysis is valid for any type of the modes, in particular, for TM and hybrid ones.

In circuit theory, a section of the waveguide can be described as a four-port (see Figure 2). The 4×4 scattering matrix $[S]$ of this multiport relates the components of the incident and reflected waves in two orthogonal ports at the input and the output of the waveguide section as follows: $(V_1^r, V_2^r, V_3^r, V_4^r)^t = [S] \cdot (V_1^i, V_2^i, V_3^i, V_4^i)^t$ where t means transposition, r and i denote reflected and incident waves. The voltages V^i and V^r can be defined by E_z and E_x components of the corresponding electromagnetic waves.

Our problem is to calculate the structure of the matrix $[S]$ which follows from the physical symmetry of the structure. This symmetry is defined not only by geometry of the problem but includes also the Time reversal operator T [4].

Three possible orientation of dc magnetic field $\mathbf{B}_0$ are shown in Figure 1: the Voigt configuration with $\mathbf{B}_0||x$, Figure 1(a), the transversal magnetization with $\mathbf{B}_0||z$, Figure 1(b) and the Faraday configuration with $\mathbf{B}_0||y$, Figure 1(c). It is convenient to discuss the properties of the magnetized waveguides using the theory of magnetic groups.

For the Voigt configuration, the magnetic group of symmetry is $C_{2v}(C_s)$ (in Schoenflies notations [4]) which contains the following elements: e — identity; σ_x — plane of symmetry $x = 0$; $T\sigma_y$ — plane of symmetry $y = 0$ combined with the Time reversal T; TC_{2z} — rotation by π around the axis z also combined with T.

For the transversal magnetization, the group is $C_{2v}(C_2)$ with the following elements of symmetry: e — identity; C_{2z} — rotation by π around the axis z; $T\sigma_x$ — plane of symmetry $x = 0$ combined with the Time reversal T; $T\sigma_y$ — plane of symmetry $y = 0$ combined with T.

Finally, for the Faraday geometry, the magnetic group is $C_{2v}(C_s)$ as in case of the Voigt configuration but the elements of symmetry in this case are as follows: e — identity; σ_y — plane of symmetry $y = 0$; $T\sigma_x$ — plane of symmetry $x = 0$ combined with the Time reversal T; TC_{2z} — rotation by π around the axis z combined with T.

Notice, that in the above magnetic groups, the "pure" Time reversal operator is absent. Thus, all the waveguides in Figure 1 can be nonreciprocal. The elements of the groups corresponding to geometrical transformations are called unitary elements. The combined geometrical transformations and the Time reversal T are known as antiunitary elements.

Our method of calculation is based on the commutation relations for the scattering matrix $[S]$ and the 4×4 matrix representation $[R]$ of generators of the groups [4]. These commutation relations are $[R][S] = [S][R]$ and $[R][S] = [S]^t[R]$ for unitary and antiunitary elements of the corresponding magnetic group, respectively. It should be noted that the transposition of the matrix $[S]$ in the last commutation relation is a consequence of the presence of the Time reversal T in antiunitary elements.

The reflection σ_y (see Figure 2) interchanges the ports 1 and 2, and also the ports 3 and 4. The rotation by π around the axis z interchanges the ports 1 and 2. It also interchanges the ports 3 and 4 but with changing the sign. Therefore, the matrix representations $[R]$

for these elements can be written as follows:

$$[R]_{\sigma_y} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad [R]_{C_{2z}} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}. \tag{1}$$

In Table 1, we present two chosen generators for every magnetic group, the corresponding commutation relations and the calculated scattering matrices. These matrices allow one to analyze a possibility of realization of nonreciprocal effects in the waveguides: difference in phase, amplitude and polarization for the waves propagating in opposite directions.

Table 1. Symmetry description of SPP waveguides with different orientation of dc magnetic field and their scattering matrices.

Orientation of $\mathbf{B}_0$	Generators	Commutation relations	Scattering matrix
$\mathbf{B}_0 \parallel x$ Figure 1(a)	$T\sigma_y$, TC_{2z}	$[R]_{\sigma_y}[S] = [S]^t[R]_{\sigma_y}$ $[R]_{C_{2z}}[S] = [S]^t[R]_{C_{2z}}$	$\begin{pmatrix} S_{11} & S_{12} & 0 & 0 \\ S_{21} & S_{11} & 0 & 0 \\ 0 & 0 & S_{33} & S_{34} \\ 0 & 0 & S_{43} & S_{33} \end{pmatrix}$
$\mathbf{B}_0 \parallel z$ Figure 1(b)	$T\sigma_y$, C_{2z}	$[R]_{C_{2z}}[S] = [S][R]_{C_{2z}}$ $[R]_{\sigma_y}[S] = [S]^t[R]_{\sigma_y}$	$\begin{pmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{11} & -S_{14} & -S_{13} \\ -S_{13} & S_{14} & S_{33} & S_{34} \\ -S_{14} & S_{13} & S_{34} & S_{33} \end{pmatrix}$
$\mathbf{B}_0 \parallel y$ Figure 1(c)	σ_y, TC_{2z}	$[R]_{\sigma_y}[S] = [S][R]_{\sigma_y}$ $[R]_{C_{2z}}[S] = [S]^t[R]_{C_{2z}}$	$\begin{pmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{11} & S_{14} & S_{13} \\ -S_{13} & -S_{14} & S_{33} & S_{34} \\ -S_{14} & -S_{13} & S_{34} & S_{33} \end{pmatrix}$

In this paper, we are not interested in polarization effects. Our interest lies in nonreciprocal phase shift between port 1 and port 2. We can see from Table 1 that this effect, i.e., the condition $S_{21} \neq S_{12}$ can exist only in the Voigt geometry. Moreover, there is no coupling in this geometry between ports 1 and 2 from one side and ports 3 and 4 from the other side. Therefore, in the following we restrict ourselves by consideration of this simple case.

2.2. Dispersion Relation for SPPs in Voigt Geometry

When a medium of a multilayer nanoplasmonic waveguide exhibits MO activity, theoretical description of SPP modes is more complex as compared with nonmagnetic case because a coupling of different electromagnetic field components is possible. The nonzero off-diagonal elements of the antisymmetric permittivity tensor $[\varepsilon]$ stipulate this coupling.

For magnetization along the x-direction, the tensors $[\varepsilon]_d$ for a MO dielectric and $[\varepsilon]_m$ for a metal have the same structure:

$$[\varepsilon]_d = \begin{pmatrix} \varepsilon'_d & 0 & 0 \\ 0 & \varepsilon_d & -j\zeta_d \\ 0 & j\zeta_d & \varepsilon_d \end{pmatrix}, \quad [\varepsilon]_m = \begin{pmatrix} \varepsilon'_m & 0 & 0 \\ 0 & \varepsilon_m & -j\zeta_m \\ 0 & j\zeta_m & \varepsilon_m \end{pmatrix}, \tag{2}$$

where j is the imaginary unit.

The Helmholtz equation for the longitudinal electric field component $E_y^{d,m}$ for dielectric (denoted by d) and metal (designated by m) media with harmonic variation $\exp(-j\beta y)$ along the axis of propagation (the factor $\exp(j\omega t)$ is suppressed) is written as follows:

$$\frac{\partial^2 E_y^{d,m}}{\partial z^2} + \left(k_0^2 \varepsilon_{\mathit{eff}}^{d,m} - \beta^2\right) E_y^{d,m} = 0, \tag{3}$$

where $k_0 = \omega/c$ is the wavenumber in free space, c being the velocity of light in vacuum, β the complex wave number, and

$$\varepsilon_{\mathit{eff}}^{d,m} = \frac{\varepsilon_{d,m}^2 - \zeta_{d,m}^2}{\varepsilon_{d,m}}. \tag{4}$$

The Helmholtz Equation (3) has the same structure as the corresponding equation for the nonmagnetized media. Thus, the Voigt configuration allows propagation of TM SPP mode with the components E_y, E_z and H_x [2]. But now the wave number β depends not only on the diagonal elements ϵ_d and ϵ_m of the tensors but also on the off-diagonal ones ζ_d and ζ_m.

The solution of Equation (3) for semi-infinite MO dielectric and semi-infinite metal can be written as follows:

$$\begin{aligned} E_y^d(z) &= A \exp\left(-\sqrt{\beta^2 - k_0^2 \varepsilon_{\mathit{eff}}^d}\, z\right), \quad z \geq 0, \\ E_y^m(z) &= B \exp\left(\sqrt{\beta^2 - k_0^2 \varepsilon_{\mathit{eff}}^m}\, z\right), \quad z \leq 0, \end{aligned} \tag{5}$$

where A and B are arbitrary constants. The boundary conditions for our problem are continuity of the electric field E_y and magnetic field H_x at the interface $z = 0$. Using these conditions, after some mathematical manipulations we come to the following biquadratic equation for the wave number β:

$$\begin{aligned} &\left[4(\varepsilon_d \varepsilon_m)^2 - (U - 2\zeta_d \zeta_m + V)^2\right] \beta^4 \\ &-2(\varepsilon_d V + \varepsilon_m U)[2\varepsilon_d \varepsilon_m - (U - 2\zeta_d \zeta_m + V)] k_0^2 \beta^2 \\ &+ \left[4\varepsilon_d \varepsilon_m U V - (\varepsilon_d V + \varepsilon_m U)^2\right] k_0^4 = 0, \end{aligned} \tag{6}$$

where

$$U = \varepsilon_m^2 + \zeta_m^2, \quad V = \varepsilon_d^2 + \zeta_d^2.$$

Solution of (6) gives the desired dispersion relation:

$$\beta = \pm k_0 \sqrt{\frac{(\varepsilon_d V + \varepsilon_m U)[2\varepsilon_d \varepsilon_m - (V - 2\zeta_d \zeta_m + U)] \pm \sqrt{M}}{4(\varepsilon_d \varepsilon_m)^2 - (V - 2\zeta_d \zeta_m + U)^2}}, \tag{7}$$

where

$$\begin{aligned} M &= (\varepsilon_d V + \varepsilon_m U)^2 [2\varepsilon_d \varepsilon_m - (V - 2\zeta_d \zeta_m + U)]^2 \\ &\quad -[4(\varepsilon_d \varepsilon_m)^2 - (V - 2\zeta_d \zeta_m + U)^2][4\varepsilon_d \varepsilon_m VU - (\varepsilon_d V + \varepsilon_m U)^2]. \end{aligned}$$

In one limiting case when only MO material is magnetized and metal is described by scalar parameters, i.e., $\zeta_m \to 0$, Equation (7) is transformed to that obtained in [2]:

$$\beta = \pm k_0 \sqrt{\frac{\varepsilon_m(\varepsilon_d \varepsilon_m + \varepsilon_d^2 + \zeta_d^2 \pm 2\varepsilon_m \zeta_d \sqrt{\varepsilon_d \varepsilon_m / [(\varepsilon_d - \varepsilon_m)^2 + \zeta_d^2]})}{(\varepsilon_d + \varepsilon_m)^2 + \zeta_d^2}}. \tag{8}$$

Changing in ζ_d the subscript d by m and making substitution ε_d by ε_m and vice versa, we obtain from (8) the dispersion relation for the structure where only metal is magnetized.

In another limiting case where both dielectric and metal are described by scalar parameters, expression (7) is simplified to the well-known formula [1]:

$$\beta = \pm k_0 \sqrt{\frac{\varepsilon_d \varepsilon_m}{\varepsilon_d + \varepsilon_m}}. \tag{9}$$

Let us return to relation (7). Notice first of all that with opposite signs of ζ_d and ζ_m, this equation can be used for the calculation of structure on Figure 1(a) with opposite directions of magnetization of MO dielectric and metal. Secondly, Equation (7) contains ζ_d and ζ_m only squared or as a product of them. Thus, simultaneous change of the signs of ζ_d and ζ_m does not change this equation.

The $\pm$ signs in front of the square root of M correspond to two possible signs of dc magnetic field $\mathbf{B}_0$. The $\pm$ signs in front of k_0 in (7) set two possible directions of wave propagation. Thus, formally we have four solutions of biquadratic Equation (6). But in fact, only two different solutions exist for the waveguide as it was discussed also in [2]. An explanation of this from the symmetry point of view is as follows. The Time reversal operator T applied to magnetic field and to propagation constant changes their signs. This fact and symmetry of Maxwell equation with respect to T lead to the following conclusion: the propagation constant for one direction of propagation with a fixed orientation of dc magnetic field equals to the propagation constant for the opposite sense of propagation with the reversed dc magnetic field. But for a fixed direction of dc magnetic field $\mathbf{B}_0$, the forward and backward traveling waves have different phase velocities, i.e., the waveguide is nonreciprocal.

3. NUMERICAL EXAMPLES

Using formula (7), we give below two illustrative examples: the first one is a combination of yttrium iron garnet (YIG) and silver (Ag). In the second example one medium is YIG and the other medium is cobalt (Co). The applied external magnetic field corresponds to the Voigt configuration shown in Figure 1(a).

The tensor elements for silver at $\lambda = 1.55\,\mu\text{m}$ are $\varepsilon_m = -97.87 + j2.69$ (obtained from [5]) and $j\zeta_m = -0.8678 + j0.0474$ (obtained by the Drude-Zener model with dc magnetic field $B_0 = 10T$). For cobalt, we use the following parameters: $\varepsilon_m = -8.2 + j59.75$, $j\zeta_m = -1.4858 + j0.9832$ [2]. For YIG, we use the values of the elements of the tensor $[\varepsilon]_d$ typical for the region of $\lambda = 1.55\,\mu\text{m}$: $\varepsilon_d = 4.84$ and $\zeta_d = 0.005$ [2]. We do not take into account the dielectric losses in MO material because they are small in comparison to those of the metal.

In Table 2 we present the calculated difference $\text{Re}\Delta\beta = (\text{Re}\beta^+ - |\text{Re}\beta^-|)$ between phase constants of forward $\text{Re}\beta^+$ and backward $\text{Re}\beta^-$ waves (the symbol Re means the real part) for $\lambda = 1.55\,\mu\text{m}$. The values of $\text{Re}\Delta\beta$ are normalized to the phase constant of the nonmagnetized structure $\text{Re}\beta_0$.

From Table 2 we see, firstly, that one can obtain greater values of $\text{Re}\Delta\beta$ when the effect of external magnetization on the plasma at the metallic surface is taken into account. Secondly, because of high

Table 2. Calculated parameters of waveguides in the Voigt geometry.

Structures	$\text{Re}\Delta\beta/\text{Re}\beta_0$	$\text{Im}\beta^+$ (dB/μm)	$\text{Im}\beta^-$ (dB/μm)
YIG + Ag only YIG is magnetized	0.05231	0.0180	0.0971
YIG + Ag both are magnetized	0.05243	0.0583	0.0599
YIG + Co only YIG is magnetized	1.1490	3.1485	3.2009
YIG + Co both are magnetized	2.5273	3.1691	3.2230

magnetic properties of Co, the difference in propagation constant is much higher for combination YIG + Co in comparison with that for YIG + Ag. This implies an increase of phase shifting between the forward and backward SPP waves. But the losses defined by Imβ^+ and Imβ^- (see Table 2) in case of YIG + Co are higher than for YIG + Ag.

The problem of high ohmic losses of ferromagnetic metals can be resolved by using ad hoc created materials. For example, in [6], a nanocomposition of Au-Co is suggested which allows one to combine high conductivity and high MO activity in one material. Another known drawback of metals as MO materials is high value of the required dc magnetic field for their magnetization. One method of solving this problem is the use of metal films with a reduced thickness [7].

4. CONCLUSION

In this paper, we presented an analytical investigation of SPP propagation at the interface of two semi-infinite magnetized media. Using the theory of magnetic groups, we calculated scattering matrices for the Voigt and the Faraday geometry and for the transversal magnetization. For the Voigt configuration, we obtained the exact analytical dispersion relation. We also showed numerically that phase shifting between forward and backward traveling SPP modes is increased significantly when the magnetic properties of the metal are taken into account. The obtained results can be useful for projects of nonreciprocal and control nanoplasmonic components such as isolators, circulators, modulators and switches.

ACKNOWLEDGMENT

This work was supported by the Brazilian agency CNPq.

REFERENCES

1. Bozhevolnyi, S. I., *Plasmonic Nanoguides and Circuits*, Pan Stanford Publishing, London, 2009.
2. Sepulveda, B., L. M. Lechuga, and G. Armelles, "Magnetooptic effects in surface-plasmon-polaritons slab waveguides," *Journal of Lightwave Technology*, Vol. 24, 945–955, 2006.
3. Pistora, J., et al., "Surface plasmon resonance sensor with a magneto-optical structure," *Optica Applicata*, Vol. 40, No. 4, 883–895, 2010.

4. Barybin, A. A. and V. A. Dmitriev, *Modern Electrodynamics and Coupled-mode Theory: Application to Guided-wave Optics*, Chapter 2, Rinton Press, Princeton, New Jersey, 2002.
5. Johnson, P. B. and R. W. Christy, "Optical constants of the noble metals," *Physical Review B*, Vol. 22, 1099–1119, 1983.
6. Yang, K., et al., "Surface-plasmon resonance and magneto-optical enhancement on Au-Co nanocomposite thin films," *Journal of Applied Physics*, Vol. 107, 103924, 2010.
7. Temnov, V. V., et al.,"Active magneto-plasmonics in hybrid metalferromagnet structures," *Nature Photonics*, Vol. 4, 107–111, 2010.