Progress In Electromagnetics Research M, Vol. 17, 13–27, 2011

'A NOVEL DESIGN ALGORITHM' AND PRACTICAL REALIZATION OF RECTANGULAR PATCH ANTENNA LOADED WITH SNG METAMATERIAL

M. R. C. Mahdy, M. R. A. Zuboraj, A. A. N. Ovi and M. A. Matin

Department of Electrical and Electronic Engineering
Bangladesh University of Engineering and Technology
Dhaka, Bangladesh

Abstract—In this paper, for the very first time, a general algorithm for designing rectangular microstrip patch antenna, partially loaded with SNG (Single Negative) (MNG (μ Negative) and ENG (ε Negative)) metamaterial has been proposed to achieve better radiation performance. Then, applying our proposed algorithm, theoretically we have predicted novel dual band miniaturized rectangular patch antennas (loaded with MNG metamaterial) for two different bands using unconventional interface resonance mode under fundamental TM_{010} mode. Then we have proposed a complete design of magnetic inclusions, presenting full wave numerical simulations of the structure, which effectively supports the theoretical expected resonant modes as well as satisfactory radiation pattern performance. Prior to our current work, impossibility of sub-wavelength or electrically small rectangular patch antenna has been demonstrated using ENG metamaterial. However, in this paper, we have indicated a direction towards the real-life implementation of possible miniaturized rectangular patch antennas partially loaded with MNG metamaterial. The algorithm proposed in this paper is the key to choose the appropriate material parameter to design all such antennas.

Received 20 November 2010, Accepted 31 January 2011, Scheduled 7 February 2011
Corresponding author: Mahdy Rahman Chowdhury Mahdy (mahdybuet@gmail.com).

1. INTRODUCTION

Microstrip patch antennas, though popular for several applications, are difficult to miniaturize, since their resonant frequency is determined by the dominant modes of the patch cavity. Nevertheless, numerous miniaturization techniques using shorting posts, active loading or high permittivity dielectrics [1] have been conceived to lower the resonant frequency of patch antennas without increasing their size. Unfortunately, shorted posts, etched slots, and parasitic elements are often empirical (based on a cut-and-try process), and the additional parts will inevitably cause deformed radiation pattern and high cross polarization as a side effect [2]. Moreover, dual frequency patch antennas provide an alternative to large bandwidth planner antennas. In radar and communication system applications, patch antennas have attracted much interest due to their light weight and low cost. A unique radiating structure is desirable to accomplish these operations [3]. Probably, metamaterial based antennas can be the best solution for all these demands stated above. In [4] and [5] possibility of sub-wavelength patch using only circular patch has been demonstrated. But yet solution of reduced size rectangular patch antenna has not been reported in literature. In this contribution we have reported partial solution with proper practical realization to reduce the size of rectangular patch antenna applying our novel algorithm.

At first, Engheta proposed a sub-wavelength cavity resonator formed by a pair of conventional DPS (Double Positive) and DNG (Double Negative) materials with the DNG as a phase compensator [6]. However, in the quasi static limit, when the retardation effects are negligible due to the small dimensions of such components and only one of the two constitutive parameters interact with the field depending on its polarization, even single negative (SNG) materials, i.e., ENG or MNG, may be utilized to achieve similar effects. Later, Alù et al. [4] have proposed design method to obtain sub-wavelength rectangular patch antennas using DPS-ENG bi-layer. But it has been shown that such patches (rectangular), even if working at resonances in the sub-wavelength region, do not radiate energy efficiently in free space (broadside null radiation pattern; Figure 2 of [7]). It has been predicted that these rectangular antennas partially loaded with metamaterial can only be good resonators but may not be good radiators [4].

In this paper, at first (sec-2) we will give an algorithm for the modification of radiation pattern using MNG and ENG metamaterial. Then applying the algorithm, we shall theoretically predict the novel design of miniaturized rectangular microstrip patch antennas partially loaded with MNG metamaterial for dual band applications.

Later, using spiral ring resonators we shall propose a complete design of magnetic inclusions for real-life implementation of possible miniaturized rectangular patch antennas partially loaded with MNG metamaterial. So, the overall work has been demonstrated here along with our proposed novel algorithm, code based simulation and from the point of view of practical-life implementation.

2. PROPOSED ALGORITHM FOR DESIGNING

In Figure 1, the geometry of an un-slotted patch antenna with metamaterial block is shown. It consists of a metallic patch with transverse dimensions ($L_p \times W_p$) placed over a ground plane (distance h). The underneath substrate (which is $L_p \times W_p \times h$ in volume) is inhomogeneous, filled with two isotropic and homogeneous materials with permittivity and permeability ε_1, μ_1 and ε_2, μ_2. The filling ratio of the DPS is η (i.e., the length of DPS block under the patch is ηL). The problem is proper choice of necessary parameters to design such an antenna to get not only interface resonance but also good radiation pattern. To solve these problems, in this section, we have proposed an algorithm (with the help of some of our novel approximations) for a rectangular Microstrip patch antenna partially loaded with

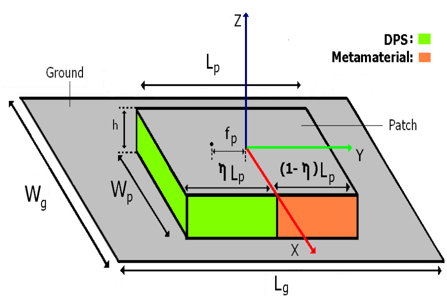
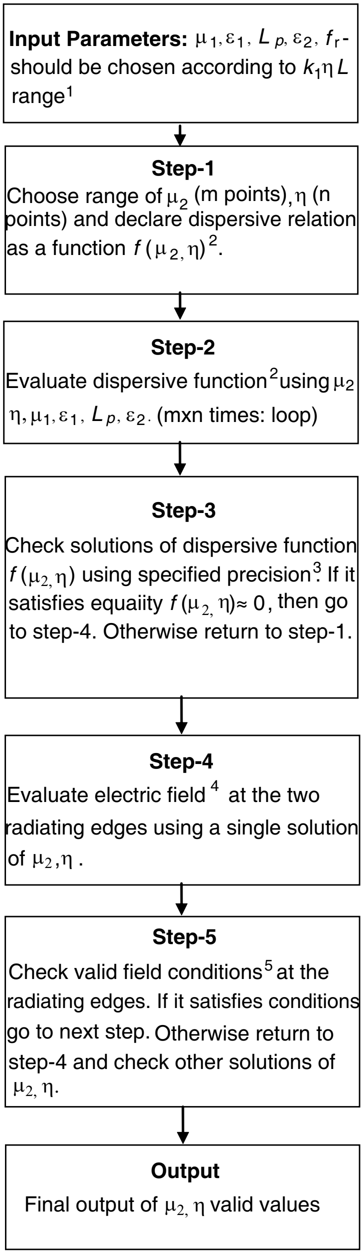


Figure 1. Geometry of a rectangular microstrip patch antenna partially loaded with metamaterial (ENG, MNG or DNG). f_p = feed position from DPS and metamaterials interface. f_r = chosen resonant frequency. Detailed information and constitutive parameters are: $\mu_{1r} = 1$. Case 1: $L_p = 50\,\text{mm}$, $W_p = 40\,\text{mm}$, $L_g = 75\,\text{mm}$, $W_g = 75\,\text{mm}$, $f_p = 21.4356\,\text{mm}$, $h = 5\,\text{mm}$, $\eta = 0.8176$, $\varepsilon_{1r} = 3.8$, $\mu_{1r} = 1$ and $\varepsilon_{2r} = 5.1$, $\mu_{2r}\,(\text{at}\,f_r) = -0.1091$ in case of MNG loading. Case 2: $L_p = 30\,\text{mm}$, $W_p = 24\,\text{mm}$, $L_g = 50\,\text{mm}$, $W_g = 50\,\text{mm}$, $f_p = 10.647\,\text{mm}$, $h = 3\,\text{mm}$, $\eta = 0.7439$, $\varepsilon_{1r} = 4.4$, $\mu_{2r}\,(\text{at} f_r) = -0.1091$ and $\varepsilon_{2r} = 4.4$ in case of MNG loading.



PROPOSED ALGORITHM FOR DESIGNING RECTANGULAR PATCH ANTENNA LOADED WITH SNG METAMTERIAL

FOR MNG METAMATERIAL LOADING

1. $k_1\eta L$ Range for MNG metamaterial loading:
$$135^0 < k_1\eta L < 180^0$$
2. Dispersive function, $f(\mu_2,\eta)$. [4]. [9] and [10]:
$$\sqrt{\frac{\epsilon_1}{\mu_1}}\tan(k_1\eta L_p)+\sqrt{\left|\frac{\epsilon_2}{\mu_2}\right|}\tan(|k_2|(1-\eta)L_p)$$
= precision
3. Precision range for dispersive equation: Solution of function within 0.000001 to –0.000001.
4. Electric fields at radiating edges (according to field equations of [9], [10]):
$$E_{z_1}=E_0\cosh(|k_2|(1-\eta)L_p)$$
$$E_{z_2}=E_0\cosh(k_1\eta L_p)$$
5. Edge electric fields ratio(r) must be -1<r<-0.7, where r= E_{z_2}/E_{z_1} (our approximation).

FOR ENG METAMATERIAL LOADING

Only a few changes:

μ_2 is an input and ε_2 is as variable. Outputs will be μ_2 and η.

1. $k_1\eta L$ Range for MNG metamaterial loading:
$$180^0 < k_1\eta L < 225^0$$
2. Dispersive function, $f(\varepsilon_2,\eta)$:
$$\sqrt{\frac{\epsilon_1}{\mu_1}}\tan(k_1\eta L_p)-\sqrt{\left|\frac{\epsilon_2}{\mu_2}\right|}\tan(|k_2|(1-\eta)L_p)$$
= precision

Other conditions are same as MNG.

Figure 2. Proposed algorithm.

(SNG) metamaterial. Using our algorithm, mode modification of rectangular microstrip antenna will be possible for better radiation performance. Later, in the following sections, the generality of the proposed algorithm will be proved using 'CST MICROWAVE STUDIO' based [8] realistic simulated results considering relevant material dispersion, presence of antenna feed and other losses. Then, 'practical realization' of such patch antennas will prove the acceptance of this newly proposed algorithm.

However, in case of designing such dual band patch antennas (conventional TM_{010} with unconventional $TM_{0\delta 0}$ or conventional TM_{010} with modified TM_{020}) several other considerations should also be taken account for satisfactory performance. Firstly, we must be careful about the size of the substrate. The size of the substrate should be equal to the patch (for both SNG and DNG metamaterial case). Moreover, the feed position of the antenna should be optimized properly for impedance matching. If anyone gets better radiation pattern with poor S-parameter performance, feed position optimization can be a very helpful tool. According to [4] and [9], the power taken from the feed is expected to be lost in ohmic losses or trapped into the unconventional waves travelling along the interface $y = 0$ (where $\partial E/\partial y$ flips its sign). At this $y = 0$ interface, due to the excitation of the surface plasmons, standing waves take place. Moreover, if the substrate in extended to the size of the ground plane, strong surface wave is observed at the interface between the air and the metamaterial block outside the patch. The surface wave is an unwanted radiation source that disturbs the pattern and consequently good broadside radiation cannot be achieved. Last but not the least; substrate height should also be optimized properly so that unwanted surface waves or impedance mismatching cannot take place in such a metamaterial loaded antenna.

Now applying our proposed algorithm, we will design two theoretically predicted novel dual band miniaturized rectangular patch antennas using MNG metamaterial. Moreover, using our proposed algorithm novel design of dual band rectangular patch antenna loaded with ENG metamaterial may also be possible. But we predict that ENG metamaterial may not be utilized for size miniaturization purpose.

3. MNG METAMATERIAL FOR SIZE REDUCTION WITH DUAL BAND APPLICATION

Prior to our work, the antenna proposed by Alù et al. [4] give broadside null for unconventional plasmonic mode using ENG metamaterial. But here, we are going to introduce the theoretical and practical implementation concept of MNG metamaterial to excite unconventional $TM_{0\delta 0}$ mode $(0 < \delta < 1)$. Using this unconventional mode, at first, it has been possible to theoretically predict our proposed dual band rectangular microstrip patch antennas with satisfactory radiation performance. Then, we have implemented this theoretically predicted antenna using SRs inclusion. Such unconventional $TM_{0\delta 0}$ mode gives satisfactory radiation pattern due to the proper use of our 'Algorithm of Better Radiation'. In this section, we will introduce two theoretically predicted MNG based antennas for dual band application for two different bands using interface resonant mode $(0 < \delta < 1)$.

In our proposed dual band antennas maximum tolerance is 3° deviation from broadside direction in one plane (principal E-plane only) for only one mode ($TM_{0\delta 0}$ $(0 < \delta < 1)$ mode). If the deviation is taken above 3° (i.e., 4° or 5°) further size reduction may be possible for close choice of f_r, near TM_{010} mode (like here). But by choosing much lower value of f_r (simultaneously all parameters will be changed then), further size reduction may be possible of such antennas if main lobe deviation is acceptable up to 7°. (In [9], 7° deviation from broadside has been accepted for TM_{010} mode). However, valuable frequency shifting phenomenon may not be avoidable for all the cases. Feed positions must be optimized carefully to get satisfactory bandwidth performance.

3.1. Theoretical Design

At first, for L band (low frequency region), consider a rectangular microstrip patch antenna with a transverse dimension $L = 50$ mm and $W = 40$ mm. If $\varepsilon_1 = 3.8$ and $\mu_1 = 1$ is used, then TM_{010} is at 1.57 GHz using DPS only. We will use an unconventional $TM_{0\delta 0}$ mode below the fundamental TM_{010} mode. In the case of Alu et al. [4], such mode was produced using ENG metamaterial but it gave broadside null radiation pattern. But here, using MNG metamaterial, we have produced such a mode with satisfactory radiation pattern performance. To introduce modified unconventional $TM_{0\delta 0}$ mode $(0 < \delta < 1)$ with 50 mm × 40 mm patch (Using MNG metamaterial), at first f_r has been chosen 1.48 GHz (lower than TM_{010} mode at 1.57 GHz, using DPS only). But 'interface resonance' has been found at 1.224 GHz.

Table 1. Chosen parameters after applying our algorithm.

Chosen resonant frequency, f_r	μ_2 (at f_r)	η	$\frac{E_{z2}}{E_{z1}}$ (at edge boundaries)
1.48 GHz	−0.1091	0.8176	−0.78

This valuable 'shifting phenomenon' has also been found with $TM_{0\delta 0}$ modes ($0 < \delta < 1$) in different sized patches for application in other bands. This valuable shifting may not be avoidable for this specific mode. Later, this mode at 1.224 GHz will show quite satisfactory broadside radiation pattern. Here, $k_1 \eta L$ will be chosen greater than 135° and less than 180° for getting this 'interface resonance' with MNG metamaterial. Then after applying our algorithm we have found possible choices of η and μ_2 (at f_r) which has been shown in Figure 3. In Table 1, we have shown the specific parameters those have been determined by using our proposed novel algorithm.

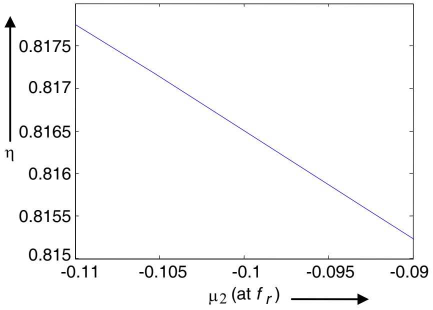


Figure 3. The possible choices of filling ratio and corresponding permeability of the MNG metamaterial (at $f = f_r$).

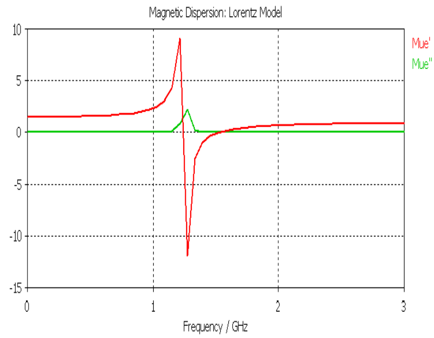


Figure 4. Lorentz dispersive model for proposed antenna (design-1).

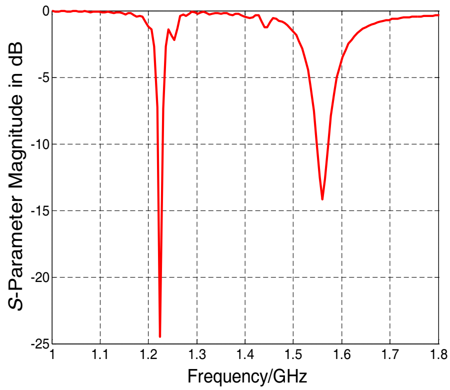


Figure 5. S-parameter performance of our proposed antenna (Figure 1, case 1). TM_{010} at 1.56 GHz and $TM_{0\delta 0}$ at 1.224GHz. Radiation performance for both bands are satisfactory.

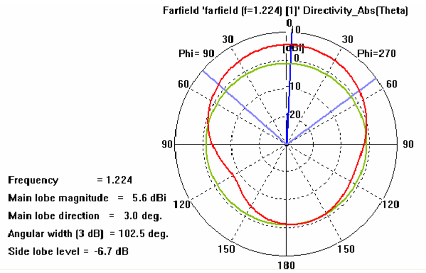


Figure 6. Principal E-plane radiation pattern (Polar plot) of $\mathrm{TM}_{0\delta 0}$ $(0 < \delta < 1)$ of our proposed antenna. of $\mathrm{TM}_{0\delta 0}$ $(0 < \delta < 1)$ of our proposed antenna. Main lobe deviation is only 3° from broadside direction.

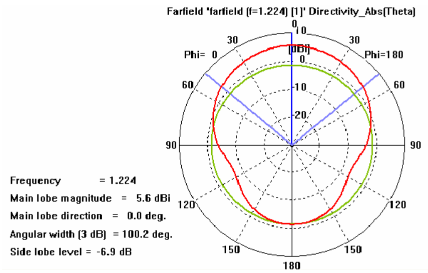


Figure 7. Principal H-plane radiation pattern (Polar plot) of $\mathrm{TM}_{0\delta 0}$ $(0 < \delta < 1)$ of our proposed antenna. Main lobe deviation is 0° from broadside direction.

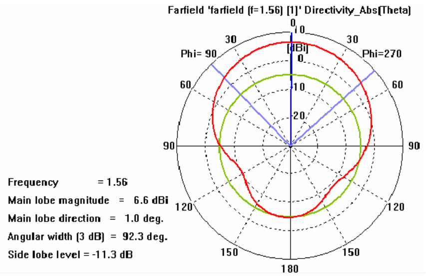


Figure 8. Principal E-plane radiation pattern (Polar plot) of TM_{010} of our proposed antenna. Main lobe deviation is only 1° from broadside direction.

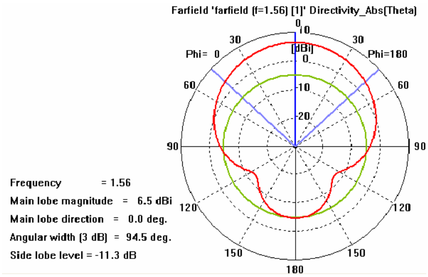


Figure 9. Principal H-plane radiation pattern (Polar plot) of TM_{010} of our proposed antenna. Main lobe deviation is 0° from broadside direction.

Finally, we have chosen the parameters given in Figure 1 (case 1) finally for partially loading with MNG metamaterial. Using these optimized parameters (Figure 1) and Lorentz model, we get the desired broadside radiation patterns (Figures 6 to 9). Actually, metamaterials are inherently dispersive and lossy [4]. So, without using dispersive lossy model (i.e., Lorentz model), the simulated theoretical results cannot give proper realistic results. Here we have used Lorentz model [5, 8, 9] for MNG:

$$\mu(\omega) = \mu_\infty + \frac{(\mu_s - \mu_\infty)\omega_0^2}{\omega_0^2 - \omega^2 + j\omega\epsilon}$$

where, $\mu_\infty = 1.0$, $\mu_s = 1.488$, $\omega_0 = 7853981634\,\text{rad/s}$, $\delta = 50\,\text{MHz}$.

The radiation pattern found from simulation tells that at broadside, theoretically it gets maximum radiation. The directivity of the antenna at broadside is 5.57 dBi for unconventional resonant mode at 1.224 GHz (Figure 6). So, the algorithm proposed in Section 2 is properly justified here with mathematical calculation, code based simulation and 'CST Microwave Studio' based realistic simulation [8]. Hence such MNG metamaterial based rectangular microstrip patch antennas can be used for miniaturized novel designs. Lorentz model, S-parameter performance and radiation pattern have been shown in Figure 4 to Figure 9.

3.2. Theoretically Predicted Second Design

Here we are considering another patch with size 30 mm × 24 mm with $\epsilon_1 = 4.4$, $\mu_1 = 1$ with TM_{010} at 2.62 GHz. Following the same procedure like previous design at first we have chosen $k_1\eta L$ above 135° and less than 180°. Here f_r has been chosen 2.5 GHz. Here, again frequency shifting phenomenon has taken place. Unconventional $\text{TM}_{0\delta0}$ mode $(0 < \delta < 1)$ has been found at 2.225 GHz. Later, this unconventional mode has shown satisfactory radiation performance at 2.225 GHz. Here, necessary information and relevant figures using our code and 'CST MICROWAVE STUDIO' [8] based simulated results are given below. Finally we have chosen the parameters given in Figure 1,

Table 2. Performance parameters of proposed antenna-2..

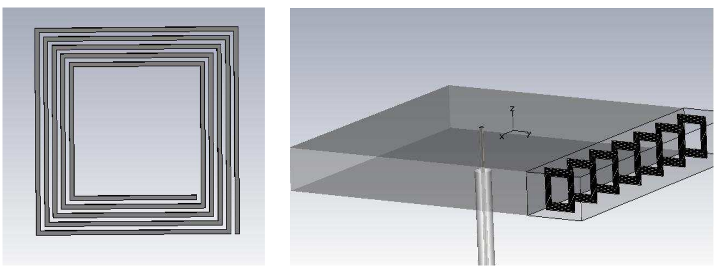
Patch Size (mm^2)	TM_{010} Resonance Freq (GHz)	TM_{010} radiation pattern directivity	TM_{010} radiation pattern's main lobe direction (degree)	
30×24	2.59	6.63 dBi	1	
TM_{010} S-parameter's magnitude (dB)	$\text{TM}_{0\delta0}$ Resonance Freq (GHz)	$\text{TM}_{0\delta0}$ radiation pattern directivity	$\text{TM}_{0\delta0}$ radiation pattern's main lobe direction (degree)	$\text{TM}_{0\delta0}$ S-Parameter's magnitude (dB)
−22.969	2.225	6.184 dBi	3	−13.3

case 2. Lorentz model parameters are: $\mu_\infty = 1.0$, $\mu_s = 1.2032$, $\omega_0 = 14451326210\,\text{rad/s}$, $\delta = 50\,\text{MHz}$. 'CST MICROWAVE STUDIO' [8] based final simulated results have been listed in Table 2. According to the simulated results, performance of the proposed antenna-2 is quite satisfactory (From the point of view of s-parameter and radiation pattern performance).

4. PRACTICAL REALIZATION OF PROPOSED ANTENNA-1

In the above sections, application of MNG metamaterial in designing miniaturized antenna has been mentioned with theoretical modeling and simulated radiation performance. But the real challenge remains behind the practical implementation of these antennas with μ-negative loading. In the following section, practical realization of antenna (in Figure 1 (case 1)) has been proposed and demonstrated.

Practical realization of artificial μ-negative material was first demonstrated by Pendry et al. [11]. After this work, different methods have been proposed to design materials with negative permeability. Split Ring Resonator (SRR), Spiral Resonator (SR), Multiple Split Ring Resonator (MSRR), Complementary Split Ring Resonator (CSRR) and Labyrinth Resonator (LR) are some well known inclusion methods [5, 12, 13]. For the implementation of MNG loading practically, both radiation and S-parameter performance issues should be meticulously handled so that there remains minimum anomaly between theoretical and practical realization of antenna performance. Our theoretical modeling of antenna assumes isotropic loading of MNG



(a) (b)

Figure 10. (a) Actual Spiral Resonator used in our design (Dimension details are given in Figure 11). (b) Implementation of SR arrays underneath the patch to obtain MNG behavior at the desired frequency for $50\,\text{mm} \times 40\,\text{mm}$ antenna (Figure 1, case 1).

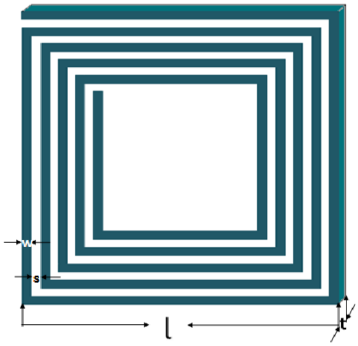


Figure 11. Geometrical dimensions of SR: $w = 0.1\,\mathrm{mm}$, $s = 0.1\,\mathrm{mm}$, $t = 0.1\,\mathrm{mm}$, $l = 4.8\,\mathrm{mm}$. Number of cells used = 7 in our actual design (Figure 10(a)).

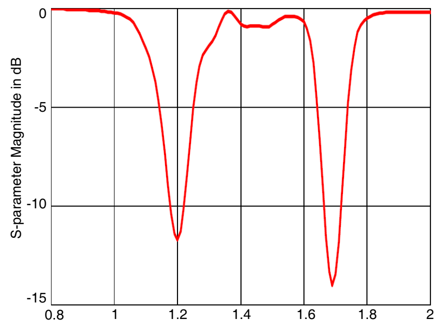
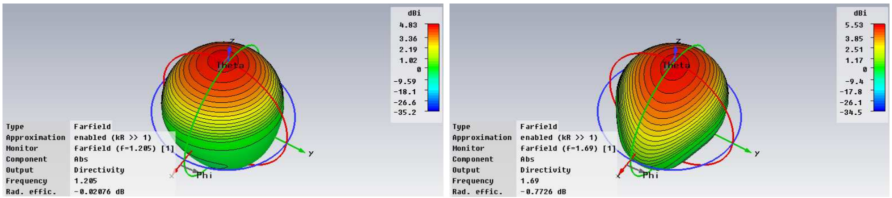


Figure 12. S-parameter performance of our proposed antenna (Figure 1, case 1). TM_{010} at 1.69 GHz and $TM_{0\delta 0}$ is at 1.205 GHz.

metamaterial. But when inclusions are placed in the partially loaded region, anisotropic MNG region appears for the introduction of SR inclusions in the radiator. Nevertheless, it is enough to guarantee a correct operation of the antenna [5]. Considering these facts, Bilotti et al. [5] showed a circular patch antenna with μ-negative loading by using inclusion of SRs. To promote same field distribution inside the antenna, incident magnetic field direction must align with the orientation of the magnetic field of inductive loops so as to provide changing effective permeability to the medium in that direction [5]. Incident magnetic energy induces currents in the circuit loops and causes coupling of energy into the resonators. Eventually it changes the relative permeability of the medium in that direction [14]. In other directions, relative permeability experienced by other magnetic field components will be that of free space (μ_0). Although SRRs or MSRRs can be used in this purpose, we have used SRs (Figure 10) for the following reasons [15]: (1) SR's are easy to manufacture using well-known and relatively cheap technologies. (2) The absence of the necessity of fabricating narrow slots between the stripes. (3) SRs allow for a significant potential reduction in the electrical size of the metamaterial unit cell which contributes to form a continuous medium of metamaterial rather than as a discrete periodic structure.

Distributed capacitance and inductance inside the SRs can be modeled as equivalent LC circuit [16]. The modeling proposed in [17] describes the behavior of the SRs as negative permeability medium with proper orientation [17]. The maximum linear dimension of SR in



(a) (b)

Figure 13. Three dimensional radiation pattern of (a) $TM_{0\delta0}$ mode at 1.205 GHz and (b) TM_{010} mode at 1.69 GHz. Directivities are quite satisfactory.

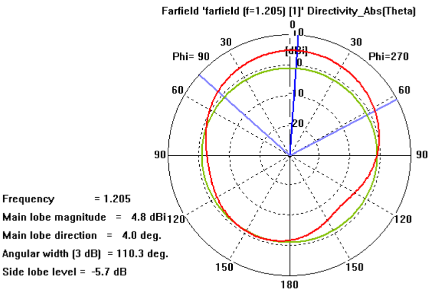


Figure 14. Principal E-plane radiation pattern (Polar plot) of $TM_{0\delta0}$ $(0 < \delta < 1)$ of our proposed antenna. Main lobe deviation is only 4° from broadside direction.

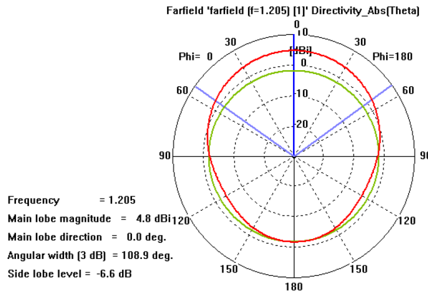


Figure 15. Principal H-plane radiation pattern (Polar plot) of $TM_{0\delta0}$ $(0 < \delta < 1)$ of our proposed antenna. Main lobe deviation is 0° from broadside direction.

our design is on the order of $\lambda_0/44$. Increasing the number of turns in SR causes reduction in quality factor and eventually increases the bandwidth [16].

We have used spiral resonators to partially load our patch antenna. According to [5], we have used the spiral resonators from the concept of near field analysis. All our findings using 'CST MICROWAVE STUDIO' [8] based realistic simulations have been shown in Figure 12 to Figure 17. The medium does not remain completely isotropic when these inclusions are loaded practically. As a result, the $|S_{11}|$ is affected (Figure 12). Due to the proper placement of the resonators and choosing the turn numbers properly according to [5] and [16], the $|S_{11}|$ of $TM_{0\delta0}$ $(0 < \delta < 1)$ mode shifts slightly to the left. But the $|S_{11}|$ of TM_{010} mode behaves differently. It shifts to the right due to anisotropic effect and other effects. Here 'other effects' means the

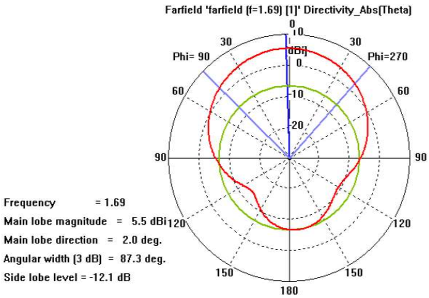


Figure 16. Principal E-plane radiation pattern (Polar plot) of TM_{010} of our proposed antenna. Main lobe deviation is only 2° from broadside direction.

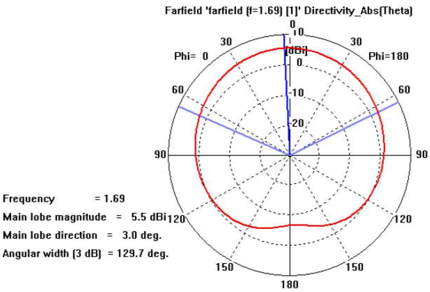


Figure 17. Principal H-plane radiation pattern (Polar plot) of TM_{010} of our proposed antenna. Main lobe deviation is 3° from broadside direction.

inclusion parameters are not fully optimized. However, the results those we have found with practical inclusions (conductivity of copper $= 5.8 \times 10^7$, feed loss) are realistic. Moreover, slightly right shift does not totally change the performance of the antenna. Then if we consider the radiation pattern case, the radiation patterns (Figure 14 to Figure 17; polar plots) change slightly from the theoretically predicted results given in Section 3.1. The 3-dimensional radiation patters (Figure 13) are clearly assuring the maximum broadside radiation for both bands. change slightly from the theoretically predicted results. So we think that the results that we have found in this contribution are very impressive and promising for real life implementation.

5. CONCLUSION

In this paper, ultimately we have found both theoretical and practical point of view that MNG loading can be useful in miniaturization of microstrip antennas with appropriate filling ratio and choice of appropriate inclusions. In this approach, a simple but organized algorithm has been proposed to reduce the complexity of designing microstrip antenna with partially loading of SNG metamaterials. Moreover, a practical way of implementing 'MNG metamaterial loaded rectangular patch antenna' found from the result of algorithm has also been discussed and demonstrated. In comparison, theoretical and practical modelling of the proposed antennas have insignificant differences in performance parameters. Therefore, the proposed antennas can be used efficiently in wireless communication systems, radio sensors, RFID tags, RADAR applications and so on.

ACKNOWLEDGMENT

The authors are thankful to Md. Gaffar, Mohammad Asif Zaman & Sajid Muhaimin Choudhury of BUET and Assistant Prof. Golam Dastegir Al-Quaderi of University of Dhaka, Department of Physics.

REFERENCES

1. Balanis, C. A., *Modern Antenna Handbook*, Wiley & Sons, Hoboken, NJ, 2008.
2. Guha, D., M. Biswas, and Y. M. M. Antar, "Microstrip patch antenna with defected ground structure for cross polarization suppression," *IEEE Antennas Wireless Propag. Lett.*, Vol. 4, 455–458, 2005.
3. Maci, S. and G. B. Gentili, "Dual-frequency patch antennas," *IEEE Antennas and Propagation Magazine*, Vol. 39, No. 6, December 1997.
4. Alù, A., F. Bilotti, N. Engheta, and L. Vegni, "Subwavelength, compact, resonant patch antennas loaded with metamaterials," *IEEE Transactions on Antennas and Propagation*, Vol. 55, No. 1, January 2007.
5. Bilotti, F., A. Alù, and L. Vegni, "Design of miniaturized metamaterial patch antennas with μ-negative loading," *IEEE Transactions on Antennas and Propagation*, Vol. 56, No. 6, 1640–1647, June 2008.
6. Engheta, N., "An idea for thin sub-wavelength cavity resonators using metamaterials with negative permittivity and permeability," *IEEE Antennas Wireless Propag. Lett.*, Vol. 1, No. 1, 10–13, 2002.
7. Zheng, K. S., W. Y. Tam, and D. B. Ge, "Broadside subwavelength microstrip antennas partially loaded with metamaterials," *2008 International Workshop Metamaterials*, November 2008.
8. CST Microwave Studio 9.0 CST of America [Online]. Available: http://www.cst.com.
9. Xiong, J., H. Li, Y. Jin, and S. He, "Modified TM_{020} mode of a rectangular patch antenna partially loaded with metamaterial for dual-band applications," *IEEE Antennas and Wireless Propag. Lett.*, Vol. 8, 2009.
10. Xu, W., L.-W. Li, H. Y. Yao, T. S. Yeo, and Q. Wu, "Left-handed material effects on waves modes and resonant frequencies: Filled waveguide structures and substrate-loaded patch antennas," *Journal of Electromagnetice Waves and Applications*, Vol. 19, No. 15, 2033–2047, 2005.

11. Pendry, J. B., A. J. Holden, D. J. Robbins, and W. J. Stewart, "Magnetism from conductors and enhanced nonlinear phenomena," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 47, No. 11, 2075–2084, November 1999.
12. Pan, W., D. F. Wang, and F. Y. Meng, "Design, verification and applications in antenna and resonator cavity miniaturization of compact left-handed metamaterial," *I. J. Wireless and Microwave Technologies*, Vol. 1, 10–18, 2009.
13. Meng, F. Y., Q. Wu, and J. H. Fu, "Miniaturized rectangular cavity resonator based on anisotropic metamaterials bi-layer," *Microwave and Optical Technology Letters*, Vol. 50, No. 8, August 2008.
14. Buell, K., H. Mosallaei, and K. Sarabandi, "A substrate for small patch antennas providing tunable miniaturization factors," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 54, No. 1, January 2006.
15. Baena, J. D., R. Marques, and F. Medina, "Artificial magnetic metamaterial design by using spiral resonators," *Phys. Rev. B*, Vol. 69, 014402(1)–(5), 2004.
16. Bilotti, F., A. Toscano, and L. Vegni, "Design of spiral and multiple split-ring resonators for the realization of miniaturized metamaterial samples," *IEEE Transactions on Antennas and Propagation*, Vol. 55, No. 8, 2258–2267, August 2007.
17. Bilotti, F., A. Toscano, L. Vegni, K. Aydin, K. Boratay, and E. Ozbay, "Equivalent-circuit models for the design of metamaterials based on artificial magnetic inclusions," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 55, No. 12, December 2007.