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THE NONLINEAR ABSORPTION OF A STRONG ELECTROMAGNETIC WAVES CAUSED BY CONFINED ELECTRONS IN A CYLINDRICAL QUANTUM WIRE

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Abstract—The nonlinear absorption of a strong electromagnetic wave caused by electrons confined in cylindrical quantum wires is theoretically studied by using the quantum kinetic equation for electrons. An analytic expression of the nonlinear absorption coefficient of a strong electromagnetic wave caused by electrons confined in a cylindrical quantum wire with a parabolic potential for electron-optical phonon scattering is obtained. The dependence of the nonlinear absorption coefficient on the intensity E_0 and the frequency Ω of the external strong electromagnetic wave, the temperature T of the system and the radius R of the wires is strong and nonlinear. Analytic expression is numerically calculated and discussed for a $GaAs/GaAsAl$ quantum wire. The results are compared with those for normal bulk semiconductors and quantum wells to show the differences.

1. INTRODUCTION

In recent years, the optical properties in bulk semiconductors, as well as low-dimensional systems, have been investigated [1–11]. The linear absorption has been studied in normal bulk semiconductors 1, 2], in two dimensional systems [3–6] and in quantum wires [7, 8]. The nonlinear absorption of a strong electromagnetic wave (EMW) was considered in normal bulk semiconductors [9], quantum wells [10, 16], doping superlattices [17, 18]. Nonlinear optical absorption has also been studied in quantum dots [19]. However, in cylindrical quantum wires, the nonlinear absorption of a strong EMW is still open for studying. In one dimensional systems, the motion of electrons is restricted in two

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dimensions, so they can flow freely in one dimension. The confinement of electrons in these systems changes the electron mobility remarkably. This results in a number of new phenomena, which concern a reduction of the sample dimensions. These effects, for example, electron-phonon interaction and scattering rates [11, 12], the linear and nonlinear optical properties [13, 14], differ from those in bulk semiconductors, as well as two-dimensional systems, and the nonlinear absorption of strong EMW is not an exception. The nonlinear absorption of a strong EMW in a quantum wire will be different from the nonlinear absorption of a strong EMW in bulk semiconductors and two-dimensional. In this paper, we use the quantum kinetic equation for electrons to theoretically study the nonlinear absorption coefficient of a strong EMW by electrons confined in a cylindrical quantum wire (CQW) with a parabolic potential. The problem is considered for electron-optical phonon scattering. Numerical calculations were carried out for specific GaAs/GaAsAl quantum wires.

2. THE NONLINEAR ABSORPTION COEFFICIENT OF A STRONG EMW IN A CQW

We consider a wire of GaAs with a circular cross section with a radius R and a length L_z embedded in AlAs. The carriers (electrons) are assumed to be confined by symmetric parabolic potential of the form

$$V = \frac{1}{2} m\omega_0^2 r^2, \tag{1}$$

where m is the effective mass of electron, and ω_0 is the effective frequency of potential well. The total wave function of electrons in cylindrical coordinates (r, ϕ, z) and the electron energy spectrum obtained directly from solving Schrodinger equation can be written as

$$\psi_{n,\ell,\vec{p}}(r) = \frac{e^{i\vec{p}z}}{\sqrt{L_z}} \sqrt{\frac{2n!}{(n+|\ell|)!}} \frac{1}{a_0} e^{-\frac{r^2}{2a_0^2}} \left(\frac{R}{a_0}\right)^{|\ell|} \left(\frac{r^2}{a_0^2}\right)^{|\ell|} L_n^{|\ell|}\left(\frac{r^2}{a_0^2}\right), \tag{2}$$

$$\varepsilon_{n,\ell}(\vec{p}) = \frac{p_z^2}{2m} + \omega_0\,(2\,n + |\ell| + 1), \tag{3}$$

where $n = 0,\ \pm 1\ \pm 2, \ldots$ is the azimuthal quantum number, $\ell = 1,\ 2,\ 3, \ldots$ is the radial quantum number; $\vec{p} = (0, 0, p_z)$ is the electron wave vector (along the wire's z axis); $L_n^{|\ell|}(x)$ is a generalized lagrange polynomial; $a_0 = 1/\sqrt{m\omega_0}$ (in this paper, we select $\hbar = 1$).

The Hamiltonian of the electron-optical phonon system in a cylindrical quantum wire in the presence of a laser field, $\vec{E}(t) =$

$\vec{E}_0 \sin(\Omega t)$, can be written as

$$H = \sum_{n,l,\vec{p}} \varepsilon_{n,l} \left(\vec{p} - \frac{e}{c}\vec{A}(t)\right) a^+_{n,l,\vec{p}}\, a_{n,l,\vec{p}} + \sum_{\vec{q}} \omega\, b^+_{\vec{q}} b_{\vec{q}} + \sum_{n,l,n',\ell',\vec{p},\vec{q}} C_{\vec{q}} I_{n,l,n',\ell'}(\vec{q}) a^+_{n,l,\vec{p}+\vec{q}}\, a_{n',\ell',\vec{p}} \left(b_{\vec{q}} + b^+_{-\vec{q}}\right), \quad (4)$$

where e is the electron charge; c is the velocity of light; $\vec{A}(t) = \frac{c}{\Omega}\vec{E}_0 \cos(\Omega t)$ is the vector potential; $\vec{E}_0$ and Ω are the intensity and the frequency of the EMW; $a^+_{n,\ell,\vec{p}}$ ($a_{n,\ell,\vec{p}}$) is the creation (annihilation) operator of an electron; $b^+_{\vec{q}}$ ($b_{\vec{q}}$) is the creation (annihilation) operator of a optical phonon for a state having wave vector $\vec{q}$; ω is the frequency of a optical phonon; and $C_{\vec{q}}$ is the electron-optical phonon interaction constant, which can be taken as [1, 5, 6]

$$|C_{\vec{q}}|^2 = e^2\omega \left(1/\chi_\infty - 1/\chi_0\right)/2\epsilon_0 q^2 V_0, \quad (5)$$

where ϵ_0 is the permittivity of free space; V_0 is the normalization volume, and χ_∞; and χ_0 are the high-frequency dielectric constants and low-frequency dielectric constants, respectively. $I_{n,l,\acute{n},\acute{l}}(\vec{q})$ is the electron form factor and can be written as

$$I_{n,\ell,n'\ell'}(\vec{q}) = \frac{2}{R^2}\int_0^R \psi_{n,\ell}(r) e^{iqr} \psi^*_{n',\ell'}(r) r dr. \quad (6)$$

The carrier current density $\vec{j}(t)$ and the nonlinear absorption coefficient of a strong electromagnetic wave α take the form [9, 10, 15]

$$\vec{j}(t) = \frac{e}{m}\sum_{n,\ell,\vec{p}} \left(\vec{p} - \frac{e}{c}\vec{A}(t)\right) n_{n,\ell,\vec{p}}(t), \quad (7)$$

$$\alpha = \frac{8\pi}{c\sqrt{\chi_\infty} E_0^2} \left\langle \vec{j}(t)\vec{E}_0 \sin\Omega t\right\rangle_t \quad (8)$$

where $n_{n,\ell,\vec{p}}(t)$ is the electron distribution function; $\langle X\rangle_t$ means the usual thermodynamic average of X at moment t; and χ_∞ is the high-frequency dielectric constant. In order to establish analytical expressions for the nonlinear absorption coefficient of a strong EMW by electrons confined in a CQW, we use the quantum kinetic equation for particle number operator of an electron $n_{n,\ell,\vec{p}}(t) = \langle a^+_{n,\ell,\vec{p}} a_{n,\ell,\vec{p}}\rangle_t$

$$i\frac{\partial n_{n,\ell,\vec{p}}(t)}{\partial t} = \left\langle \left[a^+_{n,\ell,\vec{p}} a_{n,\ell,\vec{p}}, H\right]\right\rangle_t \quad (9)$$

From Eq. (9), using the Hamiltonian in Eq. (4) and realizing calculations, we obtain quantum kinetic equation for the electrons confined in a CQW. Using the first-order tautology approximation method: $n_{n,\ell,\vec{p}}(t') \approx \overline{n}_{n,\ell,\vec{p}}$, $n_{n,\ell,\vec{p}+\vec{q}}(t') \approx \overline{n}_{n,\ell,\vec{p}+\vec{q}}$, $n_{n,\ell,\vec{p}-\vec{q}}(t') \approx \overline{n}_{n,\ell,\vec{p}-\vec{q}}$ (This approximation has been applied to a similar exercise in bulk semiconductors [15] and quantum wells [10]) to solve this equation, we obtain the expression of electron distribution function, $n_{n,\ell,\vec{p}}(t)$:

$$\begin{aligned} n_{n,\ell,\vec{p}}(t) \;=\; & -\sum_{\vec{q},n',\ell'} |C_{\vec{q}}|^2 |I_{n,\ell,n',\ell'}(\vec{q})|^2 \sum_{k,l=-\infty}^{\infty} \frac{1}{l\Omega} e^{-il\Omega t} J_k\left(\frac{e\vec{E}_0\vec{q}}{m\Omega^2}\right) \\ & \times J_{k+l}\left(\frac{e\vec{E}_0\vec{q}}{m\Omega^2}\right)\Bigg\{-\frac{\bar{n}_{n,\ell,\vec{p}}(N_{\vec{q}}+1)-\bar{n}_{n',\ell',\vec{p}+\vec{q}}N_{\vec{q}}}{\varepsilon_{n',\ell'}(\vec{p}+\vec{q})-\varepsilon_{n,\ell}(\vec{p})+\omega-k\Omega+i\delta} \\ & -\frac{\bar{n}_{n,\ell,\vec{p}}N_{\vec{q}}-\bar{n}_{n',\ell',\vec{p}+\vec{q}}(N_{\vec{q}}+1)}{\varepsilon_{n',\ell'}(\vec{p}+\vec{q})-\varepsilon_{n,\ell}(\vec{p})-\omega-k\Omega+i\delta} \\ & +\frac{\bar{n}_{n',\ell',\vec{p}-\vec{q}}(N_{\vec{q}}+1)-\bar{n}_{n,\ell,\vec{p}}N_{\vec{q}}}{\varepsilon_{n,\ell}(\vec{p})-\varepsilon_{n',\ell'}(\vec{p}-\vec{q})+\omega-k\Omega+i\delta} \\ & +\frac{\bar{n}_{n',\ell',\vec{p}-\vec{q}}N_{\vec{q}}-\bar{n}_{n,\ell,\vec{p}}(N_{\vec{q}}+1)}{\varepsilon_{n,\ell}(\vec{p})-\varepsilon_{n',\ell'}(\vec{p}-\vec{q})-\omega-k\Omega+i\delta}\Bigg\}, \end{aligned} \tag{10}$$

where $N_{\vec{q}}$ ($\bar{n}_{n,\ell,\vec{p}}$) is the time independent component of the phonon (electron) distribution function; $J_k(x)$ is the Bessel function; and the quantity δ is infinitesimal and appears due to the assumption of an adiabatic interaction of the electromagnetic wave. We insert the expression for $n_{n,\ell,\vec{p}}(t)$ into the expression for $\vec{j}(t)$ and then insert the expression for $\vec{j}(t)$ into the expression for α in Eq. (8). Using the properties of Bessel function and realizing the calculations, we obtain the nonlinear absorption coefficient of a strong EMW by confined electrons in a CQW as

$$\begin{aligned} \alpha \;=\; & \frac{8\pi^3 e^2 \Omega k_b T}{c\sqrt{\chi_\infty}E_0^2\epsilon_0 V_0}\left(\frac{1}{\chi_\infty}-\frac{1}{\chi_0}\right)\sum_{n,\ell,n',\ell'}\sum_{\vec{q},\vec{p}}|I_{n,\ell,n',\ell'}(\vec{q})|^2\frac{1}{q^2} \\ & \times\sum_{k=-\infty}^{\infty}\left[\bar{n}_{n,\ell,\vec{p}}-\bar{n}_{n',\ell',\vec{p}+\vec{q}}\right]kJ_k^2\left(\frac{e\vec{E}_0\vec{q}}{m\Omega^2}\right)\Big\{\delta(\varepsilon_{n',\ell'}(\vec{p}+\vec{q}) \\ & -\varepsilon_{n,\ell}(\vec{p})+\omega-k\Omega)+\delta\left(\varepsilon_{n',\ell'}(\vec{p}+\vec{q})-\varepsilon_{n,\ell}(\vec{p})-\omega-k\Omega\right)\Big\}, \end{aligned} \tag{11}$$

where V_0 is the normalization volume, and $\delta(x)$ is the Dirac delta function. In this paper, we only consider the absorption close to its threshold because the absorption in other cases (the absorption far away from its threshold) is very small. In this case the condition

$|k\Omega - \omega| \ll \bar{\varepsilon}$ must be satisfied [9, 10]. We restrict the problem to the case of absorbing a photon and consider the electron gas to be non-degenerate:

$$\bar{n}_{n,\ell,\vec{p}} = n_0^* \exp\left(-\frac{\varepsilon_{n,\ell}(\vec{p})}{k_b T}\right), \text{ with } n_0^* = \frac{n_0 (e\pi)^{\frac{3}{2}}}{V_0 (m_0 k_b T)^{\frac{3}{2}}} \tag{12}$$

where m_0 is the mass of a free electron; n_0 is the electron density in a CQW; and k_b is the Boltzmann constant. Using the Bessel function and the energy spectrum of an electron in a CQW, we have an explicit formula of the nonlinear absorption coefficient in a CQW with a parabolic potential:

$$\alpha = \frac{\sqrt{2\pi} e^4 n_0^* (k_b T)^{3/2}}{4c\epsilon_0 \sqrt{m\chi_\infty} \Omega^3 V_0} \left(\frac{1}{\chi_\infty} - \frac{1}{\chi_0}\right) \sum_{n,\ell,n',\ell'} |I_{n,\ell,n',\ell'}(\vec{q})|^2$$
$$\times \Bigg\{ \exp\left(\frac{B(+\omega)}{k_b T}\right) \left[\exp\left(\frac{-\Omega+\omega}{k_b T}\right) - 1\right] \left[1 + \frac{3e^2 E_0^2 k_b T}{8m\Omega^4}\left(1 + \frac{B^+(\omega)}{2k_b T}\right)\right]$$
$$+ \exp\left(\frac{B(-\omega)}{k_b T}\right) \left[\exp\left(\frac{-\Omega-\omega}{k_b T}\right) - 1\right] \left[1 + \frac{3e^2 E_0^2 k_b T}{8m\Omega^4}\left(1 + \frac{B(-\omega)}{2k_b T}\right)\right] \Bigg\}, \tag{13}$$

where $B(\pm\omega) = \omega_0(2n' - 2n + |\ell'| - |\ell|) \pm \omega - \Omega$. From the analytic expression for the nonlinear absorption coefficient of a strong EMW caused by confined electrons in CQWs with an parabolic potential (Eq. (13)), we can see that when the term proportional to the quadratic intensity of the EMW (E_0^2) tends toward zero, the nonlinear result will become a linear result.

3. NUMERICAL RESULTS AND DISCUSSIONS

In order to clarify the results that have been obtained, in this section, we numerically calculate the nonlinear absorption coefficient of a strong EMW for a $GaAs/GaAsAl$ CQW. The nonlinear absorption coefficient is considered as a function of the intensity E_0 and the energy of the strong EMW, the temperature T of the system, and the parameters of the CQW. The parameters used in the numerical calculations [6, 13] are $\xi = 13.5\,\text{eV}$, $\rho = 5.32\,\text{gcm}^{-3}$, $v_s = 5378\,\text{ms}^{-1}$, $\epsilon_0 = 12.5$, $\chi_\infty = 10.9$, $\chi_0 = 13.1$, $m = 0.066 m_0$, m_0 being the mass of free electron, $\hbar\omega = 36.25\,\text{meV}$, $k_b = 1.3807 \times 10^{-23}\,\text{j/K}$, $n_0 = 10^{23}\,\text{m}^{-3}$, $e = 1.60219 \times 10^{-19}\,\text{C}$, $\hbar = 1.05459 \times 10^{-34}\,\text{j} \cdot \text{s}$.

Figure 1 shows the dependence of the nonlinear absorption coefficient of a strong EMW on the wire's radius at different values of temperature T of the system. It can be seen from this figure

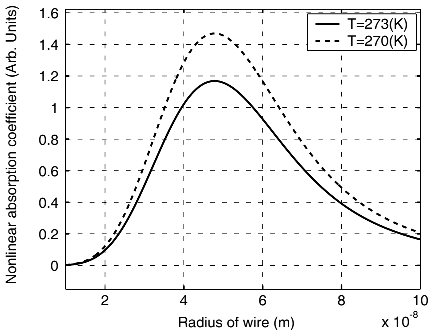

Figure 1. Dependence of α on radius of wire at different values of temperature T.

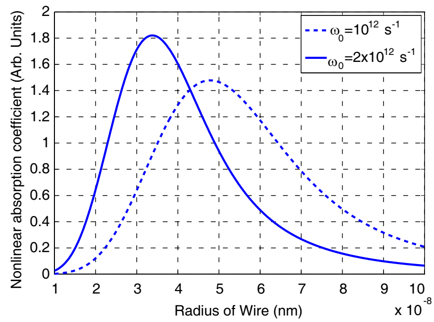

Figure 2. Dependence of α on radius of wire at different values of ω_0.

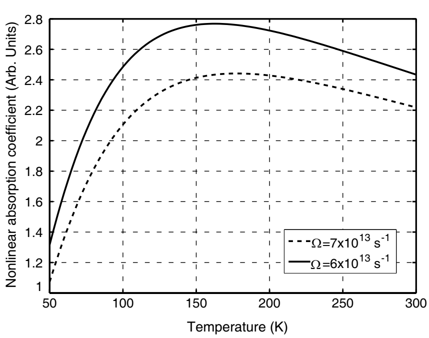

Figure 3. Dependence of α on T at different values of Ω.

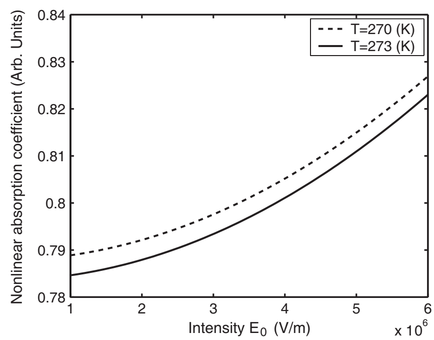

Figure 4. Dependence of α on E_0 at different values of T.

that the absorption coefficient depends strongly and nonlinearly on the radius R of wire. The absorption has the same maximum values (peaks) at $R \sim 48\,\text{nm}$. At $T = 270$ and $T = 273$, the maximum values of absorption coefficient are $1.47\,\text{m}^{-1}$ and $1.17\,\text{m}^{-1}$ respectively. The radius R of wire at which α has a maximum is not changed as the temperature T of the system is varied. However, Figure 2 shows that the radius R of wire at which α has a maximum changed if the effective frequency of potential well ω_0 is varied. For example, at $\omega_0 = 10^{12}\,\text{s}^{-1}$ and $\omega_0 = 2 \times 10^{12}\,\text{s}^{-1}$ the peaks correspond to $R \sim 48\,\text{nm}$ and $R \sim 34\,\text{nm}$, respectively. The nonlinear absorption coefficient depends strongly on the effective frequency of potential well ω_0. At $\omega_0 = 10^{12}\,\text{s}^{-1}$ and $\omega_0 = 2 \times 10^{12}\,\text{s}^{-1}$, the maximum values of absorption coefficient are $1.47\,\text{m}^{-1}$ and $1.82\,\text{m}^{-1}$ respectively. In addition, it can be seen from Figure 1 and Figure 2 that the radius at which the nonlinear absorption coefficient has a maximum value only depends on the structural parameter of wire (the effective frequency of potential well ω_0). Figure 3 shows the dependence of the nonlinear absorption coefficient α on the temperature T of the system at different values of frequency Ω of the external strong EMW. It can be seen from this figure that the nonlinear absorption coefficient α depends strongly and nonlinearly on T. The nonlinear absorption coefficient α is increased when the temperature T increases. But when α has the maximum value, if the temperature continues to increase, α decreases. This fact was not seen in bulk semiconductors [9] as well as quantum wells [10]. This can be explained that: At high temperatures, conditions $\varepsilon_{n+1} - \varepsilon_n \gg k_b T$ no longer is satisfied. The dependence of α due to the confinement of electrons in one-dimensional systems is significantly reduced. The temperature T of the system at which α has a maximum is changed as frequency Ω of the external strong EMW is varied. For example, at $\Omega = 6\times10^{13}\,\mathrm{s}^{-1}$ and $\Omega = 7\times10^{13}\,\mathrm{s}^{-1}$ the peaks correspond to $T \sim 160\,\mathrm{K}$ and $180\,\mathrm{K}$, respectively. Figure 4 shows the dependence of the nonlinear absorption coefficient α on the intensity E_0 of EMW at different values of the temperature T of the system. It can be seen from this figure that the nonlinear absorption coefficient depends strongly and nonlinearly on the intensity E_0 of EMW. The nonlinear absorption coefficient α increases when the intensity E_0 of EMW increases. Figure 5 presents the dependence of α on the EMW energy at different values of the effective frequency of potential well ω_0. It is seen that α has the same maximum values (peaks). The EMW energy at which α has a maximum is $\hbar\Omega = 0.03625\,\mathrm{eV}$ ($\Omega \equiv \omega$) and not changed as the effective frequency of potential well ω_0 is varied. This means that α depends strongly on the frequency Ω of the EMW, and

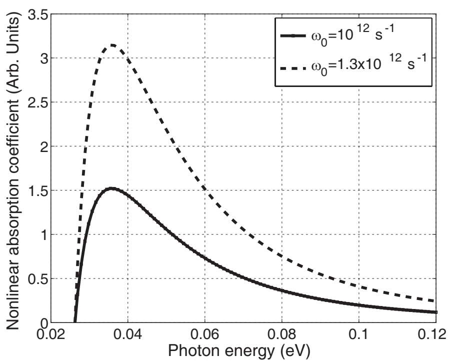

Figure 5. Dependence of α on EMW energy at different values of ω_0.

resonance conditions are determined by the EMW energy. Maximum value of α varies when changing the effective frequency of potential well ω_0. At $\omega_0 = 10^{12}\,\mathrm{s}^{-1}$ and $\omega_0 = 1.3 \times 10^{12}\,\mathrm{s}^{-1}$, the maximum values of absorption coefficient are $1.5\,\mathrm{m}^{-1}$ and $3.2\,\mathrm{m}^{-1}$ respectively. In addition, it can be seen from Figures 1–5 that, different from normal bulk semiconductor [9] and quantum wells [10], the nonlinear absorption coefficient α in quantum wires is bigger. It is explained that when electrons is confined in a quantum wire, the electron energy spectrum continues to be quantized. So the absorption of a strong electromagnetic wave is better. This fact is also reflected in the expressions of the nonlinear absorption coefficient (Eq. (13)). Besides the sum over quantum n (as in quantum well), the expressions of the nonlinear absorption coefficient in quantum wire have the sum over the quantum number ℓ.

4. CONCLUSION

In this paper, we have obtained analytical expression for the nonlinear absorption of a strong EMW by confined electrons in CQW for the case electron-optical phonon scattering and numerically calculate for a $GaAs/GaAsAl$ quantum wire. The analytical results and numerical results obtained for a $GaAs/GaAsAl$ CQW show that α depends strongly and nonlinearly on the intensity E_0 and frequency Ω of the external strong EMW, the temperature T of the system and the radius R of wires. Results also show that the nonlinear absorption coefficient has maximum values at a value determined by wire's radius, at which the nonlinear absorption coefficient has a maximum value only dependent on the structural parameter of wire (the effective frequency of potential well ω_0). In the dependence of α on the temperature T, α also has the same maximum value (peaks). From this peak, if the temperature continues to increase, α decreases because at high temperatures, conditions $\varepsilon_{n+1} - \varepsilon_n \gg k_b T$ no longer are satisfied. The dependence of α due to the confinement of electrons in one-dimensional systems is significantly reduced. These new contributions are not seen in other articles [9, 10, 16–19]. The nonlinear absorption coefficient in quantum wire is larger than the nonlinear absorption coefficient in normal bulk semiconductors [9] and two dimensional systems [10, 18]. The results show a geometrical dependence of α due to the confinement of electrons in one dimensional systems. In addition, from the analytic result, we see that when the term in proportion to quadratic the intensity of the EMW (E_0^2) (in the expression of the nonlinear absorption coefficient of a strong EMW) tends toward zero, the nonlinear result will turn back to a linear result.

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