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# UNIFORM PLANE WAVE REFLECTION FROM PEC PLANE EMBEDDED IN A NONLINEAR MEDIUM

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**Abstract**—Reflection from a perfect electric conductor (PEC) plane of infinite dimensions embedded in a second order nonlinear medium is studied. The reflected wave has two parts, one due to linear behavior, the other due to nonlinear behavior of the medium. The expressions of the reflection coefficients for parallel and perpendicular polarization cases are obtained. The reduction of reflection coefficient to a linear medium case is also reported. Dependence of the said coefficients on incident electric field intensity and the angle of incidence is also plotted.

## 1. INTRODUCTION

All mediums are basically nonlinear. For example, a nonlinear phenomena like photon-photon scattering in a magnetized vacuum has been reported [1]. The experiment of Franken et al. to generate second harmonic is marked as the birth of nonlinear optics [2]. Electromagnetic phenomena are nonlinear in the sense that they occur when the response of a material system to an applied electromagnetic field depends in a nonlinear manner on the strength of the said field. For example, an applied field produces nonlinear polarization in the medium which causes parametric or nonparametric phenomena [4]. The former is governed by the constitutive formulation and the latter by the Maxwell's postulates [3]. Many problems related to diffraction, cloaking, invisibility, focussing systems and scattering problems etc. involve the PEC boundaries [4–8]. Some authors have also worked on reflection of plane wave from a PEC planar interface embedded in a medium [7–11]. Ellipsometry is based on the reflection and is used for

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the investigation of the dielectric properties (complex refractive index or dielectric function) of thin films [12–15]. Reflectance measurements are also used to find optical properties such as optic axis, complex refractive index, optical constants etc. [16–19].

In the present work the behavior of a uniform plane wave is studied when it strikes an infinite PEC plane which is embedded in a second order nonlinear optical medium. It is assumed that the said medium is isotropic, lossless and has no dispersion. This work is carried out to study PEC reflector placed in a nonlinear medium to find high frequency field at the caustic using Maslov's method. This method requires the initial values of the reflected fields [7–9].

There are two canonical cases of this problem, and the electric field is either parallel polarized or perpendicularly polarized. The reflection coefficients for both cases have been derived which can be used to study the PEC reflectors and scatterer. Perpendicular polarization case is discussed in Section 2 and parallel polarization in Section 3. The results are shown and discussed in Section 4. Some applications are also mentioned in Section 4.1. The paper is concluded in Section 5.

## 2. PERPENDICULAR POLARIZATION

Before coming to the original problem, the form of the wave equation is considered for the propagation of light through a nonlinear optical medium. The Maxwell's equations in SI units can be written as [4]

$$\nabla \cdot \tilde{D} = \tilde{\rho}, \tag{1}$$

$$\nabla \cdot \tilde{B} = 0, \tag{2}$$

$$\nabla \times \tilde{E} = -\frac{\partial \tilde{\mathrm{B}}}{\partial t}, \tag{3}$$

$$\nabla \times \tilde{H} = \frac{\partial \tilde{D}}{\partial t} + \tilde{J}. \tag{4}$$

We use a tilde ( ˜ ) to denote a quantity that varies rapidly in time. It is assumed that the region of interest has no free charges and currents, so that

$$\begin{aligned} \tilde{\rho} &= 0, \\ \tilde{J} &= 0. \end{aligned} \tag{5}$$

The material is also nonmagnetic, so that

$$\tilde{B} = \mu_0 \tilde{H} \tag{6}$$

The electric flux density is related to electric field and polarization in a nonlinear medium as

$$\tilde{D} = \epsilon_0 \epsilon_r(\omega_s)\tilde{E} + \tilde{P}, \tag{7}$$

where the polarization vector $\tilde{P}$ depends nonlinearly upon the electric field strength $\tilde{E}$, and $\epsilon_r(\omega_s)$ is the nonlinear relative permeability of the medium. To derive the optical wave equation, we take the curl of the curl-$\tilde{E}$ Maxwell equation (3), swapping the order of space and time derivatives on the right-hand side of the resulting equation, then using Equations (4), (5) and (6) to replace $\nabla \times \tilde{B}$ by $\mu_0 \frac{\partial \tilde{D}}{\partial t}$ and obtain the equation

$$\nabla \times \nabla \times \tilde{E} + \mu_0 \frac{\partial^2 \tilde{D}}{\partial t^2} = 0,$$

Now using the definition of $\tilde{D}$ given in Equation (7) to eliminate it from the above equation. We also replace $\mu_0$ by $\frac{1}{\epsilon_0 c^2}$ on the left side of the said equation and obtain the following expression

$$\nabla^2 \tilde{E} - \frac{\epsilon(\omega_s)}{c^2} \frac{\partial^2 \tilde{E}}{\partial t^2} = \frac{1}{\mu_0} \frac{\partial^2 \tilde{P}}{\partial t^2} \tag{8}$$

Now consider an infinite PEC plane which is embedded in a medium having second order nonlinear optical properties. The PEC plane is placed on $z = 0$ in a $yz$ plane as shown in Figure 1. The medium is nonlinear for $z < 0$ and $z > 0$. The incident electric field is perpendicular to $xz$ plane, which can be defined as [4]

$$\tilde{E}_i(r, t) = E_i(\omega_i) e^{-i\omega_i t}$$

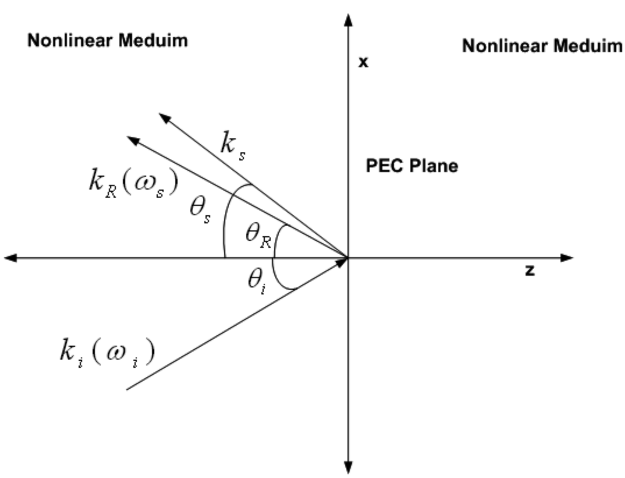


**Figure 1.** Oblique incidence on a PEC plane embedded in a second order nonlinear medium.

where

$$E_i(\omega_i) = A_i(\omega_i)\hat{y}e^{ik_i(\omega_i)(x\sin\theta_i + z\cos\theta_i)}$$

and

$$k_i(\omega_i) = \sqrt{\epsilon(\omega_i)}\omega_i/c$$

As the medium is a second order nonlinear optical type, the reflected fundamental wave will create a nonlinear polarization at frequency $\omega_s = 2\omega_i$ which can be represented as

$$\tilde{P}(r,t) = Pe^{-i\omega_s t}$$

where

$$P = pe^{ik_s(\omega_s)\cdot r}$$

and

$$p = \hat{p}\epsilon_0\chi_{\mathit{eff}}^{(2)}\Gamma^2(\omega_s)A_i^2(\omega_i)$$

Where $\chi_{\mathit{eff}}^{(2)}$ is the nonlinear susceptibility of the medium. This nonlinear polarization will cause a radiation of the second-harmonic frequency $\omega_s$. The propagation of reflected electric field $\tilde{E_R}$ in the medium can be expressed as follows using Equation (8) and putting the expression for the nonlinear polarization

$$\nabla^2 E_R(\omega s) + \left[\epsilon(\omega_s)\omega_s^2/c^2\right] E_R(\omega s) = -\left(\omega_s^2/\mu_0\right) p_\perp e^{ik_s\cdot r}$$

Where $p_\perp$ is the component of $P$ perpendicular to the plane of incidence. The solution of the above equation consists of a particular solution plus a general solution. The general solution is obtained by setting its right-hand side equal to zero. All of the appropriate boundary conditions can be met by assuming that the homogeneous solution is an infinite plane wave of amplitude $\Gamma(\omega_s)A_i(\omega_i)$ and wavevector $k_R(\omega_s)$. As PEC plane will offer a discontinuity to the incident wave so the amplitude of reflected electric field will differ from incident field by a factor $\Gamma(\omega_s)$ the reflection coefficient which will be found after applying the boundary conditions. The reflected field will has a nonlinear part also because of nonlinear polarization of the medium. Thus the solution to the above nonlinear wave equation can be written as [4]

$$\begin{aligned} E_R(\omega s) &= \Gamma(\omega_s)A_i(\omega_i)\hat{y}e^{ik_R(\omega_s)(x\sin\theta_R - z\cos\theta_R)} \\ &\quad + \frac{\hat{y}\left(\omega_s^2/\mu_0\right)}{|k_s|^2 - |k_R(\omega_s)|^2}\epsilon_0\chi_{\mathit{eff}}^{(2)}\Gamma^2(\omega_s)A_i^2(\omega_i)e^{ik_s(x\sin\theta_s - z\cos\theta_s)} \end{aligned}$$

where

$$k_R(\omega_s) = \sqrt{\epsilon(\omega_s)}\omega_s/c$$

and

$$k_s(\omega_s) = 2k_R(\omega_s) \tag{9}$$

By applying the following boundary condition at $z = 0$ to the tangential components

$$(E_i + E_R)_{z=0}^{tangential} = 0 \tag{10}$$

For the basic frequency

$$A_i(\omega_i)\hat{y}e^{ik_i(\omega_i)(x\sin\theta_i)} + \Gamma(\omega_s)A_i(\omega_i)\hat{y}e^{ik_R(\omega_s)(x\sin\theta_i)} = 0 \tag{11}$$

and for double frequency

$$A_i(\omega_i)\hat{y}e^{ik_i(\omega_i)(x\sin\theta_i)} + \Gamma(\omega_s)A_i(\omega_i)\hat{y}e^{ik_R(\omega_s)(x\sin\theta_i)} + \frac{\hat{y}\left(\omega_s^2/\mu_0\right)}{|k_s|^2 - |k_R(\omega_s)|^2}\epsilon_0\chi_{eff}^{(2)}\Gamma^2(\omega_s)A_i^2(\omega_i)e^{ik_s(x\sin\theta_s)} = 0 \tag{12}$$

To meet the boundary conditions at each point along the interface, it is necessary that the wavevectors of the incident wave, the reflected wave and reflected second-harmonic wave have identical components along the plane of the interface. We thus require that

$$k_x^i(\omega_i)\sin\theta_i = k_x^R(\omega_s)\sin\theta_R = k_x^s\sin\theta_s \tag{13}$$

let

$$B = \frac{\left(\omega_s^2/\mu_0\right)}{|k_s|^2 - |k_R(\omega_s)|^2}\epsilon_0\chi_{eff}^{(2)}$$

By examining Equations (11) and (13), it can be seen that for the basic frequency the reflection coefficient $\Gamma(\omega_s)$ is $-1$. From Equations (12) and (13)

$$A_i(\omega_i) + \Gamma(\omega_s)A_i(\omega_i) + B\Gamma^2(\omega_s)A_i^2(\omega_i) = 0$$

By simplifying

$$1 + \Gamma(\omega_s) + B\Gamma^2(\omega_s)A_i(\omega_i) = 0 \tag{14}$$

Solution of the above quadratic equation gives the reflection coefficient as

$$\Gamma_{1,2}(\omega_s) = \frac{-1 \pm \sqrt{1 - 4BA_i(\omega_i)}}{2BA_i(\omega_i)}$$

By examining Equation (14) it can bee seen that for a linear medium assumption the last term in the above equation will be zero as there will be no contribution from nonlinear polarization in reflected field and this will give the reflection coefficient $\Gamma(\omega_s)$ equal to $-1$ as earlier which is true for a PEC plane.

## 3. PARALLEL POLARIZATION

For parallel polarization case, the incident electric field vector lies in the $xz$ plane, and can be written as

$$E_i(\omega_i) = A_i(\omega_i)(\hat{x}\cos\theta_i - \hat{z}\sin\theta_i)e^{ik_i(\omega_i)(x\sin\theta_i + z\cos\theta_i)}$$

The reflected field will be

$$E_R(\omega s) = \Gamma(\omega_s)A_i(\omega_i)(\hat{x}\cos\theta_R + \hat{z}\sin\theta_R)e^{ik_R(\omega_s)(x\sin\theta_R - z\cos\theta_R)} + \left[\begin{array}{c} \frac{\hat{x}\cos\theta_s\left(\omega_s^2/\mu_0\right)}{|k_s|^2-|k_R(\omega_s)|^2}\epsilon_0\chi_{\mathit{eff}}^{(2)}\Gamma^2(\omega_s)A_i^2(\omega_i) \\ +\frac{\hat{z}\sin\theta_s\left(\omega_s^2/\mu_0\right)}{|k_s|^2-|k_R(\omega_s)|^2}\epsilon_0\chi_{\mathit{eff}}^{(2)}\Gamma^2(\omega_s)A_i^2(\omega_i) \end{array}\right] e^{ik_s(x\sin\theta_s - z\cos\theta_s)}$$

After applying the boundary condition given in Equation (10)

$$\hat{x}A_i(\omega_i)\cos\theta_i e^{ik_i(\omega_i)(x\sin\theta_i)} + \hat{x}\Gamma(\omega_s)A_i(\omega_i)(\cos\theta_R)e^{ik_R(\omega_s)(x\sin\theta_i)} + \hat{x}\cos\theta_s\frac{\left(\omega_s^2/\mu_0\right)}{|k_s|^2 - |k_R(\omega_s)|^2}\epsilon_0\chi_{\mathit{eff}}^{(2)}\Gamma^2(\omega_s)A_i^2(\omega_i)e^{ik_s(x\sin\theta_s)} = 0$$

As $\theta_i = \theta_R$, from Equation (9) and the condition given in Equation (13) we can write

$$\cos\theta_s = \sqrt{1 - \frac{\sin^2\theta_i}{4}}$$

and

$$A_i(\omega_i)\cos\theta_i + \Gamma(\omega_s)A_i(\omega_i)(\cos\theta_i) + B\Gamma^2(\omega_s)A_i^2(\omega_i)\sqrt{1 - \frac{\sin^2\theta_i}{4}} = 0$$

$B$ is same as defined in Section 2, after simplification

$$\cos\theta_i + \Gamma(\omega_s)(\cos\theta_i) + B\Gamma^2(\omega_s)A_i(\omega_i)\sqrt{1 - \frac{\sin^2\theta_i}{4}} = 0 \tag{15}$$

Solving for $\Gamma(\omega_s)$ gives

$$\Gamma_{1,2}(\omega_s) = \frac{-\cos\theta_i \pm \sqrt{\cos^2\theta_i - 4BA_i(\omega_i)\sqrt{1 - \frac{\sin^2\theta_i}{4}}\cos\theta_i}}{2BA_i(\omega_i)\sqrt{1 - \frac{\sin^2\theta_i}{4}}}$$

In the case of normal incidence $\cos\theta_i$ will be one and $\sin\theta_i$ will be zero then

$$\Gamma_{1,2}(\omega_s) = \frac{-1 \pm \sqrt{1 - 4BA_i(\omega_i)}}{2BA_i(\omega_i)}$$

This is the same result as obtained for the perpendicular polarization case. Similarly, as discussed in previous section for the case of a linear medium, the last term in Equation (15) will be zero, and the reflection coefficient $\Gamma(\omega_s)$ will be $-1$.

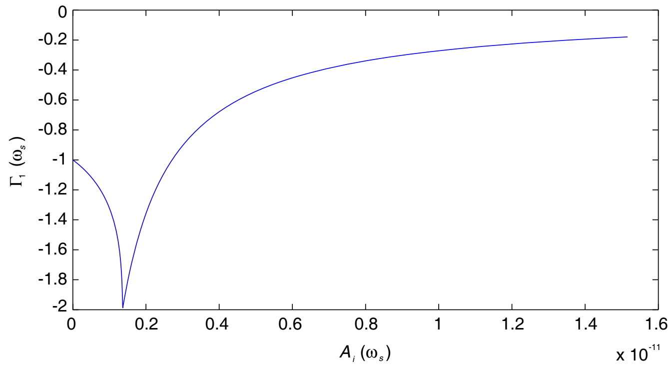


**Figure 2.** Reflection coefficient $\Gamma_1(\omega_s)$ for perpendicular polarization as a function of incident electric field $A_i(\omega_i)$.

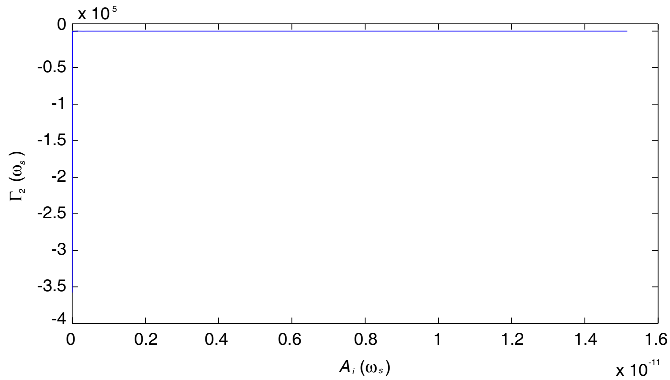


**Figure 3.** Reflection coefficient $\Gamma_2(\omega_s)$ for perpendicular polarization as a function of incident electric field $A_i(\omega_i)$.

## 4. RESULT AND DISCUSSION

By using the value of $\chi_{\mathit{eff}}^{(2)} \simeq 6.9 \times 10^{-12}$ given in [4], the reflection coefficient $\Gamma_{1,2}(\omega_s)$ is plotted against the incident electric field $A_i(\omega_i)$, for perpendicular case in Figures 2 and 3 and for parallel case in Figures 4 and 5. It is assumed that the wave is incident at an angle of $30^\circ$, $\omega_i$ is equal to $1 \times 10^{16}$ rad/s and the permeability of the medium $\epsilon$ does not depend upon frequency of the incident or reflected wave, i.e., the medium has no dispersion. The $\epsilon$ is taken as a scalar and is given by

$$\epsilon = \epsilon_0 \left(1 + \chi_{\mathit{eff}}^{(2)}\right)$$

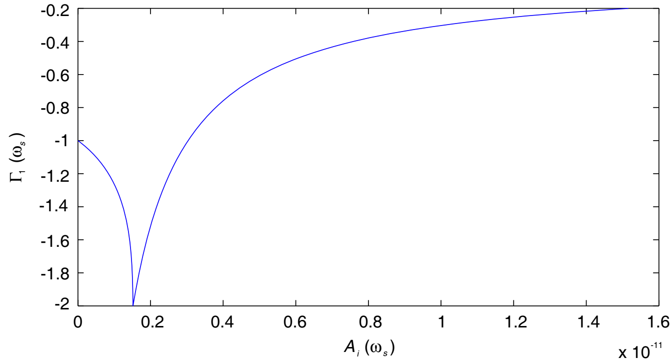


**Figure 4.** Reflection coefficient $\Gamma_1(\omega_s)$ for parallel polarization as a function of incident electric field $A_i(\omega_i)$.

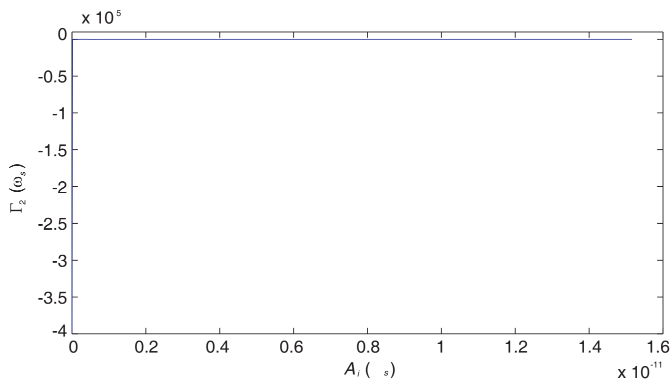


**Figure 5.** Reflection coefficient $\Gamma_2(\omega_s)$ for parallel polarization as a function of incident electric field $A_i(\omega_i)$.

As the angle of incidence is fixed the behaviors of $\Gamma_{1,2}(\omega_s)$ are similar in both perpendicular and parallel polarization cases. $\Gamma_1(\omega_s)$ first decreases then increases. $\Gamma_2(\omega_s)$ increases as $A_i(\omega_i)$ increases in both cases. In our case, the boundary is a PEC plane; the incident field is reflected back in the medium; and some of the incident energy is used to give birth to nonlinear part of the reflected field.

The reflection coefficients in the parallel polarization case also depends on the angle of incidence. In Figures 6 and 7 the dependence of $\Gamma_{1,2}(\omega_s)$ on angle of incidence has been shown graphically. The amplitude of incident electric field $A_i(\omega_i)$ is assumed as $1.5177 \times 10^{-13}$

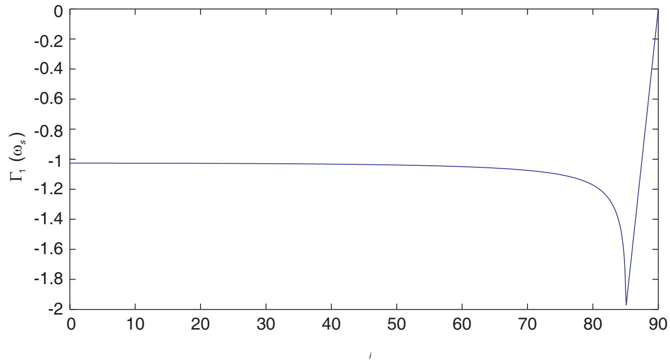


**Figure 6.** Reflection coefficient $\Gamma_1(\omega_s)$ for parallel polarization as a function of incident angle $\theta_i$.

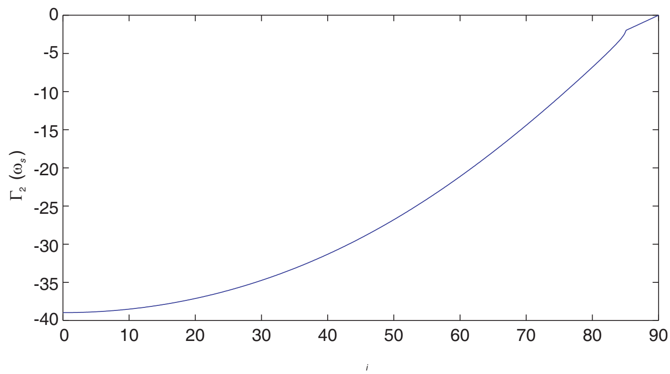


**Figure 7.** Reflection coefficient $\Gamma_2(\omega_s)$ for parallel polarization as a function of incident angle $\theta_i$.

for these plots. $\Gamma_1(\omega_s)$ decreases from $-1$ to $-2$ then goes to zero at the angle 90° and $\Gamma_2(\omega_s)$ increases and goes to zero at the angle 90° because there is no reflection at that angle and surface wave will generate [26].

### 4.1. Application

Waveguide is basically two planes of PEC or any other dielectric media which are facing each other. The results can be used to solve the waveguides filled with a nonlinear medium. Our work is a preliminary one to find the field at a focal point of reflector (embedded in a nonlinear medium) such as circular, spherical or parabolic reflector using Maslov's Method or any other method as geometrical or physical optics. Many authors have explored the reflection and transmission of waves at the interface of linear and nonlinear medium [20–25]. The main difference in our formulism is that the wave reflected completely due to PEC plane, i.e., no transmission.

## 5. CONCLUSIONS

The problem of oblique incidence on a PEC plane embedded in a second order nonlinear optical medium has been analyzed. The effects of the medium's nonlinearity on the reflection coefficients have been revealed. As the nonlinear polarization depends on the incident electric field, the reflection coefficients also varies with the amplitude of the said field. This dependence is shown graphically. The change in the amplitude of reflection coefficients with respect to angle of incidence is also plotted. This change has to be incorporated in finding the field at the focal point of a reflector. It is necessary to sum all the reflected fields to find the intensity of the field at the caustic. This work is carried out to find the said field. Our results can be used to find fields in nonlinear dielectric filled planar waveguides, cavities etc.

It is assumed that the medium is isotropic, lossless and non-dispersive. The reflection coefficients have negative values because of a PEC boundary. As the governing equations of the reflection coefficients are quadratic in nature so the amplitude of incident electric field should be less than $1/(4\frac{(\omega_s^2/\mu_0)}{|k_s|^2-|k_R(\omega_s)|^2}\epsilon_0\chi_{\mathit{eff}}^2)$ otherwise the reflection coefficient will become imaginary and reflected wave will be nonuniform [26].

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