Progress In Electromagnetics Research, Vol. 113, 429–441, 2011

IMPROVED DIFFERENTIAL EVOLUTION STRATEGY FOR ANTENNA ARRAY PATTERN SYNTHESIS PROBLEMS

R. Li, L. Xu, X.-W. Shi, N. Zhang, and Z.-Q. Lv

Science and Technology on Antenna and Microwave Laboratory
P. O. Box 223, Xidian University, Xi'an 710071, P. R. China

Abstract—An Improved Differential Evolution (IDE) algorithm is proposed for optimization problems. With the novel mutant operation adopting sub-optimal individual, the convergence of Differential Evolution (DE) algorithm is accelerated without increasing the risk of premature. Five typical test functions are minimized using DE and IDE algorithms, and the results show the superior performance of IDE algorithm. Furthermore, the algorithm is applied to pattern synthesis of two antenna arrays. Broad nulls are formed in radiation pattern of a linear array to suppress broad-band interferences. In a microstrip patch array, the sidelobe level of array is decreased about 12.9 dB and the mainlobe can scan to the desired angle.

1. INTRODUCTION

Recently, pattern synthesis becomes a hot research topic. Scholars deal with designing the weights of elements in array to realize the desired array pattern [1–21]. With population based stochastic methods, the array synthesis problem can be solved effectively. Genetic Algorithm (GA), Simulated Annealing (SA), Particle Swarm Optimization (PSO) and Differential Evolution (DE) belong to these kinds of methods and have been applied to array synthesis problems, where complex excitations are optimized to realize maximizing the directivity, minimizing the sidelobe levels (SLLs), and placing null constrains, etc. Compared with other methods, DE method shows an excellent convergence performance.

Proposed by Storn and Price [22], DE algorithm is a simple yet powerful population-based stochastic search technique, which is good

Received 18 January 2011, Accepted 11 February 2011, Scheduled 14 February 2011
Corresponding author: Rui Li (ruili@mail.xidian.edu.cn).

at solving multi-variable function optimization problem particularly. It has been successfully applied to diverse fields such as mechanical engineering [23, 24], communication [25], array synthesis problems [26–32], ect. [33, 34].

Despite DE's excellent performance, the convergence speed of it still seems slow for some problems. At present, there are many improvements for the differential evolution algorithm and specific improvement measures focus on the evolution operation, parameter settings and other improvements [30, 32, 35]. The algorithm's operations include mutation, crossover and selection operator, most improvements concentrate in the mutation operation. In [30], all individuals are sorted according to their fitness value, and the mutant vector is generated only from the superior half group. But the computational cost is increased significantly. A best of random differential mutation is proposed in [32], but introducing the optimization parameters increases the possibility of premature.

In this paper, we develop an Improved Differential Evolution (IDE) algorithm, in which the convergence rate is improved by adopting the sub-optimal vector in the mutant operation. Additional computational cost caused by the sub-optimal individual finding is small. Furthermore, IDE method introduces no extra control parameters. As a challenge, IDE method is applied to some array synthesis problems to verify the effectiveness.

The paper is organized as follows. The improvement of DE method is described in Section 2. The numerical simulations are presented in Section 3. Conclusion is given in Section 4.

2. IMPROVED DE ALGORITHM

For completeness and better readability, the procedure of DE method is briefly depicted in this section. Then the improvement of DE method is analyzed.

2.1. Standard DE

DE is a simple, efficient and robust evolutionary algorithm. It aims at evolving a population of M-dimensional parameter vectors, so-called individuals, towards the global optimum. M stands for the number of parameters. DE algorithm involves four stages, namely, initialization, mutation, crossover and selection.

Initialization: It utilizes NP M-dimensional parameter vectors as a population for each generation G.

$$\mathbf{x}_{i,G}, \quad i = 1, \ldots, NP \tag{1}$$

where NP is individual number in a group.

Differential Mutation: Differential mutation generates a mutant vector $v_{i,G}$ for each individual according to the following formula:

$$\mathbf{v}_{i,G} = \mathbf{x}_{p1,G} + F(\mathbf{x}_{p2,G} - \mathbf{x}_{p3,G}) \tag{2}$$

$p1, p2, p3 \in [1, NP], p1 \neq p2 \neq p3 \neq i$. And F is a real and constant factor satisfied $F \in [0, 2]$ which controls the amplification of the differential variation $(\mathbf{x}_{p2,G} - \mathbf{x}_{p3,G})$.

Crossover: After the mutation, crossover operator is applied to generate a trial individual $\mathbf{u}_{ji,G}$

$$\mathbf{u}_{ji,G} = \begin{cases} \mathbf{v}_{ji,G}, & \text{if} \quad (r(j) \leq CR) \quad \text{or} \quad j = r(i) \\ \mathbf{x}_{ji,G}, & \text{otherwise} \end{cases} \tag{3}$$

where $r(j)$ is a real random number uniform in the range $[0, 1]$ and *CR,* a constant in $[0, 1]$, is the crossover probability.

Selection: The fitness value of each trial vector $\mathbf{u}_{i,G}$ is compared to that of its corresponding target vector $\mathbf{x}_{i,G}$ in the current population. If the trial vector has less or equal fitness value than the corresponding target vector, $\mathbf{x}_{i,G}$ will be set to $\mathbf{u}_{i,G}$ and enter the population of the next generation. Otherwise, the old target vector $\mathbf{x}_{i,G}$ will be retained in the population for the next generation.

2.2. Improved DE (IDE)

In natural biological evolution, weak individuals are easily eliminated, and elites in the society have more chance to inherit their excellent characteristic to their offspring through genetic transmission. Inspired by that, competition manipulation among the individuals of the same generation is adopted in IDE algorithm to enhance the convergence speed.

Actually, increasing population diversity leads to a reliable result but convergence is slowed down. On the other hand, guidance provided by elite individuals accelerates convergence but it increases the risk of premature. To balance two contradictory aspects: diversity and guidance, the sub-optimal vector is introduced to the novel mutant operation. In IDE algorithm, the objective function values of all individuals are evaluated. After that, the mutant vector is generated with sub-optimal vector and other two individuals in the group to improve the convergence speed. IDE algorithm introduces no extra control parameters to the algorithm, and the computational cost of the additional sub-optimal individual selection is negligible.

The new mutant operation is as following

$$\mathbf{v}_{i,G} = \mathbf{x}_{p1,G} + F(\mathbf{x}_{sub-opt,G} - \mathbf{x}_{p2,G}) \tag{4}$$

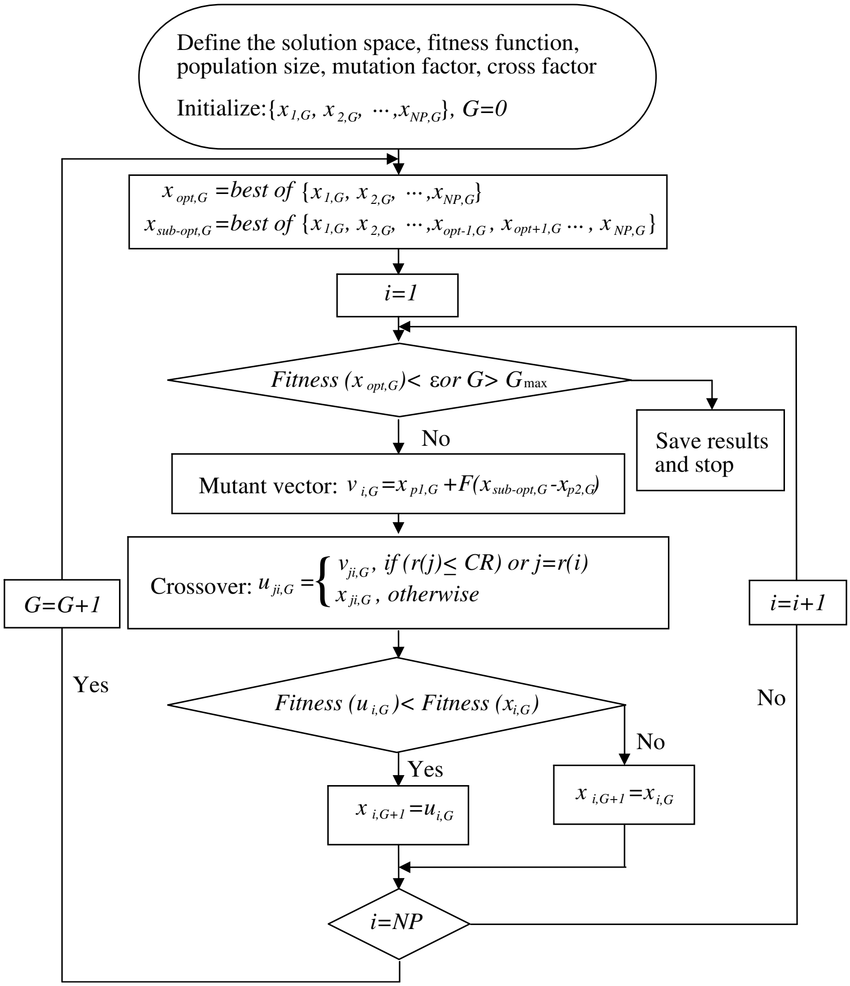


Figure 1. The flowchart of IDE algorithm.

$p1, p2 \in [1, NP]$, $p1 \neq p2 \neq i$. Here, the sub-optimal value is introduced to the differential variation $(\mathbf{x}_{sub-opt,G} - \mathbf{x}_{p2,G})$. To avoid overly fast converging to a local optimum domain, the optimal value is not used in the differential mutation. Fig. 1 presents the flowchart of IDE with the modification techniques.

2.3. Fitness Function

$$fitness = U(F_0(\theta) - F_d(\theta))[\alpha|MSLL - DSLL| + \beta|NULL - DNULL|] \quad (5)$$

where $U(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$, $MSLL$ is maximum sidelobe level, $DSLL$ is desired sidelobe level, $NULL$ is null depth, and $DNULL$ is desired null depth. α and β are weight parameters.

3. SIMULATION

In this section, we carry out some representative simulation experiments to demonstrate the superior of IDE method. As ED has been compared to other algorithms [22], here we only focus on the comparison of DE and IDE algorithms.

3.1. Test Function

Our function testbed contains five 20-dimensional test functions. The simulation results for these functions are shown in Table 1. Initial parameter ranges (IPR) are given. At the beginning of the optimization, the initial parameter values are drawn randomly from the IPR. We executed 20 test runs with randomly chosen initial parameter vectors for each test function and each minimization. And the iteration number is set to 400. The group number is 40, $F = 0.6$, $CR = 0.6$.

The results in Table 1 clearly show that in all test cases IDE exhibits superior performance when compared to DE reported best results. The optimal value found by IED is smaller than DE. It means the method proposed in this paper has a better converge speed.

Table 1. Best fitness value optimized by DE and IDE.

	DE	**IDE**	**IPR**
Spherical	$4.8261e-012$	$1.8615e-016$	[−100,100]
Rosenbrock	4.3833	2.4739	[−6,6]
Ackley	0.6971	0.3955	[−40,40]
Griewank	0.2453	0.0238	[−200,200]
Rastrigin	18.0250	1.9324	[−20,20]

3.2. A Linear Array

Usually, the directional pattern nulls formed by beamforming algorithm are extremely sharp. However, in practical communication systems, the high speed jammer motion or broadband signal may bring the jammers out of the nulls. To illustrate the broad-band interference suppression capability of IDE, the pattern having a broad null located at $45^\circ \sim 50^\circ$ is designed. 32 isotropic elements are spaced 0.5λ in a uniform linear array. The desired sidelobe level $DSLL = -20\,\mathrm{dB}$ and the null depth level $DNULL = -40\,\mathrm{dB}$. The group number is 40, $F = 0.6$, $CR = 0.8$, $\alpha = 2$, $\beta = 1$. Here we only synthesize the excitation amplitudes. Because of the symmetrical array, only 16 amplitudes are to be optimized. The excitation amplitude distribution is symmetric with respect to the center of the array.

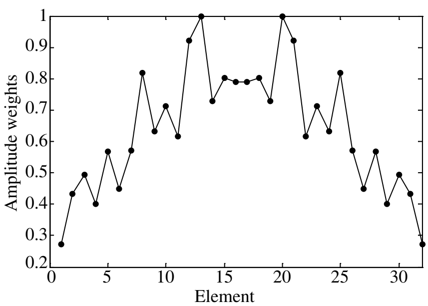


Figure 2. Amplitude weights by IDE for the linear array.

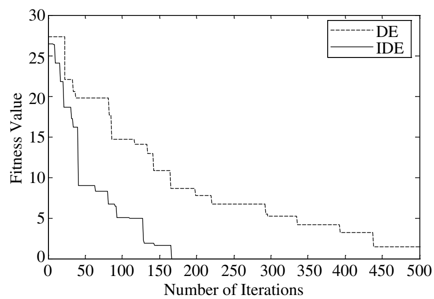


Figure 3. Fitness value versus the number of iterations using DE and IDE.

The excitation amplitude weights derived using IDE are given in Fig. 2. The corresponding convergence curves are plotted in Fig. 3. Obviously, convergence speed of IDE is faster than DE. The normalized radiation pattern in dB obtained at the end of the optimization process is given in Fig. 4. From the figure we can seen that both DE and IDE methods satisfy the requirements. But the optimized side-lobe levels of IDE are about $2\,\mathrm{dB}$ lower than that of DE. The null width is accurately produced by IDE. The shape of the main-beam is not optimized, only the beam-width is limited to $-5^\circ \sim 5^\circ$.

3.3. A Circular Microstrip Antenna Array

In this example, we will address the synthesis problem for a circular microstrip antenna array. The radiation pattern of each element in the array is introduced instead of the point sources in the last example.

By this way, more practical results are obtained.

Antenna array is an important part of smart antenna system. The circular array attracts attention ahead of other shapes since it is symmetric geometry.

A circularly-polarized microstrip patch is designed for receiver system. The basic structure of the antenna is depicted in Fig. 5. In order to resonate at the center frequency of 1.575GHz, yielding its dimensions as $l = 36.8$ mm, $r = 35$ mm, $c = 4$ mm, $lf = 7$ mm. The antenna is built up on substrate with $\varepsilon_r = 6.15$ and thickness of 3.18 mm. Ten patch elements are placed along a circle radius $R = 184$ mm and separated by a uniform angle 36°. The circle radius is defined as to accommodate the inter-element distance of $d = 0.6\lambda_0$. As shown in Fig. 6(a), the antenna placed symmetric in the array. Due

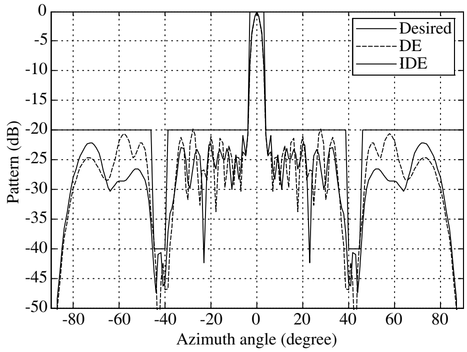


Figure 4. Radiation patterns for a 32-element uniform linear array.

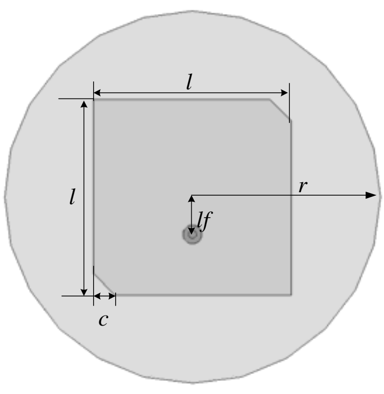
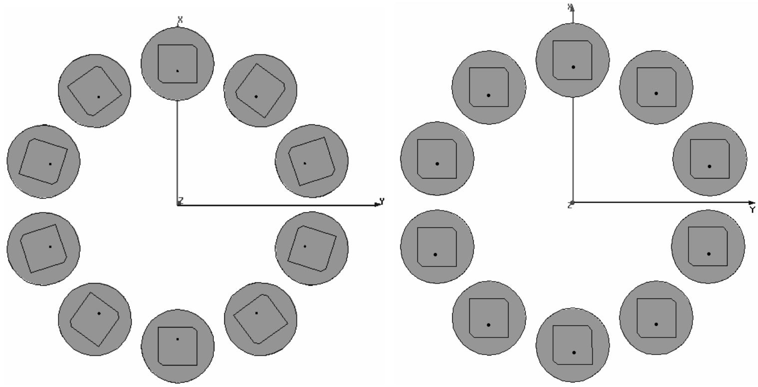


Figure 5. Geometry of the circularly-polarized microstrip patch antenna.

to the polarization of elements, the contribution to radiation pattern in far field of elements at the symmetric location will be counteracted at direction $\theta = 0°$. To overcome the polarization counteraction, the elements are rotated as shown in Fig. 6(b).

All radiation patterns in the following example are obtained using HFSS11. The couplings mutual among the elements are considered in the synthesis. The group number is 40, $F = 0.6$, $CR = 0.9$, $\alpha = 1$, $\beta = 0$. The low sidelobe synthesis ability of IDE is examined in this example. Fig. 6 depicts the gain pattern in x-z plane of this array. The highest sidelobe level with the same excitation is -10 dB.



(a) Before rotation (b) After rotation

Figure 6. Geometry of the array before and after rotation.

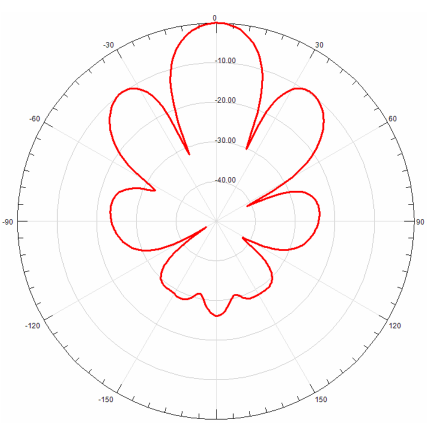


Figure 7. Radiation patterns of the array before synthesis.

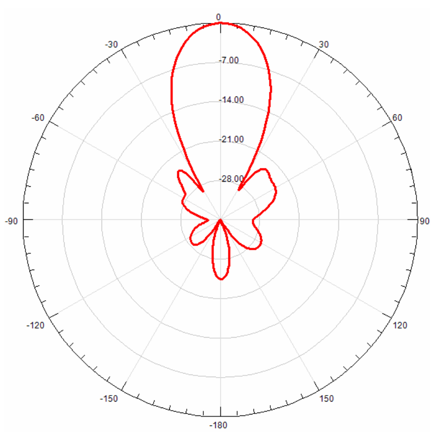


Figure 8. Synthesized radiation pattern for low sidelobe.

The synthesized result is given in Fig. 7. With the excitation weights obtained by IDE, the highest sidelobe level drops to −22.9 dB and overall sidelobe levels are substantially lower. The optimal amplitude and phase weights are shown in Table 2. The result shows that the algorithm proposed in this paper reduces the sidelobe level for 12.9 dB.

Table 2. Excitation amplitude and phase of each element for low sidelobe synthesis.

Element	Amplitude	Phase	Element	Amplitude	Phase
1	0.9444	−38.3936	6	0.9444	−38.3936
2	0.2253	−63.7946	7	0.2253	−63.7946
3	1.8221	−12.8283	8	1.8221	−12.8283
4	1.0997	−81.4198	9	1.0997	−81.4198
5	0.2691	−3.3572	10	0.2691	−3.3572

Then we synthesize a beam scan radiation pattern. Here, the maximum radiation direction desired for this array is at $\theta = -30^\circ$. From the radiation pattern shown in Fig. 9 we can see the radiation can focus on the desired direction with the optimal solutions found by IDE. The corresponding excitation amplitude and phase weights are given in Table 3.

The results shown in Figs. 6–8 confirm that IDE presented in this paper is able to approach the desired pattern by controlling the element excitations of the circular array.

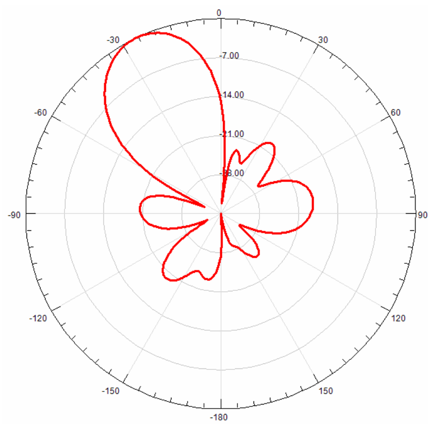


Figure 9. Synthesized radiation pattern for beam scan at $\theta = -30^\circ$.

Table 3. Excitation amplitude and phase of each element for beam scan synthesis.

Element	Amplitude	Phase	Element	Amplitude	Phase
1	0.6536	64.801	6	1.131	−109.751
2	2.3224	−125.199	7	0.9565	137.2963
3	3.2041	−18.6127	8	2.8167	68.6576
4	0.4063	13.2671	9	0.2316	123.3881
5	1.2917	141.7415	10	0.6677	−71.6369

4. CONCLUSION

In this paper, IDE with a novel mutant operation adopting sub-optimal individual is proposed. By this way, the convergence speed is improved without increasing the risk of premature. The new algorithm introduces no extra control parameters to the algorithm, and the computational cost of the additional sub-optimal individual search is negligible. The simulation results of test functions show the superior performance of IDE. With the optimal solution found by IED, the broad nulls are realized to suppress broad-band interferences in a linear array. Then it is applied to the low sidelobe and beam scan synthesis of a microstrip antenna array. With the optimal weights found by IDE, the sidelobe level is decreased 12.9 dB and the mainlobe can scan to the desired angle.

ACKNOWLEDGMENT

This work was supported by the National Science Foundation of China under Grant No. 60801039.

REFERENCES

1. Xu, Z., H. Li, Q. Z. Liu, and J. Y. Li, "Pattern synthesis of conformal antenna array by the hybrid genetic algorithm," *Progress In Electromagnetics Research*, Vol. 79, 75–90, 2008.
2. Qu, Y., G. Liao, S. Q. Zhu, and X. Y. Liu, "Pattern synthesis of planar antenna array via convex optimization for airborne forward looking radar," *Progress In Electromagnetics Research*, Vol. 84, 1–10, 2008.
3. Lu, Z. B., A. Zhang, and X. Y. Hou, "Pattern synthesis of cylindrical conformal array by the modified particle swarm

optimization algorithm," *Progress In Electromagnetics Research*, Vol. 79, 415–426, 2008.

4. Li, W. T., X. W. Shi, L. Xu, and Y. Q. Hei, "Improved GA and PSO culled hybrid algorithm for antenna array pattern synthesis," *Progress In Electromagnetics Research*, Vol. 80, 461–476, 2008.
5. Li, W. T., X. W. Shi, and Y. Q. Hei, "An improved particle swarm optimization algorithm for pattern synthesis of phased arrays," *Progress In Electromagnetics Research*, Vol. 82, 319–332, 2008.
6. Lee, K. C. and J. Y. Jhang, "Application of electromagnetism-like algorithm to phase-only syntheses of antenna arrays," *Progress In Electromagnetics Research*, Vol. 83, 279–291, 2008.
7. He, Q. Q. and B. Z. Wang, "Design of microstrip array antenna by using active element pattern technique combining with taylor synthesis method," *Progress In Electromagnetics Research*, Vol. 80, 63–76, 2008.
8. Zhang, F., F. S. Zhang, C. Lin, G. Zhao, and Y. C. Jiao, "Pattern synthesis for planar array based on elements rotation," *Progress In Electromagnetics Research Letters*, Vol. 11, 55–64, 2009.
9. Mangoud, M. A.-A. and H. M. Elragal, "Antenna array pattern synthesis and wide null control using enhanced particle swarm optimization," *Progress In Electromagnetics Research B*, Vol. 17, 1–14, 2009.
10. Laviada-Martinez, J., R. G. Ayestaran, M. R. Pino, F. Las-Heras Andrés, and R. Mittra, "Synthesis of phased arrays in complex environments with the multilevel characteristic basis function method," *Progress In Electromagnetics Research*, Vol. 92, 347–360, 2009.
11. Carro Ceballos, P. L., J. De Mingo Sanz, and P. G. Dúcar, "Radiation pattern synthesis for maximum mean effective gain with spherical wave expansions and particle swarm techniques," *Progress In Electromagnetics Research*, Vol. 103, 355–370, 2010.
12. Lim, S. H., J. H. Han, S. Y. Kim, and N. H. Myung, "Azimuth beam pattern synthesis for airborne SAR system optimization," *Progress In Electromagnetics Research*, Vol. 106, 295–309, 2010.
13. Liu, Y., Z. P. Nie, and Q. H. Liu, "A new method for the synthesis of non-uniform linear arrays with shaped power patterns," *Progress In Electromagnetics Research*, Vol. 107, 349–363, 2010.
14. Morabito, A. F., A. R. Lagana, and T. Isernia, "On the optimal synthesis of ring symmetric shaped patterns by means of uniformly spaced planar arrays," *Progress In Electromagnetics Research B*,

Vol. 20, 33–48, 2010.

15. Singh, U., H. Kumar, and T. S. Kamal, "Linear array synthesis using biogeography based optimization," *Progress In Electromagnetics Research M*, Vol. 11, 25–36, 2010.
16. Ayestarán, R. G., J. Laviada, and F. Las-Heras, "Realistic antenna array synthesis in complex environments using a MOM-SVR approach," *Journal of Electromagnetic Waves and Applications*, Vol. 23, No. 1, 97–108, 2009.
17. Chou, H. T., K. L. Hung, and C. Y. Chen, "Utilization of a yagi antenna director array to synthesize a shaped radiation pattern for optimum coverage in wireless communications," *Journal of Electromagnetic Waves and Applications*, Vol. 23, No. 7, 851–861, 2009.
18. He, Q. Q., H. D. He, and H. Lan, "An efficient pattern synthesis method for cylindrical phased array antennas," *Journal of Electromagnetic Waves and Applications*, Vol. 23, No. 4, 473–482, 2009.
19. Zhang, Y. J., S. X. Gong, and Y. X. Xu, "Radiation pattern synthesis for arrays of conformal antennas mounted on an irregular curved surface using modified genetic algorithms," *Journal of Electromagnetic Waves and Applications*, Vol. 23, No. 10, 1255–1264, 2009.
20. Eirey-Pérez, R., M. Álvarez-Folgueiras, J. A. Rodriguez-González, and F. Ares-Pena, "Arbitrary footprints from arrays with concentric ring geometry and low dynamic range ratio," *Journal of Electromagnetic Waves and Applications*, Vol. 24, No. 13, 1795–1806, 2010.
21. Rocca, P., L. Poli, G. Oliveri, and A. Massa, "Synthesis of time-modulated planar arrays with controlled harmonic radiations," *Journal of Electromagnetic Waves and Applications*, Vol. 24, Nos. 5–6, 827–838, 2010.
22. Storn, R. and K. V. Price, "Differential evolution-A simple and efficient heuristic for global optimization over continuous spaces," *J. Global Optim.*, Vol. 11, 341–359, 1997.
23. Rogalsky, T., R. W. Derksen, and S. Kocabiyik, "Differential evolution in aerodynamic optimization," *Proc. 46th Annu. Conf. of Can. Aeronaut. Space Inst.*, 29–36, Montreal, QC, Canada, May 1999.
24. Joshi, R. and A. C. Sanderson, "Minimal representation multisensory fusion using differential evolution," *IEEE Trans. Syst. Man Cybern. A, Syst. Humans*, Vol. 29, No. 1, 63–76, Jan. 1999.

25. Storn, R., "On the usage of differential evolution for function optimization," *Proc. Biennial Conf. North Amer. Fuzzy Inf. Process. Soc.*, 519–523, Berkeley, CA, 1996.
26. Yang, S., Y. B. Gan, and A. Qing, "Sideband suppression in time-modulated linear arrays by the differential evolution algorithm," *IEEE Antennas Wireless Propag. Lett.*, Vol. 1, 173–175, 2002.
27. Kurup, D. G., M. Himdi, and A. Rydberg, "Synthesis of uniform amplitude unequally spaced antenna arrays using the differential evolution algorithm," *IEEE Transactions on Antennas and Propagation*, Vol. 51, No. 9, 2210–2217, 2003.
28. Caorsi, S., A. Massa, M. Pastorino, and A. Randazzo, "Optimization of the difference patterns for monopulse antennas by a hybrid real/integercoded differential evolution method," *IEEE Transactions on Antennas and Propagation*, Vol. 53, No. 1, 372–376, 2005.
29. Massa, A., M. Pastorino, and A. Randazzo, "Optimization of the directivity of a monopulse antenna with a subarray weighting by a hybrid differential evolution method," *IEEE Antennas Wireless Propag. Lett.*, Vol. 5, 155–158, 2006.
30. Chen, Y., S. Yang, and Z. Nie, "The application of a modified differential evolution strategy to some array pattern synthesis problems," *IEEE Transactions on Antennas and Propagation*, Vol. 56, No. 7, 1919–1927, 2008.
31. Guo, J.-L. and J.-Y. Li, "Pattern synthesis of conformal array antenna in the presence of platform using differential evolution algorithm," *IEEE Transactions on Antennas and Propagation*, Vol. 57, No. 9, 2615–2621, 2009.
32. Lin, C., A. Qing, and Q. Feng, "Synthesis of unequally spaced antenna arrays by using differential evolution," *IEEE Transactions on Antennas and Propagation*, Vol. 58, No. 8, 2553–2561, 2010.
33. Chiou, J.-P., "A variable scaling hybrid differential evolution for solving large-scale power dispatch problems," *Generation, Transmission & Distribution, IET*, Vol. 3, No. 2, 154–163, 2009.
34. Qin, A. K., V. L. Huang, and P. N. Suganthan, "Differential evolution algorithm with strategy adaptation for global numerical optimization," *IEEE Transactions on Evolutionary Computation*, Vol. 13, No. 2, 398, 2009.
35. Fan, Z., J. Liu, T. Sorensen, and P. Wang, "Improved differential evolution based on stochastic ranking for robust layout synthesis of MEMS components," *IEEE Transactions on Industrial Electronics*, Vol. 56, No. 4, 937–948, 2009.