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COUPLED WAVES IN THE PERIODIC COMPOSITE MAGNETIC-SEMICONDUCTING MEDIA

M. N. Shaikh and R. Ali

Department of Physics
University of the Punjab
Quaid-i-Azam Campus, Lahore 54590, Pakistan

Abstract—The Alfven-spin and helicon-spin waves are analyzed in both sinusoidal periodic and layered periodic structures. These periodic structures are composed of a single composite medium having the properties of both magnetic and semiconducting materials. Numerical analysis of the dispersion relations presented for these periodic structures shows band-gap effects. The idea of these band-gap effects could be utilized in the design of periodic structures operating at microwave frequencies. Extreme cases for the decoupled independent modes in the absence of magnetization or carriers are also discussed.

1. INTRODUCTION

The realization of the band-gap effects to the problems of wave propagation is obtained by changing the longitudinal properties of the medium. The periodic structures that exhibit the band-gap effects can be described by two simple one-dimensional models, i.e., (i) a sinusoidal periodic medium and (ii) a layered periodic medium. In these cases, the modulation or inhomogeneity is characterized along one coordinate about an average value of number density or dielectric constant etc. (see, e.g., [1, 2] and references therein). Although the literature survey reveals that the topic of the wave propagation in periodic and multilayered media has been extensively addressed but still some progress is going on in this direction. For example, Choubani et al. [3] derived a method based on Hill's equation and matrix concept for the analysis of electromagnetic wave propagation in multilayered structures with arbitrary profiles. Gürel and Öncü [4]

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Corresponding author: Rashid Ali (Rashid_Ali2003@hotmail.com).

investigated for the first time the electromagnetic wave propagation through a plasma layer with equi-width sub-layers as an alternative to the previous multilayered designs. He considered a linearly varying electron density distribution with positive and negative slopes through a magnetized plasma layer. The references [1–4] are limited to the linear investigations, however, there has been a considerable progress in the nonlinear theory of wave propagation in the layered media and superlattices. For example, Shah et al. [5] and Ali and Shah [6] used Kronig-Penney model to investigate helicon solitons in a layered medium. The nonlinear wave propagation in the sinusoidal periodic media is analyzed less extensively. The periodic structures consisting of semiconductor and ferromagnetic layers combine different physical properties which may be useful for obtaining magnetic systems in semiconductor electronics and spintronics. In an external magnetic field, the characteristics of these structures can easily be changed. Shramkova [7] investigated the specific features of transverse electric wave propagation in a structure fabricated by periodic alternating ferrite and semiconducting layers. Similarly, the study of coupled waves, e.g., Alfven-spin or helicon-spin waves in the composite materials like magnetic-semiconducting media, has been the subject of investigation for different experimental and theoretical reasons. Work has been done by various authors in this direction [8–11].

In this paper, an analysis is presented to describe the parallel propagating circularly polarized Alfven-spin and helicon-spin waves in both sinusoidal periodic and layered periodic structures. These periodic structures are composed of a single composite magnetic-semiconducting medium. Standard analytical techniques for the periodic structures are followed in these investigations (see, e.g., Achar [1], and references therein). This work may be of some considerable interest because of its possible practical applications in the design of structures operating at microwave frequencies. In Section 2, the Alfven-spin wave in the sinusoidal and layered periodic structures is investigated. It has been shown that coupling between the spin system and the electron-hole system in a magnetic-semiconducting medium can support Alfven-spin wave.

2. ALFVEN-SPIN WAVE IN THE PERIODIC STRUCTURES

The parallel propagating circularly polarized Alfven-spin wave is considered in the presence of an externally applied uniform magnetic field B_o directed along the z-axis. By neglecting exchange interactions [11], the following basic set of equations is considered for the propagation of Alfven-spin wave in the hydrodynamic approximation:

$$\frac{\partial}{\partial z}E_{\pm} = \pm i\frac{\partial}{\partial t}B_{\pm}, \tag{1}$$

$$\frac{\partial}{\partial z}H_{\pm} = \mp iJ_{\pm} \mp i\frac{\partial}{\partial t}D_{\pm}, \tag{2}$$

$$\frac{\partial}{\partial t}M_{\pm} = \pm i\mu_o\gamma\left[H_oM_{\pm} - M_oH_{\pm}\right], \tag{3}$$

$$\frac{\partial}{\partial t}V_{e\pm} = -\frac{e}{m_e^*}E_{\pm} \pm i\frac{e}{m_e^*}\left(B_oV_{e\pm}\right), \tag{4}$$

$$\frac{\partial}{\partial t}V_{h\pm} = \frac{e}{m_h^*}E_{\pm} \mp i\frac{e}{m_h^*}\left(B_oV_{h\pm}\right), \tag{5}$$

where $B_{\pm} = \mu_o(H_{\pm} + M_{\pm})$, $J_{\pm} = -en_e(z)V_{e\pm} + en_h(z)V_{h\pm}$, $D_{\pm} = \varepsilon(z)E_{\pm} = \varepsilon_o\acute{\varepsilon}(z)E_{\pm}$ and $B_o = \mu_o(H_o + M_o)$. The subscripts e and h refer to the electron and hole, respectively. Here for the periodic modulation, the number densities $n_{e,h}(z)$ and the dielectric constant $\acute{\varepsilon}(z)$ are characterized along the z-axis. The perpendicular fluctuating quantities have all been expressed in the form $a_{\pm} = a_x \pm ia_y$. In the above set of equations $E_{\pm}$, $B_{\pm}$, $H_{\pm}$, $D_{\pm}$, $M_{\pm}$, $V_{e\pm}$ and $V_{h\pm}$ are the fluctuating electric field, magnetic induction, magnetic field, electric induction, magnetization and the electron and hole velocities, respectively. Magnetic susceptibility, gyromagnetic ratio, effective mass of electron and hole are given by μ_o, γ, m_e^* and m_h^*, respectively.

In order to discuss the Alfven-spin wave for a sinusoidal periodic structure, the number densities and dielectric constant are modulated along the z-axis and they are periodic of the forms given by the following Equations (6) and (7). The layered periodic structure will be discussed later. Since in the semiconductor plasma, number densities of electron and hole are equal, therefore, $n_e(z) = n_h(z) = n(z)$.

$$n(z) = n_o[1 + \bar{n}\cos(Qz)], \tag{6}$$

$$\acute{\varepsilon}(z) = \acute{\varepsilon}[1 + \bar{\varepsilon}\cos(Qz)], \tag{7}$$

where $\bar{n} = \frac{\Delta n}{n_o} \ll 1$ and $\bar{\varepsilon} = \frac{\Delta\acute{\varepsilon}}{\acute{\varepsilon}} \ll 1$ are modulation factors. Here Δn, $\Delta\acute{\varepsilon}$ are the modulation in number density and modulation in dielectric constant, respectively, whereas n_o is the average uniform number density. The quantity $2\pi/Q$ is taken as the period of modulation or period of inhomogeneity. To obtain the expression for Bloch wave type solution along the z-axis, a periodic time dependence of frequency ω for all time dependant quantities is assumed. Therefore, using (6) and (7) in Equations (1)–(5) and skipping the algebraic details, the equation for $H_\pm$ is obtained by eliminating all other fluctuating quantities as

$$\frac{d^2}{dz^2}H_\pm+\left[\left[\frac{e^2n\omega\mu_o}{(\omega\pm\omega_{ce})(\omega\mp\omega_{ch})}\pm\frac{e^2n\omega\mu_o^2\gamma M_o}{(\omega\pm\omega_{ce})(\omega\mp\omega_{ch})(\omega\pm\mu_o\gamma H_o)}\right]\right.$$
$$\cdot\left[\left\{-\frac{(\omega\pm\omega_{ce})}{m_h^*}-\frac{(\omega\mp\omega_{ch})}{m_e^*}\right\}[1+\bar{n}\cos(Qz)]\right]$$
$$\left.+\left[\left\{\omega^2\mu_o\varepsilon\pm\frac{\omega^2\mu_o^2\gamma M_o\varepsilon}{(\omega\pm\mu_o\gamma H_o)}\right\}[1+\bar{\varepsilon}\cos(Qz)]\right]\right]H_\pm=0, \tag{8}$$

where $\omega_{ce}=eB_o/m_e^*$, $\omega_{ch}=eB_o/m_h^*$ and $\varepsilon=\varepsilon_o\acute{\varepsilon}$. Using the conditions $\omega\ll\omega_{ce}$, ω_{ch}, a simplified expression can be written as

$$\frac{d^2}{dz^2}H_\pm+\left[\frac{\omega^2}{V_A^2}[1+\bar{n}\cos(Qz)]\right.$$
$$\left.+\omega^2\mu_o\varepsilon[1+\bar{\varepsilon}\cos(Qz)]\right]\left[1\pm\frac{\mu_o\gamma M_o}{\omega\pm\mu_o\gamma H_o}\right]H_\pm=0, \tag{9}$$

where $V_A^2=B_o^2/[\mu_o n_o(m_e^*+m_h^*)]$, V_A is the Alfven wave velocity.

Let $\phi=Qz/2$, the Equation (9) in the standard form of Mathieu's equation can be written as

$$\frac{d^2}{d\phi^2}H_\pm+(\alpha_{AS}-\beta_{AS}^2\cos^2\phi)H_\pm=0, \tag{10}$$

where

$$\alpha_{AS}=\frac{4\omega^2}{Q^2}\left[\frac{1}{V_A^2}(1-\bar{n})+\mu_o\varepsilon(1-\bar{\varepsilon})\right]\left[1\pm\frac{\mu_o\gamma M_o}{\omega\pm\mu_o\gamma H_o}\right], \tag{11}$$
$$\beta_{AS}^2=-\frac{8\omega^2}{Q^2}\left[\frac{1}{V_A^2}\bar{n}+\mu_o\varepsilon\bar{\varepsilon}\right]\left[1\pm\frac{\mu_o\gamma M_o}{\omega\pm\mu_o\gamma H_o}\right]. \tag{12}$$

If $\cos^2\phi$ is a periodic function with a period of π, then $H_\pm(\phi+\pi)$ is also a solution of (10). Hence the solution $H_\pm(\phi)$ satisfies the Floquet's theorem [1] and therefore can be written as

$$H_\pm(\phi)=e^{is_{AS}\phi}P(\phi),$$

where s_{AS} depends upon α_{AS} and β_{AS}, and is the characteristic constant which plays the role of a wave vector. Here $P(\phi)$ is a periodic function of ϕ which stands for both types of circular polarizations. Now considering a Fourier series for $P(\phi)$, the solution of (10) becomes

$$H_\pm(\phi)=e^{is_{AS}\phi}\sum a_n e^{i2n\phi}, \tag{13}$$

By substituting (13) in (10), the basic recursion relation is obtained as

$$\beta_{AS}^2a_{n+1}+\left[2\beta_{AS}^2-4\alpha_{AS}+16\left(n+\frac{s_{AS}}{2}\right)^2\right]a_n+\beta_{AS}^2a_{n-1}=0. \tag{14}$$

The series (13) is convergent only for certain arbitrary chosen values of a_o, a_1 and a_{-1}. By substituting $n = 0$, the relation (14) becomes

$$s_{AS}^2 = \left[\alpha_{AS} - \frac{\beta_{AS}^2}{4}\left(2 + \frac{a_1}{a_o} + \frac{a_{-1}}{a_o}\right)\right]. \tag{15}$$

The continued fractions a_1/a_o and a_{-1}/a_o can be obtained by using the recursion relation (14). After skipping the rather messy algebra, the continued fractions are given by

$$\begin{aligned}\frac{a_1}{a_o} =& -\left[\frac{\beta_{AS}^2}{2\beta_{AS}^2-4\alpha_{AS}+16\left(1+\frac{s_{AS}}{2}\right)^2}\right]-\left[\frac{\beta_{AS}^2}{2\beta_{AS}^2-4\alpha_{AS}+16\left(2+\frac{s_{AS}}{2}\right)^2}\right]\\ &+\left[\frac{\beta_{AS}^2}{2\beta_{AS}^2-4\alpha_{AS}+16\left(3+\frac{s_{AS}}{2}\right)^2}\right], \end{aligned}\tag{16}$$

$$\begin{aligned}\frac{a_{-1}}{a_o} =& -\left[\frac{\beta_{AS}^2}{2\beta_{AS}^2-4\alpha_{AS}+16\left(1-\frac{s_{AS}}{2}\right)^2}\right]-\left[\frac{\beta_{AS}^2}{2\beta_{AS}^2-4\alpha_{AS}+16\left(2-\frac{s_{AS}}{2}\right)^2}\right]\\ &+\left[\frac{\beta_{AS}^2}{2\beta_{AS}^2-4\alpha_{AS}+16\left(3-\frac{s_{AS}}{2}\right)^2}\right]. \end{aligned}\tag{17}$$

The Equation (15) is the required dispersion relation of Alfven-spin wave in a sinusoidal periodic structure, which shows the Alfven-spin wave dispersion and its variation with modulation amplitude.

Now the propagation of Alfven-spin wave in the layered periodic structure is discussed. For this purpose, without going into mathematical details from the beginning, the sinusoidal modulation is ignored in Equation (9) and the resulting equation is

$$\frac{d^2}{dz^2}H_\pm + F_{AS}(z)H_\pm = 0, \tag{18}$$

where

$$F_{AS}(z) = \omega^2\left[\frac{1}{V_A^2} + \mu_o\varepsilon\right]\left[1 \pm \frac{\mu_o\gamma M_o}{\omega \pm \mu_o\gamma H_o}\right].$$

Equation (18) is Hill's equation, which is a generalization of Mathieu's equation and $F_{AS}(z)$ is a periodic function of z. If the layered periodic structure is composed of alternating homogeneous layers of thicknesses d_1 and d_2 of the composite magnetic-semiconducting medium having different densities and dielectric constants, the solutions of (18) can be written as

$$\begin{aligned}H_{\pm1}(z) &= Ae^{ik_{AS1}z} + Be^{-ik_{AS1}z}, \quad 0 \le z \le d_1,\\ H_{\pm2}(z) &= Ce^{ik_{AS2}z} + De^{-ik_{AS2}z}, \quad d_1 \le z \le d = d_1 + d_2,\end{aligned}$$

where d is the period of the medium. Now applying the following boundary conditions at the interface between layers 1 and 2:

$$|H_{\pm 1}|_{z=d_1} = |H_{\pm 2}|_{z=d_1}, \qquad \left|\frac{dH_{\pm 1}}{dz}\right|_{z=d_1} = \left|\frac{dH_{\pm 2}}{dz}\right|_{z=d_1},$$

$$|H_{\pm 1}|_{z=0} = e^{iq_{AS}d}\,|H_{\pm 2}|_{z=d}, \quad \left|\frac{dH_{\pm 1}}{dz}\right|_{z=0} = e^{iq_{AS}d}\left|\frac{dH_{\pm 2}}{dz}\right|_{z=d}.$$

These boundary conditions lead to the well known Kronig-Penney dispersion equation, which is given by

$$\begin{aligned}\cos(q_{AS}d) = {} & \cos(k_{AS1}d_1)\cos(k_{AS2}d_2) \\ & - \left(\frac{k_{AS1}^2 + k_{AS2}^2}{2k_{AS1}k_{AS2}}\right)\sin(k_{AS1}d_1)\sin(k_{AS2}d_2), \end{aligned} \tag{19}$$

where q_{AS} is the Bloch wave vector for the Alfven-spin wave and the values of the wave vectors k_{AS1} and k_{AS2} are given by

$$k_{AS1}^2 = \omega^2\left[\frac{1}{V_{A1}^2} + \mu_o\varepsilon_1\right]\left[1 \pm \frac{\mu_o\gamma M_o}{\omega \pm \mu_o\gamma H_o}\right], \tag{20}$$

$$k_{AS2}^2 = \omega^2\left[\frac{1}{V_{A2}^2} + \mu_o\varepsilon_2\right]\left[1 \pm \frac{\mu_o\gamma M_o}{\omega \pm \mu_o\gamma H_o}\right]. \tag{21}$$

The Equation (19) is the dispersion relation describing the propagation of Alfven-spin wave in a layered periodic structure. In Section 4, Equations (15) and (19) will be analyzed numerically for the propagation characteristics of the Alfven-spin wave in both the sinusoidal and layered periodic structures, respectively.

3. HELICON-SPIN WAVE IN THE PERIODIC STRUCTURES

In the above section, it has been shown that coupling between the spin system and the electron-hole system in a magnetic-semiconducting medium can support Alfven-spin wave. Similarly the spin system and the single component electron system in a magnetic-semiconducting medium are also capable of sustaining circularly polarized helicon-spin wave. Thus the mathematical procedure used in the investigation of helicon-spin wave for the sinusoidal and layered periodic structures is qualitatively similar to that given in Section 2, but is presented here for the sake of completeness since this is used in the numerical analysis given in the Section 4.

For the propagation of helicon-spin wave in a sinusoidal periodic structure, the Equations (1)–(4) along with (6) and (7) are solved to obtain the following expression in favour of $H_\pm$ for the Bloch wave type solution.

$$\frac{d^2}{dz^2}H_\pm+\Big[-\omega^2\mu_o\varepsilon\left(\frac{\omega_{pe}^2/\omega}{\omega\pm\omega_{ce}}\right)[1+\bar{n}\cos(Qz)] +\omega^2\mu_o\varepsilon[1+\bar{\varepsilon}\cos(Qz)]\Big]\left[1\pm\frac{\mu_o\gamma M_o}{\omega\pm\mu_o\gamma H_o}\right]H_\pm=0, \qquad (22)$$

where $\omega_{pe}^2=e^2n_o/\varepsilon m_e^*$.

If $\phi=Qz/2$ (as has been assumed earlier in Section 2), then the Equation (22) reduces to the Mathieu's equation, that is

$$\frac{d^2}{d\phi^2}H_\pm+(\alpha_{HS}-\beta_{HS}^2\cos^2\phi)H_\pm=0, \qquad (23)$$

and the values of α_{HS} and β_{HS} for the helicon-spin wave are

$$\alpha_{HS}=\frac{4\omega^2}{Q^2}\left[-\mu_o\varepsilon\left(\frac{\omega_{pe}^2/\omega}{\omega\pm\omega_{ce}}\right)(1-\bar{n})+\mu_o\varepsilon(1-\bar{\varepsilon})\right]\left[1\pm\frac{\mu_o\gamma M_o}{\omega\pm\mu_o\gamma H_o}\right], \qquad (24)$$

$$\beta_{HS}^2=-\frac{8\omega^2}{Q^2}\left[-\mu_o\varepsilon\left(\frac{\omega_{pe}^2/\omega}{\omega\pm\omega_{ce}}\right)\bar{n}+\mu_o\varepsilon\bar{\varepsilon}\right]\left[1\pm\frac{\mu_o\gamma M_o}{\omega\pm\mu_o\gamma H_o}\right]. \qquad (25)$$

Thus the dispersion relation describing the propagation of helicon-spin wave in a sinusoidal periodic structure along with its continued fractions is obtained as

$$s_{HS}^2=\left[\alpha_{HS}-\frac{\beta_{HS}^2}{4}\left(2+\frac{a_1}{a_o}+\frac{a_{-1}}{a_o}\right)\right], \qquad (26)$$

$$\frac{a_1}{a_o}=-\left[\frac{\beta_{HS}^2}{2\beta_{HS}^2-4\alpha_{HS}+16\left(1+\frac{s_{HS}}{2}\right)^2}\right]-\left[\frac{\beta_{HS}^2}{2\beta_{HS}^2-4\alpha_{HS}+16\left(2+\frac{s_{HS}}{2}\right)^2}\right] +\left[\frac{\beta_{HS}^2}{2\beta_{HS}^2-4\alpha_{HS}+16\left(3+\frac{s_{HS}}{2}\right)^2}\right], \qquad (27)$$

$$\frac{a_{-1}}{a_o}=-\left[\frac{\beta_{HS}^2}{2\beta_{HS}^2-4\alpha_{HS}+16\left(1-\frac{s_{HS}}{2}\right)^2}\right]-\left[\frac{\beta_{HS}^2}{2\beta_{HS}^2-4\alpha_{HS}+16\left(2-\frac{s_{HS}}{2}\right)^2}\right] +\left[\frac{\beta_{HS}^2}{2\beta_{HS}^2-4\alpha_{HS}+16\left(3-\frac{s_{HS}}{2}\right)^2}\right]. \qquad (28)$$

For the propagation of helicon-spin wave in the layered periodic structure, the required Hill's equation can be obtained by ignoring the sinusoidal modulation in Equation (22), i.e.,

$$\frac{d^2}{dz^2}H_\pm+F_{HS}(z)H_\pm=0, \qquad (29)$$

where

$$F_{HS}(z) = \omega^2 \mu_o \varepsilon \left[1 - \left(\frac{\omega_{pe}^2/\omega}{\omega \pm \omega_{ce}}\right)\right] \left[1 \pm \frac{\mu_o \gamma M_o}{\omega \pm \mu_o \gamma H_o}\right].$$

Thus, following the similar procedure as discussed in Section 2, the dispersion relation describing the propagation of helicon-spin wave in the periodic structure with alternating homogeneous layers having different densities and dielectric constants is given by

$$\begin{aligned} \cos(q_{HS} d) &= \cos(k_{HS1} d_1) \cos(k_{HS2} d_2) \\ &\quad - \left(\frac{k_{HS1}^2 + k_{HS2}^2}{2 k_{HS1} k_{HS2}}\right) \sin(k_{HS1} d_1) \sin(k_{HS2} d_2), \end{aligned} \tag{30}$$

where q_{HS} is the Bloch wave vector and the wave vectors k_{HS1} and k_{HS2} are given by

$$k_{HS1}^2 = \omega^2 \mu_o \varepsilon_1 \left[1 - \left(\frac{\omega_{pe1}^2/\omega}{\omega \pm \omega_{ce}}\right)\right] \left[1 \pm \frac{\mu_o \gamma M_o}{\omega \pm \mu_o \gamma H_o}\right], \tag{31}$$

$$k_{HS2}^2 = \omega^2 \mu_o \varepsilon_2 \left[1 - \left(\frac{\omega_{pe2}^2/\omega}{\omega \pm \omega_{ce}}\right)\right] \left[1 \pm \frac{\mu_o \gamma M_o}{\omega \pm \mu_o \gamma H_o}\right]. \tag{32}$$

The dispersion relations (26) and (30) will be analyzed in the following section.

4. RESULTS AND DISCUSSIONS

4.1. Numerical Analysis for the Periodic Structures

In this section, dispersion relations (15) and (19) of the Alfven-spin wave and (26) and (30) of the helicon-spin wave are numerically investigated for the sinusoidal and layered periodic structures. Numerical values associated with these investigations are chosen for a typical composite magnetic-semiconducting medium [10], whereas the propagation frequency ω is taken from 10^9 to 10^{12} Hz. Achar [1] also made an analysis to study helicon wave propagation in a sinusoidal periodic structure as an alternative to the Kronig-Penney type of layered periodic structure. Such comparison can also be made in our cases. Therefore, the same values of average number density, dielectric constant and the periodicity are chosen for these two types of periodic structures. Thus, for the sinusoidal and layered periodic structures, Figure 1 shows the modulation of number density along z-axis about an average value $n_o = 10^{20}\,\mathrm{m}^{-3}$. Here the number densities $n_{o1} = 1.99 \times 10^{20}\,\mathrm{m}^{-3}$ and $n_{o2} = 0.01 \times 10^{20}\,\mathrm{m}^{-3}$ for the layered

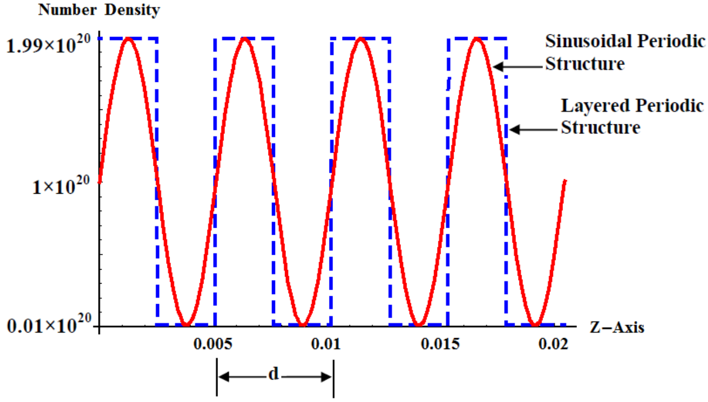


Figure 1. Schematic representation of modulation of number density for the sinusoidal and layered periodic structures.

periodic structure corresponding to the layers d_1 and d_2, respectively, also correspond to the maximum and minimum values of the number densities for the sinusoidal periodic structure. If $Q = 1.232 \times 10^3\,\mathrm{m}^{-1}$, then period of modulation $2\pi/Q$ matches d, such that $d = d_1 + d_2$, where $d_1 = d_2 = 2.55 \times 10^{-3}\,\mathrm{m}$. Similarly, assuming a modulation of the dielectric constant about an average value $\acute{\varepsilon} = 10$, such that $\acute{\varepsilon}_1 = 10.1$ and $\acute{\varepsilon}_2 = 9.9$ corresponding to the layers d_1 and d_2, respectively, also correspond to the maximum and minimum values of the dielectric constant for the sinusoidal periodic structure. From above, the modulation factors $\bar{n} = \frac{\Delta n}{n_o}$ and $\bar{\varepsilon} = \frac{\Delta\acute{\varepsilon}}{\acute{\varepsilon}}$ are clearly defined as $\bar{n} = 0.99$ and $\bar{\varepsilon} = 0.1$, respectively. The values of magnetic field, precession frequency, magnetization frequency and the effective masses for electrons and holes are given by $B_o = 0.1\,\mathrm{Tesla}$, $\mu_o\gamma H_o = 0.1\omega_{pe}$, $\mu_o\gamma M_o = 0.01\omega_{pe}$ and $m^*_{e,h} = 0.1m_{e,h}$, respectively. In this analysis the log-linear plots have been used to show comparatively the better picture of the propagation characteristics.

In order to analyze the Alfven-spin wave in the sinusoidal and layered periodic structures, Equations (15) and (19) for the wave vectors s_{AS} and q_{AS} are numerically investigated against the propagation frequency ω. The Equation (15) is presented graphically in Figures 2(a) and 2(b) for a sinusoidal periodic structure with upper and lower signs of polarization, respectively. Figure 2(a) shows a continuous propagation band or a region of propagation from 10^9 to 10^{12} Hz, whereas Figure 2(b) shows a propagation gap or a region of non-propagation near 7×10^{10} Hz. Similarly the Equation (19) is presented in Figures 3(a) and 3(b) for a layered periodic structure

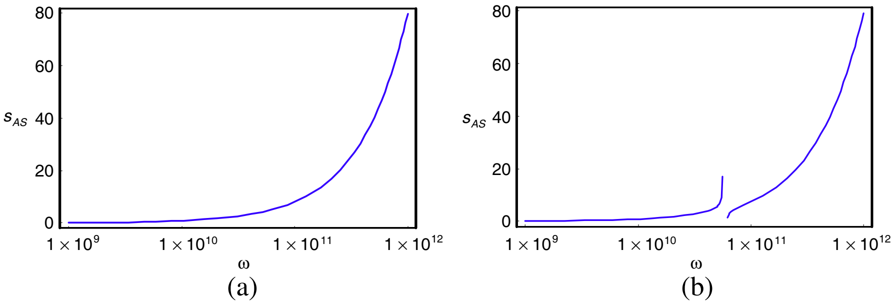


Figure 2. Propagation characteristics of coupled Alfven-spin wave for a sinusoidal periodic structure. The wave vector s_{AS} versus propagation frequency ω with (a) upper sign of polarization and with (b) lower sign of polarization.

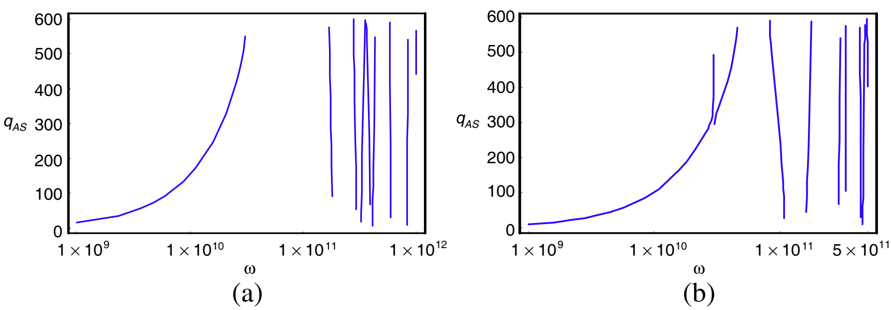
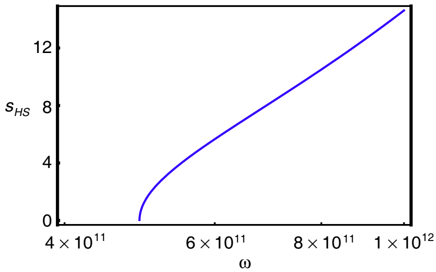


Figure 3. Propagation characteristics of coupled Alfven-spin wave for a layered periodic structure. The Bloch wave vector q_{AS} versus propagation frequency ω with (a) upper sign of polarization and with (b) lower sign of polarization.

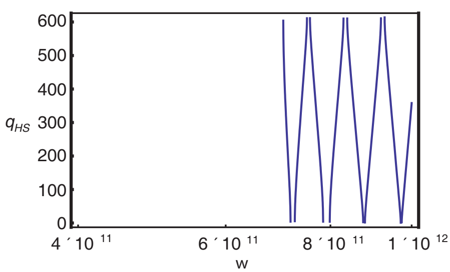
with upper and lower signs of polarization, respectively. Figures 3(a) and 3(b) show the propagation bands and gaps from 10^9 to 10^{12} Hz. It should be noted that the gaps in Figure 2(b) and Figures 3(a) and 3(b) correspond to the complex values of the s_{AS} and q_{AS}, respectively, i.e., these gaps are the frequency regions in which the wave does not propagate. Moreover, in Figure 3, the vertical lines appearing in the propagation characteristics between 10^{11} Hz to 10^{12} Hz are, in fact, the discontinuities or gaps at some higher frequency range.

In order to analyze the helicon-spin wave in the sinusoidal and layered periodic structures, Equations (26) and (30) for the wave vectors s_{HS} and q_{HS} are numerically investigated against the propagation frequency ω. The Equation (26) is presented in Figures 4(a) and 4(b) for a sinusoidal periodic structure with upper



(a) (b)

Figure 4. Propagation characteristics of coupled helicon-spin wave for a sinusoidal periodic structure. The wave vector s_{HS} versus propagation frequency ω with (a) upper sign of polarization and with (b) lower sign of polarization.



(a) (b)

Figure 5. Propagation characteristics of coupled helicon-spin wave for a layered periodic structure. The Bloch wave vector q_{HS} versus propagation frequency ω with (a) upper sign of polarization and with (b) lower sign of polarization.

and lower signs of polarization, respectively. In Figure 4(a), there is a propagation gap from 10^9 Hz onwards, up to $\sim 5 \times 10^{11}$ Hz and then there is a continuous propagation band up to 10^{12} Hz. Since there is a large propagation gap from 10^9 Hz onwards, therefore, the frequency range from 10^9 Hz up to $\sim 4 \times 10^{11}$ Hz is not shown in Figure 4(a) in order to show the propagation region clearly. For the lower signs of polarization, Figure 4(b) shows two regions of non-propagation or gaps between 10^9 Hz to 10^{12} Hz.

The Equation (30) is presented in Figures 5(a) and 5(b) for a layered periodic structure with upper and lower signs of polarization, respectively. In Figure 5(a), there is a propagation gap from 10^9 Hz onwards, up to $\sim 7 \times 10^{11}$ Hz (complete frequency range is not shown in figure), and then there are continuous propagation bands and gaps up to 10^{12} Hz. Figure 5(b) also shows continuous propagation bands and gaps from $\sim 8.6 \times 10^{11}$ Hz onwards up to 10^{12} Hz. Finally, after the above discussion, it should further be noted that the wave vectors s_{AS}, q_{AS}, s_{HS} and q_{HS} are also the functions of applied magnetic field. Thus for these periodic structures, the width of the frequency bands and gaps in the propagation characteristics can be controlled not only by varying the layer-thickness or period of modulation but also by varying the externally applied magnetic field, which is an additional feature that such a periodic structure device could be tuned.

As the values of parameters and the periodicity of these sinusoidal and layered structures are same, therefore, the study of Alfven-spin wave (or helicon-spin wave) propagation in a sinusoidal periodic structure can be compared with the layered periodic structure. First of all, the propagation characteristics of Alfven-spin wave in the sinusoidal periodic structure are compared with the layered periodic structure. In this connection, comparisons can be made between Figure 2(a) with Figure 3(a) for upper sign of polarization and Figure 2(b) with Figure 3(b) for lower sign of polarization. Figure 2(a) shows a continuous propagation band and Figure 3(a) shows the propagation bands as well as gaps from 10^9 to 10^{12} Hz. Whereas Figure 2(b) shows a propagation gap near 7×10^{10} Hz between the two regions of propagation from 10^9 to 10^{12} Hz and Figure 3(b) shows the propagation bands and gaps. Thus, for the lower sign of polarization, Alfven-spin wave shows band-gap effects in these two types of periodic structures. Similarly, when the propagation characteristics of helicon-spin wave in the sinusoidal periodic structure are compared with the layered periodic structure, they also show band-gap effects for the lower sign of polarization (see, e.g., Figures 4(b) and 5(b)). Although the comparisons of Figures 2(b) with 3(b) and Figures 4(b) with 5(b) show band-gap effects, but overall nature of the curves for these effects is not similar. Thus for the study of Alfven-spin or helicon-spin wave, a sinusoidal periodic structure can not be used as an alternative to the layered periodic structure.

4.2. Extreme Cases

Now the extreme cases for the decoupled independent modes in the absence of magnetization or carriers are discussed. These extreme cases can easily be obtained directly from Sections 2 and 3. For a semiconducting medium when there is no magnetization, (i.e., $M_o = 0$), the dispersion relations governing the Alfven wave propagation in the sinusoidal and layered periodic structures can be obtained (along with their companion equations) from (15) and (19), respectively. Similarly, the dispersion relations governing the helicon wave propagation in the sinusoidal and layered periodic structure can be obtained from Equations (26) and (30). For a magnetic medium when there are no free charges, (i.e., $e = 0$), the Equations (15) and (19) from Section 2 or Equations (26) and (30) from Section 3, give the same dispersion relation for the propagation of spin wave (without exchange interactions). Thus to analyze these three independent modes, i.e., Alfven wave, helicon wave and spin wave for the sinusoidal and layered periodic structures, the wave vectors s_A, q_A, s_H, q_H and s_S, q_S can be expressed by the following dispersion relations

$$s_j^2 = \left[\alpha_j - \frac{\beta_j^2}{4}\left(2 + \frac{a_1}{a_o} + \frac{a_{-1}}{a_o}\right)\right], \tag{33}$$

$$\cos(q_j d) = \cos(k_{j1} d_1)\cos(k_{j2} d_2) - \left(\frac{k_{j1}^2 + k_{j2}^2}{2k_{j1}k_{j2}}\right)\sin(k_{j1} d_1)\sin(k_{j2} d_2). \tag{34}$$

The subscript j stands for three different wave modes, i.e., $j = A, H, S$. In these modes, the expressions for the coefficients α_j, β_j^2 and the wave vectors k_{j1}^2, k_{j2}^2 can also be shown. Thus the expressions corresponding to the Alfven wave are given by

$$\alpha_A = \frac{4\omega^2}{Q^2}\left[\frac{1}{V_A^2}(1-\bar{n}) + \mu_o\varepsilon(1-\bar{\varepsilon})\right], \tag{35}$$

$$\beta_A^2 = -\frac{8\omega^2}{Q^2}\left[\frac{1}{V_A^2}\bar{n} + \mu_o\varepsilon\bar{\varepsilon}\right], \tag{36}$$

$$k_{A1}^2 = \omega^2\left[\frac{1}{V_{A1}^2} + \mu_o\varepsilon_1\right], \tag{37}$$

$$k_{A2}^2 = \omega^2\left[\frac{1}{V_{A2}^2} + \mu_o\varepsilon_2\right], \tag{38}$$

and the expressions for the helicon wave are given by

$$\alpha_H = \frac{4\omega^2}{Q^2}\left[-\mu_o\varepsilon\left(\frac{\omega_{pe}^2/\omega}{\omega \pm \omega_{ce}}\right)(1-\bar{n}) + \mu_o\varepsilon(1-\bar{\varepsilon})\right], \tag{39}$$

$$\beta_H^2 = -\frac{8\omega^2}{Q^2}\left[-\mu_o\varepsilon\left(\frac{\omega_{pe}^2/\omega}{\omega \pm \omega_{ce}}\right)\bar{n} + \mu_o\varepsilon\bar{\varepsilon}\right], \tag{40}$$

$$k_{H1}^2 = \omega^2\mu_o\varepsilon_1\left[1 - \left(\frac{\omega_{pe1}^2/\omega}{\omega \pm \omega_{ce}}\right)\right], \tag{41}$$

$$k_{H2}^2 = \omega^2\mu_o\varepsilon_2\left[1 - \left(\frac{\omega_{pe2}^2/\omega}{\omega \pm \omega_{ce}}\right)\right], \tag{42}$$

and for the spin wave

$$\alpha_S = \frac{4\omega^2}{Q^2}\left[\mu_o\varepsilon(1-\bar{\varepsilon})\right]\left[1 \pm \frac{\mu_o\gamma M_o}{\omega \pm \mu_o\gamma H_o}\right], \tag{43}$$

$$\beta_S^2 = -\frac{8\omega^2}{Q^2}[\mu_o\varepsilon\bar{\varepsilon}]\left[1 \pm \frac{\mu_o\gamma M_o}{\omega \pm \mu_o\gamma H_o}\right], \tag{44}$$

$$k_{S1}^2 = \omega^2\mu_o\varepsilon_1\left[1 \pm \frac{\mu_o\gamma M_o}{\omega \pm \mu_o\gamma H_o}\right], \tag{45}$$

$$k_{S2}^2 = \omega^2\mu_o\varepsilon_2\left[1 \pm \frac{\mu_o\gamma M_o}{\omega \pm \mu_o\gamma H_o}\right]. \tag{46}$$

It should be noted that with the help of the above expressions, the continued fractions a_1/a_o and a_{-1}/a_o involving in the dispersion relation (33) can also be determined in the same way as discussed in Section 2. At this place, there is no need to present the numerical analysis for these decoupled independent modes. First $j = H$ is considered and the relations (33) and (34) reduces to the previously investigated work of Achar [1], in which he showed that for a sinusoidal periodic structure the numerical solution of the dispersion relations does not show band-gap behaviour. When $j = A$ is used, the dispersion relations (33) and (34) describe the propagation of Alfven wave in a sinusoidal periodic and layered periodic structures, respectively. In this case also, the sinusoidal periodic structure does not show band-gap effect. Finally, the spin wave propagation is analyzed for $j = S$ in the two types of periodic structures. Here, the values of precession frequency and magnetization frequency can be chosen as $\mu_o\gamma H_o = 1\times10^{11}$ Hz and $\mu_o\gamma M_o = 0.1\times10^{11}$ Hz. Thus for the spin wave propagation, the dispersion relations (33) and (34) show band-gap behaviour for lower sign of polarization (figures are not presented here) and these characteristics of spin wave are very similar to the case of Alfven-spin wave as discussed in Section 2. The reasons for this similarity can also be seen by comparing the expressions (11), (12), (20) and (21) in Section 2 with expressions (43) to (46). These expressions differ only by the terms, i.e., $(1-\bar{n})/V_A^2$, $\bar{n}/V_A^2$, $1/V_{A1}^2$ and $1/V_{A2}^2$ appearing in the expressions (11), (12), (20) and (21), respectively. Thus it seems that the band-gap effects of the coupled modes in the sinusoidal periodic structure (for the lower sign of polarization) are associated primarily with the magnetic effects of the composite medium.

5. CONCLUSION

In this paper, the propagation of Alfven-spin and helicon-spin waves has been analyzed in both sinusoidal periodic and layered periodic structures. In these investigations, an idealized situation has been used in which exchange interactions and collisions have been neglected. The dispersion relations for these wave modes in the periodic structures have been obtained by Hill's equation. In the numerical analysis, for the same values of parameters and the periodicity of the two structures, these modes show band-gap effects in the layered periodic structures, whereas for the sinusoidal periodic structures these effects appear only for the lower sign of polarization. In the extreme cases, i.e., for the decoupled independent modes, numerical solutions of the Alfven wave and the helicon wave do not show band-gap effects for the sinusoidal periodic structure, whereas spin wave propagation shows band-gap effects in both types of periodic structures for the lower sign of polarization only. This shows that the band-gap effects of the Alfven-spin and helicon-spin modes in the sinusoidal periodic structure are associated with the magnetic effects of the composite medium.

In conclusion, it is clear that for a composite magnetic-semiconducting medium both types of periodic structures are useful and exhibit band-gap effects. These structures can be used as tunable devices operating at microwave frequencies whose band or gap widths can be controlled by varying the periodicity, dielectric constant and the external magnetic field etc. Further, it should be noted that these band-gap effects may have some useful applications for the propagation of coupled Alfven-spin or helicon-spin modes in a sinusoidal periodic structure, because for a single mode of helicon (or Alfven), these band-gap effects in a sinusoidal periodic structure have not earlier been observed.

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