Progress In Electromagnetics Research, Vol. 113, 269–284, 2011

NON-UNIFORM TRANSMISSION LINE TRANSFORMERS AND THEIR APPLICATION IN THE DESIGN OF COMPACT MULTI-BAND BAGLEY POWER DIVIDERS WITH HARMONICS SUPPRESSION

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Abstract—In this paper, the application of compact non-uniform transmission line transformers (NTLTs) in suppressing and controlling the odd harmonics of the fundamental frequency is presented. A design example showing the complete suppression of the odd harmonics of the fundamental frequency is given. In addition, several compact NTLTs are designed showing the possibility of controlling the existence of a fundamental frequency's odd harmonics. Moreover, multi-band operation using NTLTs is investigated. Specifically, a design example of a miniaturized triple-frequency NTLT is introduced. Based on these compact NTLTs, a 3-way triple-frequency modified Bagley power divider (BPD) with a size reduction of 50%, and a 5-way modified BPD with harmonics suppression and size reduction of 34%, are designed. For verification purposes, both dividers are simulated using the two full-wave simulators IE3D and HFSS. Moreover, the modified 5-way BPD with harmonics suppression is fabricated and measured. Both the simulation and measurement results validate the design approach.

1. INTRODUCTION

Recent advances in wireless communication applications demand high performance compact RF circuits with the ability of suppressing the fundamental frequency's harmonics and achieving multi-frequency operation. So, different techniques were proposed in the literature to suppress the unwanted presence of the odd multiples of the design frequency and achieving multi-frequency operation for microwave

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Received 11 January 2011, Accepted 30 January 2011, Scheduled 3 February 2011
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components, such as power dividers and microwave couplers. In [1], a miniaturized Wilkinson power divider (WPD) with harmonics suppression was presented. Nevertheless, parallel combinations of capacitors and inductors were considered in the design. In [2], reduced size WPD and branch line coupler with harmonics suppression were proposed using T-shaped structure, where additional microstrip stubs had to be added to the conventional design. In [3], a WPD with size reduction of 68% and higher harmonics suppression was achieved. In [4], a modified WPD with nth harmonic suppression was proposed, where a parallel combination of an inductor and a resistor was used for isolation, and extra stubs were added to the conventional transmission line transformer (TLT) sections. In [5], a multi-frequency WPD was presented. Nevertheless, as the number of operating frequencies increases, more uniform transmission line transformer (UTLT) sections were needed which results, eventually, into a larger circuit area, especially at low frequencies. In [6], a compact dual-frequency three-way unequal power divider was presented. However, multiple sections of impedance transformers were used which increases the circuit area. In [7], a miniaturized dual-band power divider with harmonics suppression for GSM applications using artificial transmission lines was introduced. In [8, 9], dual-frequency WPDs were presented, in which additional microstrip lines were added to the conventional design.

Unlike the Wilkinson power dividers (WPDs), the Bagley polygon power divider [10–13] is one of the microwave power dividers that do not use lumped elements, and can be easily extended to any number of output ports. However, one of their major drawbacks is the large area they occupy especially at low frequencies. In [10], reduced size multi-way Bagley power divider (BPD) was presented, in which additional microstrip lines (in the form of shunt stubs) were added to the conventional BPD. In [11], an optimum design of a modified 3-way rectangular BPD was presented. In [12], a general design of compact multi-way dividers based on BPDs was introduced. In [13], a compact dual-frequency 3-way BPD using composite right/left handed (CRLH) transmission lines was presented, in which parallel combinations of lumped elements (capacitors and inductors) were used.

In this paper, the design of non-uniform transmission line transformers (NTLTs) [14–18] that are capable of reducing the circuit area, suppressing and/or controlling the odd harmonics of the fundamental frequency, and achieving multi-frequency operation will be presented. Then, in contrast to the techniques proposed in [1–10] and [13] in which lumped elements and/or extra microstrip lines were incorporated into the conventional design of passive microwave components, these NTLTs are used in the design of miniaturized multi-band BPDs. Specifically, a miniaturized triple-frequency 3-way BPD and a compact single-band 5-way BPD with harmonics suppression are designed. The designed dividers are simulated using two different full-wave simulators (HFSS and IE3D). Moreover, the 5-way BPD is fabricated and measured. The results of the simulations and the measurements verify the design approach.

The paper is divided as follows: Section 2 briefly presents the theory of designing compact NTLTs. Section 3 shows the capability of NTLTs in suppressing and controlling the odd harmonics, in addition to their capability in achieving multi-band operation. In the same section, these NTLTs are incorporated into the design of compact 3-way and 5-way BPDs. Simulation and measurement results are given in the same section too. Section 4 concludes the paper.

2. DESIGN OF COMPACT NTLTS

In this section, the theory of designing compact NTLTs is briefly presented. A general simple design procedure of NTLTs was introduced in [14, 15], and their application in the design of compact Wilkinson power dividers and branch line couplers can be found in [16–18]. Figure 1 shows a typical uniform transmission line transformer (UTLT) that matches a load impedance Z_L to a source impedance Z_s, with a length of $d_0 = \lambda/4$ and a characteristic impedance Z_0. An equivalent non-uniform transmission line transformer (NTLT) of length d, with a varying characteristic impedance $Z(z)$ and propagation constant $\beta(z)$, is also shown in Figure 1. The NTLT is designed so that the magnitude of the reflection coefficient $|\Gamma|$ is enforced to be zero (or very small) at a single design frequency or several design frequencies of interest. Moreover, size reduction is achieved by choosing the length d to be smaller than d_0.

The magnitude of the reflection coefficient of the NTLT can be calculated from its $ABCD$ parameters which can be found by

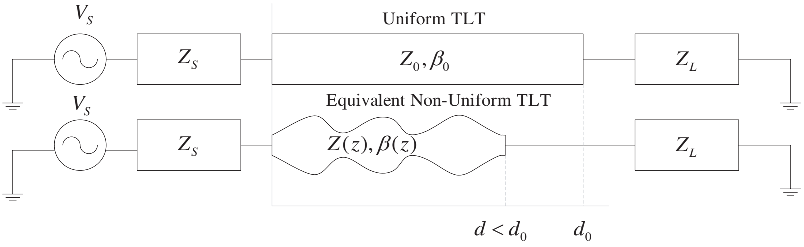


Figure 1. A uniform TLT versus a compact non-uniform TLT.

subdividing it into K uniform electrically short segments with length of Δz. The $ABCD$ parameters of the whole NTLT are obtained by multiplying the $ABCD$ parameters of each section as follows [19]:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdots \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix} \cdots \begin{bmatrix} A_K & B_K \\ C_K & D_K \end{bmatrix} \tag{1}$$

where the $ABCD$ parameters of the ith segment are [19]:

$$A_i = D_i = \cos(\Delta\theta) \tag{2a}$$

$$B_i = Z^2((i-0.5)\Delta z) C_i = jZ((i-0.5)\Delta z)\sin(\Delta\theta), \quad i=1,2,\ldots,K \tag{2b}$$

$$\Delta\theta = \frac{2\pi}{\lambda}\Delta z = \frac{2\pi}{c} f\sqrt{\varepsilon_{eff}}\Delta z \tag{2c}$$

Then, the following truncated Fourier series expansion for the normalized characteristic impedance $\bar{Z}(z) = \frac{Z(z)}{Z_0}$ is considered [14, 15]:

$$\ln\left(\bar{Z}(z)\right) = \sum_{n=0}^{N} C_n \cos\left(\frac{2\pi n z}{d}\right) \tag{3}$$

So, an optimum designed compact length NTLT has to have its reflection coefficient magnitude at the design frequencies (f_j, $j = 1, \ldots, M$) as close as possible to zero. Therefore, the optimum values of the Fourier coefficients C_n's can be obtained through minimizing the following error function [15]:

$$Error = \sqrt{\sum_{j=1}^{M} \left|\Gamma_{in}(f_j)\right|^2} \tag{4}$$

where:

$$Z_{in}(f_j) = \frac{A(f_j)\cdot Z_L + B(f_j)}{C(f_j)\cdot Z_L + D(f_j)} \tag{5a}$$

$$\Gamma_{in}(f_j) = \frac{Z_{in}(f_j) - Z_s}{Z_{in}(f_j) + Z_s} \tag{5b}$$

Also, the error function in (4) should be restricted by some constraints such as reasonable fabrication and physical matching, as follows:

$$\bar{Z}_{\min} \le \bar{Z}(z) \le \bar{Z}_{\max} \tag{6a}$$

$$\bar{Z}(0) = \bar{Z}(d) = 1 \tag{6b}$$

So, the goal is to find the Fourier coefficients values (C_n's) that minimize the above error function at the desired M frequencies. To solve the above constrained minimization problem, the MATLAB function "fmincon.m" is used.

3. EXAMPLES ON NTLTS APPLICATIONS

In this section, some examples will be presented to show the capabilities of the NTLTs presented in Section 2 in achieving size reduction, harmonics suppression or control, and multi-frequency operation. Then, these NTLTs will be used in the design of miniaturized triple-band 3-way BPD and single-band 5-way BPD.

3.1. NTLTs for Harmonics Suppression

First, a NTLT that matches a load impedance of $33.33\,\Omega$ to a source impedance of $100\,\Omega$ is designed to operate at a fundamental frequency of 1 GHz. For an FR-4 substrate with a relative permittivity (ε_r) of 4.6, and a substrate height of 1.6 mm, the length of the UTLT is 41 mm. Considering Equation (4) with $|\Gamma_{in}(f_j)| = |\Gamma_{in}(1\,\text{GHz})|$ and choosing the length of the NTLT as $d = 27\,\text{mm}$, a size reduction of 34% is achieved. Figure 2 shows S_{11} (in dB) for both the UTLT and NTLT in the frequency range from 0 to 10 GHz. It can be clearly seen that using the NTLT results in the complete suppression of the odd harmonics of the fundamental frequency. The Fourier coefficients for the designed NTLT are shown in Table 1. It should be mentioned here that the error value in Equation (4) equals 1.5×10^{-7}.

Table 1. Fourier coefficients for the designed reduced size NTLT.

C_0	C_1	C_2	C_3	C_4	C_5
0.0675	**0.2104**	**−0.9999**	**−0.3045**	**0.2079**	**−0.2929**
C_6	C_7	C_8	C_9	C_{10}	
0.4117	**0.2248**	**0.1745**	**0.2614**	**0.0392**	

3.2. NTLTs for Harmonics Control

Besides the capability of harmonics suppression, compact NTLTs can also control the presence of the fundamental frequency's odd harmonics. By choosing the wanted harmonics along with the fundamental frequency in Equation (4), one can control the existence of any harmonics. Figures 3–5 represent different scenarios considering the same fundamental frequency, the same substrate, and the same UTLT and NTLT lengths mentioned above.

Figure 3 shows the suppression of the third, fifth and ninth odd harmonics while maintaining the seventh. Also, Figure 4 shows the suppression of the third and the seventh ones, while Figure 5 shows

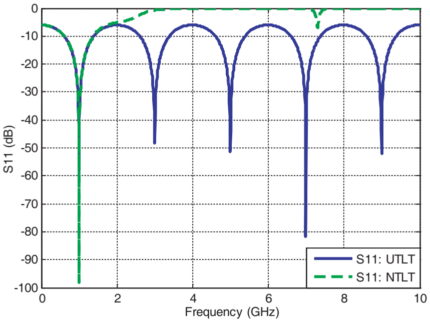


Figure 2. S_{11} (in dB) of the reduced size NTLT versus the UTLT.

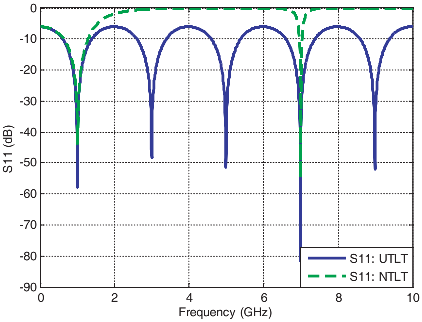


Figure 3. S_{11} (in dB) of the NTLT that suppresses the 3rd, 5th and 9th harmonics versus the UTLT response.

Table 2. Fourier coefficients for the different scenarios of the harmonics suppressing NTLTs.

$C_n's$	C_0	C_1	C_2	C_3	C_4
1st scenario	**−0.0567**	**−0.7060**	**0.5868**	**0.6973**	**0.0581**
2nd scenario	**0.0134**	**−0.1214**	**0.0864**	**−0.1381**	**0.4421**
3rd scenario	**−0.0069**	**−0.2315**	**−0.4156**	**0.4524**	**−0.2033**
C_5	C_6	C_7	C_8	C_9	C_{10}
−0.1436	**−0.0134**	**−0.0664**	**−0.1611**	**−0.1330**	**−0.0618**
0.2422	**0.5535**	**−0.5339**	**−0.4239**	**0.3248**	**−0.4451**
−0.6160	**0.0588**	**−0.0484**	**0.7335**	**0.0776**	**0.1993**

the suppression of the fifth and the seventh ones. Table 2 below summarizes the Fourier series coefficients of the three different cases. The obtained error values in Equation (4) for the three different cases were in terms of 10^{-4}.

3.3. NTLTs for Multi-band Operation

So far, using the NTLTs, harmonics suppression and control were introduced. In this section, compact triple-frequency NTLT will be presented, and is then, applied in the design of a reduced size triple-frequency 3-way modified BPD. Figure 6 shows the schematic diagram of the modified 3-way BPD proposed in [12]. Using simple transmission line theory, it can be easily shown that the length l_h can

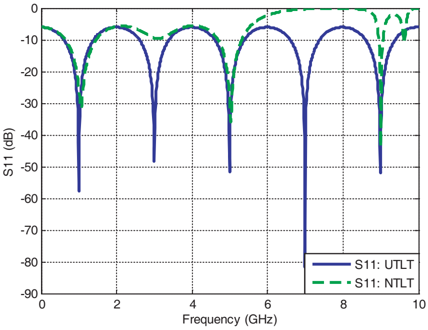


Figure 4. S_{11} (in dB) of the NTLT that suppresses the 3rd and 7th harmonics versus the UTLT response.

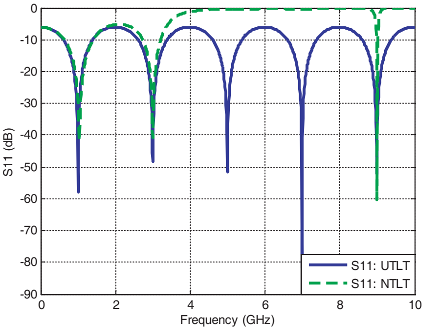


Figure 5. S_{11} (in dB) of the NTLT that suppresses the 5th and 7th harmonics versus the UTLT response.

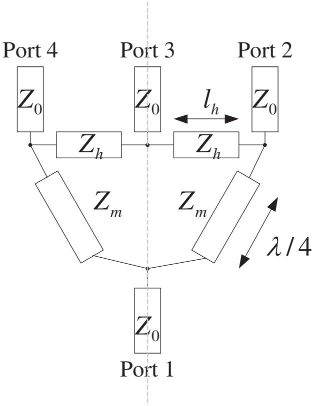


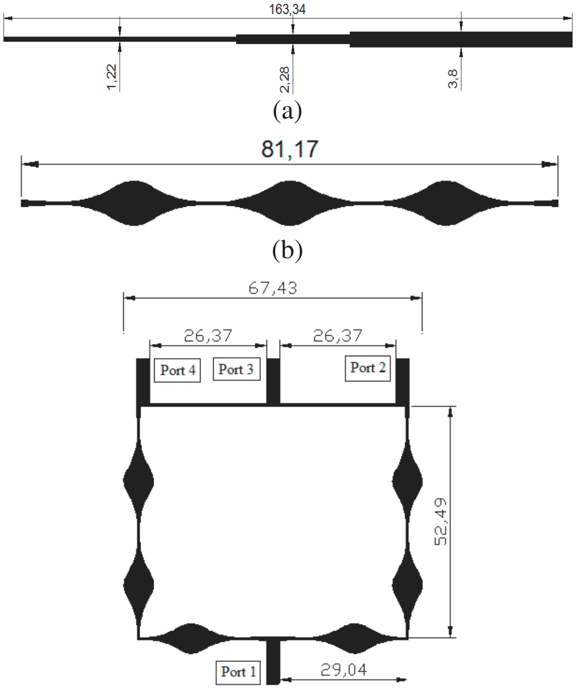
Figure 6. Schematic diagram of the 3-way modified BPD proposed in [12].

be arbitrarily chosen, while the impedances are given as: $Z_h = 2Z_0$ and $Z_m = \frac{2Z_0}{\sqrt{3}}$ [12]. Thus, each quarter-wave section matches an impedance of $\frac{2Z_0}{3}$ to $2Z_0$, resulting in a perfect match at port 1 (the input port) and equal split power division to the three output ports.

Here, assuming $Z_0 = 50\,\Omega$, these uniform quarter-wave transformers will be replaced by triple-frequency NTLTs that match a load impedance of $33.33\,\Omega$ to a source impedance of $100\,\Omega$. Specifically, considering the same substrate mentioned earlier, and operating frequencies of 0.5 GHz, 1.25 GHz and 2 GHz, each

57.735 Ω quarter-wave uniform section is replaced by an NTLT having a length of 81.2 mm (which corresponds to a quarter-wavelength at 0.5 GHz), and a non-uniform impedance bounded between $\left(0.5 \leq \bar{Z}(z) \leq 2.176\right)$ corresponding to a microstrip width variation of $(0.332\,\text{mm} \leq W(z) \leq 6.8\,\text{mm})$. For comparison purposes, Figure 7(a) shows the triple-frequency uniform 3-section transformer that has been designed using the expressions in [20], while Figure 7(b) presents the proposed layout of the triple-frequency NTLT section. Also, Figure 7(c) presents the proposed BPD after incorporating the designed NTLT sections instead of the conventional uniform ones.

It is worth mentioning here that compactness is achieved without adding any extra transmission lines and/or lumped elements. Moreover, during the optimization process, Equation (6b) was not enforced to give more freedom in the design. Comparing Figures 7(a) and 7(b), it is clear that a size reduction of 50% is achieved.

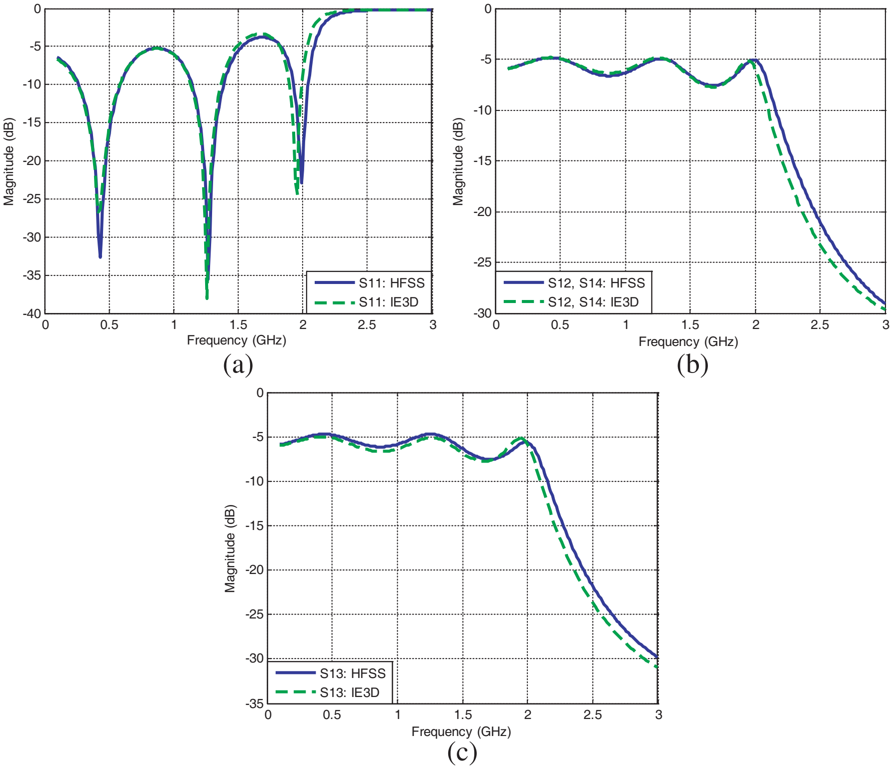


(a)

(b)

(c)

Figure 7. (a) The triple-frequency uniform 3-section transformer, (b) the proposed triple-frequency NTLT section, and (c) the proposed 3-way NTLTs-based BPD. (Dimensions are in mm).



(a) (b)

(c)

Figure 8. Simulation results for the triple-band 3-way NTLTs-based BPD.

Figure 8 shows the full-wave simulation results for the designed compact 3-way BPD (shown in Figure 7(c)) using the full-wave simulators IE3D [21] and HFSS [22]. The triple-frequency behavior is very clear. Very good input port matching is achieved around the design frequencies. Specifically, the input port matching parameter S_{11} is below -20 dB around the design frequencies. Also, the transmission parameters S_{12}, S_{13} and S_{14} vary between -4.8 dB and -5 dB at the design frequencies, which is very close to their theoretical value of -4.77 dB. It is worth mentioning here that, because of the symmetry of the BPD structure, the transmission parameter S_{14} is the same as the transmission parameter S_{12}. In general, the results are acceptable; with a small shift in the design frequencies (especially at the lower frequency 0.5 GHz) which is due to the resulting error in the optimization process and the effect of the discontinuities. The Fourier series coefficients for the triple-frequency NTLT are given in Table 3 with an error value of 0.042.

Table 3. The Fourier coefficients for the designed triple-frequency NTLT.

C_0	C_1	C_2	C_3	C_4	C_5
0.1292	**0.1516**	**0.2025**	**0.5549**	**−0.3555**	**−0.1145**
C_6	C_7	C_8	C_9	C_{10}	
−0.0658	**−0.0082**	**−0.0241**	**−0.0190**	**−0.0163**	

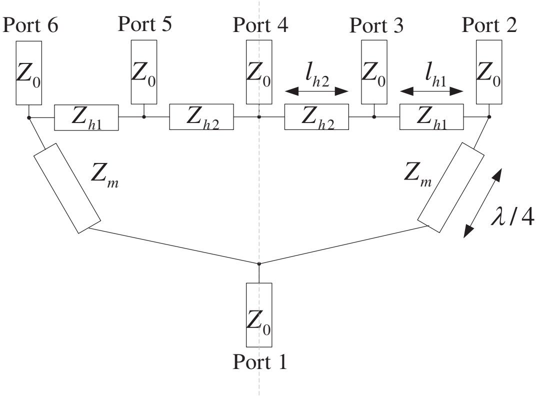
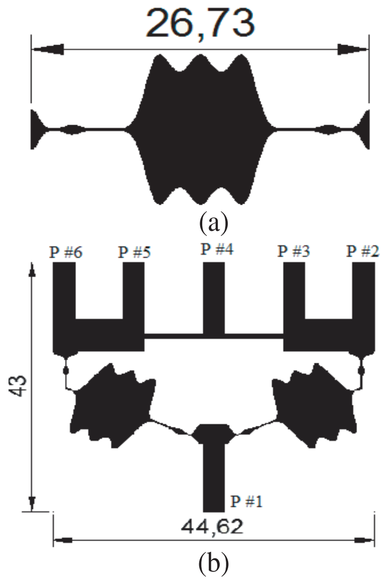


Figure 9. Schematic diagram of the modified 5-way BPD proposed in [12].

3.4. Design of a 5-way BPD with Harmonics Suppression

Figure 9 shows the schematic diagram of the modified 5-way BPD proposed in [12]. According to the analysis presented in [12], the lengths l_{h1} and l_{h2} can be arbitrarily chosen, and the impedances are given as follows: $Z_m = \frac{2Z_0}{\sqrt{5}}$, $Z_{h2} = 2Z_0$, and $Z_{h1} = 2Z_0/3$. Thus, each quarter-wave section matches an impedance of $\frac{2Z_0}{5}$ to $2Z_0$, resulting in a perfect match at port 1 (the input port) and equal split power division to the five output ports. Using the same approach presented above, the quarter-wave uniform sections will be replaced by their equivalent NTLTs.

For $Z_0 = 50\,\Omega$, and considering the same substrate mentioned above, the corresponding microstrip line widths are $W_m = 3.55$ mm, $W_{h1} = 5.511$ mm, and $W_{h2} = 0.64$ mm. Now, for a design frequency of 1 GHz, each quarter-wave section (with $d_0 = 40.04$ mm, and $Z_m = 44.72\,\Omega$) that matches a load impedance $Z_L = 20\,\Omega$ to a source impedance $Z_S = 100\,\Omega$, is replaced with an equivalent NTLT having a length of 26.73 mm, and a non-uniform impedance bounded between $\left(0.385 \leq \bar{Z}(z) \leq 3.57\right)$ corresponding to a width variation of $(0.12\,\text{mm} \leq W(z) \leq 13\,\text{mm})$. Figure 10(a) shows the designed NTLT, while Figure 10(b) presents the proposed layout of the 5-way BPD after incorporating the NTLTs instead of the uniform quarter-wave transformers.



(a)

(b)

Figure 10. (a) The designed NTLT, (b) the proposed compact 5-way BPD using NTLTs. (Dimensions in mm).

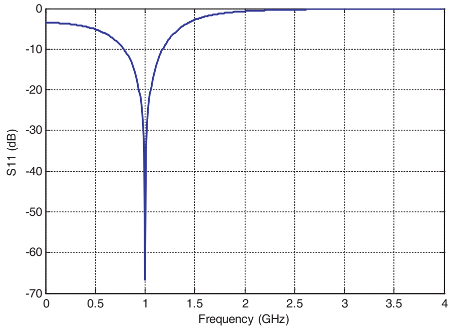
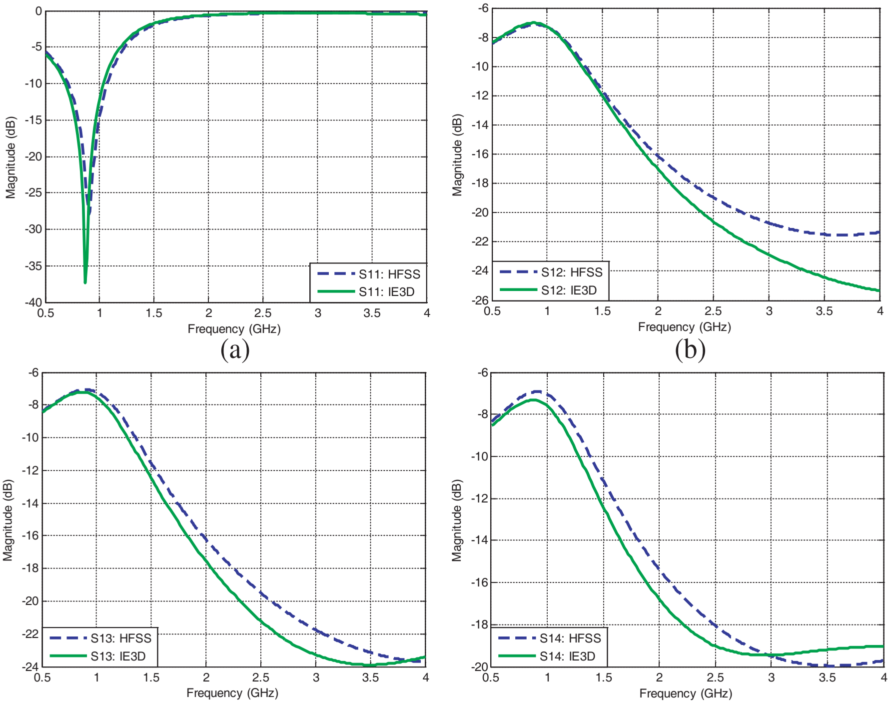


Figure 11. S_{11} of the designed single-frequency NTLT shown in Figure 10(a).

Figure 11 shows the input port matching parameter (S_{11}) for the designed NTLT that will replace each UTLT in the 5-way BPD. It is clear from Figure 11 that a perfect match is obtained at the design frequency (1 GHz). Moreover, the 3rd harmonic of the fundamental frequency is suppressed.

Figure 12 shows the full-wave simulation results of this compact NTLTs-based 5-way BPD. In Figure 12(a), the input port matching parameter S_{11} is below -25 dB with a small frequency shift of about 0.1 GHz from the design frequency. This shift could be due to the parasitic effects of the discontinuities in the structure. Figures 12(b), 12(c) and 12(d) present the transmission parameters S_{12}, S_{13}, and S_{14}, respectively, of the designed compact 5-way BPD. The values of the insertion losses are close to their theoretical value of -7 dB at 0.9 GHz. Better performance could be achieved by optimizing the structure of the divider to obtain the exact response at the design frequency. It should be mentioned here that $S_{12} = S_{16}$, and $S_{13} = S_{15}$, due to the symmetry of the structure. The suppression of the odd harmonics of



(a) (b)

(c) (d)

Figure 12. Scattering parameters of the compact 5-way BPD.

Table 4. The Fourier coefficients for the NTLT used in the design of the compact 5-way BPD.

C_0	C_1	C_2	C_3	C_4	C_5
0.3116	**0.9912**	**−0.5972**	**−0.2370**	**0.2493**	**−0.1220**
C_6	C_7	C_8	C_9	C_{10}	
−0.2223	**−0.0185**	**−0.1720**	**−0.1301**	**−0.0531**	

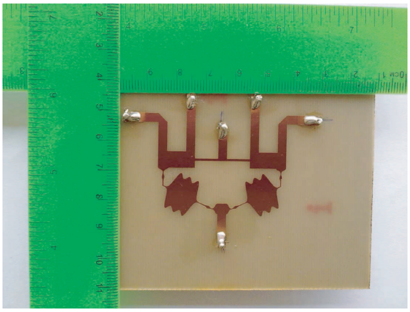


Figure 13. A photograph of the fabricated compact 5-way BPD with NTLTs.

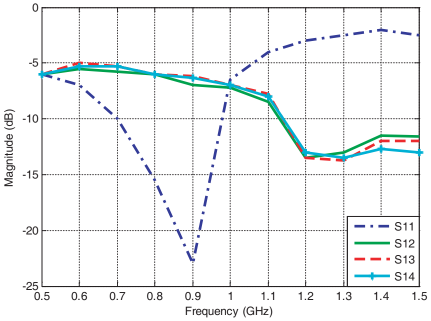


Figure 14. The measured scattering parameters of the fabricated 5-way miniaturized BPD.

the design frequency is clearly seen. This is due to the fact that the designed compact NTLT was enforced to give a reflection coefficient magnitude as close to zero as possible at the fundamental frequency only. Table 4 gives the Fourier coefficients for the designed NTLT shown in Figure 10(a). The error value in Equation (4) was around 10^{-6}.

For further validation, the designed compact 5-way BPD has been fabricated. A photograph of the fabricated divider is shown in Figure 13. Figure 14 shows the measurement results using an Agilent Spectrum Analyzer (with a built-in tracking generator extending from 0–1.5 GHz). The measured results are in good agreement with the simulation ones shown in Figure 12. The measured input port matching parameter S_{11} is below -20 dB at 0.9 GHz, and the transmission parameters vary around -7 dB at the design frequency. The small discrepancies between simulation and measurement results could be due to the connectors, fabrication process, measurement errors, and the fact that a spectrum analyzer (not a network analyzer) is used.

4. CONCLUSIONS

The concept of non-uniform transmission line transformers (NTLTs) was used in the achievement of size reduction, harmonics suppression, harmonics control, and multi-frequency operation. Based on this concept, compact Bagley power dividers (BPDs) were designed. Specifically, a triple-frequency 3-way BPD was designed and simulated, and a 5-way BPD with harmonics suppression was designed, simulated, and fabricated. The simulations and experimental results prove the validity of the design approach. Size reduction of about 50% and 34% were achieved for the 3-way and 5-way BPDs, respectively. Discrepancies between simulation and measurements are mainly due to the fabrication process and measurements errors.

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