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A NEW FAST METHOD OF PROFILE AND WAVE-FRONT REDUCTION FOR CYLINDRICAL STRUCTURES IN FINITE ELEMENTS METHOD ANALYSIS

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Abstract—We present a new accurate node's renumbering method for minimizing the profile of stiffness matrix arising in finite elements problems. This method is suitable for cylindrical structures like electrical rotating machines and is especially intended for movement consideration by the moving band method. The structure is divided into sectors classified in a special way. The nodes contained in each sector are classified according to their radius value in regressing order. We show that the performances of the method are better than the most popular ones proposed in the literature. Application for a permanent magnet synchronous machine is presented. Application for finite elements analysis of a permanent synchronous machine in motion is achieved.

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1. INTRODUCTION

The Finite element Method is widely used for the computation of the electromagnetic field in the electrical machines. It allows movement consideration and takes into account saturation and coupling with electrical circuit [1].

The finite element discretization leads to solve algebraic systems with large sparse matrices using direct or iterative methods. In the 2D case, direct methods are preferred because they produce an exact solution using a finite number of operations and they have no convergence difficulties. Moreover, they do not require an initial estimate for the solution [1].

In both cases, a nodes renumbering method for minimizing the bandwidth, the profile or the frontwidth is required in order to reduce the computation time while solving the algebraic system [2]. Most of these methods use the matrix graph levels. A graph of the matrix is built and structured into levels. The nodes of the graph are then numbered level by level.

In [3], a comparison of some renumbering methods adapted for finite elements problems is presented. It has been concluded that the GENRCM algorithm of George and Liu [5], which is a variant of the GPS algorithm [5], gives good performances in reducing the bandwidth and the profile with acceptable computation time. The GPS method is one of the fastest methods for minimizing the bandwidth of sparse matrices [6] whereas Sloan's algorithm [7] represents a better choice for profile and frontwidth reduction. We can note that recent studies show that the method can be improved [6, 8, 9]. The cost of the improved method in [6] is not important and is comparable with the original one.

Since this comparison, few methods of minimizing bandwidth and profile were published in literature. We can quote the Tabu Search [10] developed by Marti et al. It has the particularity of being the first method which is not a graph level method. In some cases, it ensures a better reduction of bandwidth and profile compared with GPS method, but it is expensive in computing time. Moreover, this method is not applicable for large systems. The GRASP-PR method [11] and the Search Annealing method [12] have also large CPU time. Other methods based on operational research like genetic algorithms [13] and colonies of ants [14] are used with the same drawback regarding the computation time.

We proposed in [15] a new mesh renumbering method which is faster and more efficient than the GPS-GENRCM algorithm. The time required for building the matrix graph is removed. We have shown that the proposed method is very suitable for moving band problems.

In this paper, we propose another new renumbering mesh method especially dedicated to cylindrical structures like electrical rotating machines. It consists in ordering the nodes in sectors according to their radius. The sectors are classed with a specific method making it simpler than the one proposed in [15]. It also provides better results in profile reduction leading to huge savings in computation time with a Cholesky direct solver. Application to electromagnetic modeling of electrical machines with movement consideration using the moving band method is presented. It is shown that the computation time is much smaller compared to the fastest methods existing today. An application for electrical machines in rotation with moving band is presented.

2. PRINCIPLE OF THE RENUMBERING METHOD

2.1. Profile Reduction for Cholesky Resolution

Let A be an N by N symmetric positive definite matrix, with entries a_{ij}. The i-th bandwidth of A is:

$$\beta_i(A) = i - \min\{j \mid a_{ij} \neq 0\} \tag{1}$$

The bandwidth of A is defined by [4]

$$\beta = \beta(A) = \max\{\beta_i(A) \mid 1 \leq i \leq N\} = \max\{|i-j| \mid a_{ij} \neq 0\} \tag{2}$$

For Cholesky resolution, we use a vector that contains the all lines bandwidths also called envelope of A. This vector is defined by:

$$Env(A) = \{i, j \mid 0 < i - j < \beta_i(A)\} \tag{3}$$

The quantity $|Env\ (A)|$ is called the profile or the envelope size of A, and is defined by:

$$|Env(A)| = \sum_{i=1}^{N} \beta_i(A) \tag{4}$$

Another quantity called frontwidth is defined by the number of rows of the envelope of A which intersect column i.

$$\omega_i(A) = \{k \mid k > i \text{ and } a_{k\ell} \neq 0\ \forall \ell \leq i\} \tag{5}$$

The number of operations required to factor A into LLt, for envelope methods, is given by:

$$N_{fact} = \frac{1}{2} \sum_{i=1}^{N} \omega_i(A)\left(\omega_i(A) + 3\right) \tag{6}$$

Using a “profile” solver, the number of operations required during the factorization of A into LLt (L is a lower matrix) is:

$$N_{op} = 2 \times (Env\,(A)\ +\ N) \tag{7}$$

To estimate factorization time, a quantity called root mean square frontwidth is introduced by [7]. This quantity is given by:

$$f = \sqrt{\frac{1}{N}\left(\sum_{i=1}^{N} \omega_i^2\right)} \tag{8}$$

The better profile reduction methods are those minimise the quantities defined in (4) and (8).

2.2. Graph Theory and Maximum Eccentricity

Let be A symmetric sparse positive definite matrix. Each no diagonal element of matrix A is represented by a segment between diagonal elements that represents the graph's nodes. A graph $G(A)$ can be associated with the matrix. This graph is defined by a set of nodes X^A and a set of segments E^A.

$$G^A = \left(X^A, E^A\right) \tag{9}$$

A graph G is valid if:

$$\forall \{x_i, x_j\} \in E^A, \quad a_{ij} = a_{ji} \neq 0, \quad i \neq j \tag{10}$$

The fastest graph renumbering methods are based on level decomposition. Eccentricity of a graph is the number of levels. The GPS method allows finding the pseudo peripherals nodes that ensures a maximum eccentricity for a graph, by searching pseudo peripheral nodes. This method provides good results, but it heavily depends on the choice of a good starting node [6].

Figure 1 highlights the application of this renumbering method and its application on a cylindrical structure. The maximum eccentricity is obtained for nodes 1 and 5 that are the peripherals nodes of the graph, Figure 1(b). Renumbering the nodes according to the levels of the graph results in a maximum eccentricity, Figure 1(c).

2.3. Proposed Method

Given a cylindrical structure, and perform a triangular Delaunay mesh. The geometry is subdivided on even number Ns of angular sectors. Each sector has an angular opening of $2\pi/Ns$ (Figure 2(a)). They are numbered, by considering these sectors as nodes of graph in way obtained in Figure 1(c).

- The sectors are numbered in the trigonometric order with odd numbers from 1 to $Ns - 1$. This operation concerns half of a structure.
- The second part of the structure is numbered from Ns to $Ns - 2$ with even numbers in the watch needles order.

We number the nodes met sector by sector (according to the classification of these sectors) by order of decreasing radius (Figure 2(b)). In this case, we consider that radius of each node corresponds to its level.

The algorithm used to carry out this classification is easy of construction. The renumbering according to the geometrical position of the nodes provides a vector permutation applied directly to the

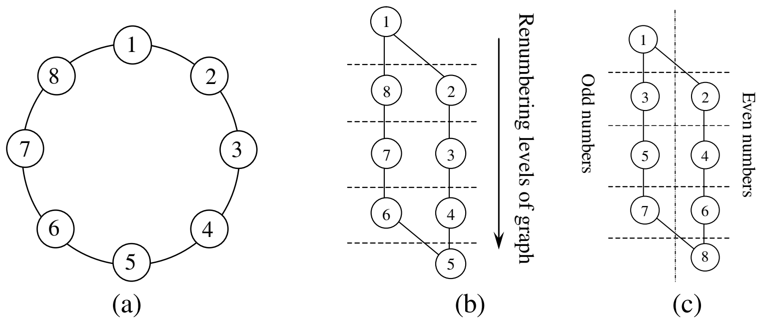

Figure 1. Maximum eccentricity for renumbering circular structure.

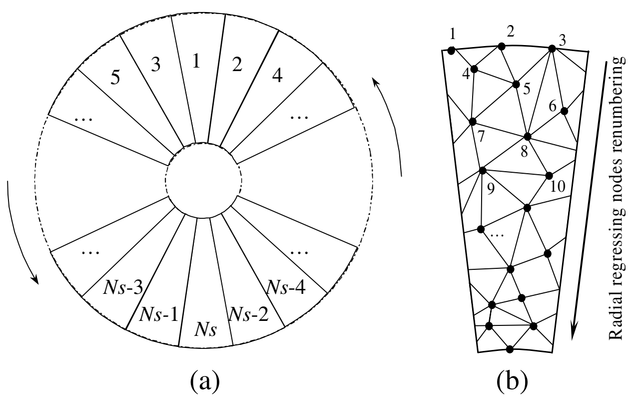

Figure 2. Decomposition of meshing cylindrical structure in sectors. (a) Renumbering the sectors according the levels like Figure 1(c). (b) Renumbering the sector nodes according their radius.

classification of the nodes of each element.

For proposed algorithm, the number of sectors is selected for reducing the profile size and the bandwidth of the stiffness matrix. We choose a number greater than 300, which insures optimal performances for the proposed method.

3. APPLYING ON ROTATING ELECTRICAL MACHINE

Three methods are applied on a six pole surface mounted permanent magnet machine (Figure 5). Four structures with different sizes of mesh are considered and given in Table 1. The performances of the proposed method are compared with those obtained with two classical renumbering methods: GPS algorithm (coded in Fortran77 by George and Liu [5]) and Sloan algorithm (also coded in Fortan77 by Sloan [7]). The proposed algorithm is coded in Fortan90.

Figure 3 compared show results of the three methods. GPS results are considered as the reference and its values (profile and rms frontwidth) are fixed to 100%. The profiles obtained with Sloan method equal 64% to 76% of those obtained with GPS method. We can

Table 1. Meshed structures renumbered with the both methods.

Structure	Number of Nodes	Number of elements
1	4645	9193
2	9193	18289
3	15440	30783
4	23872	47680

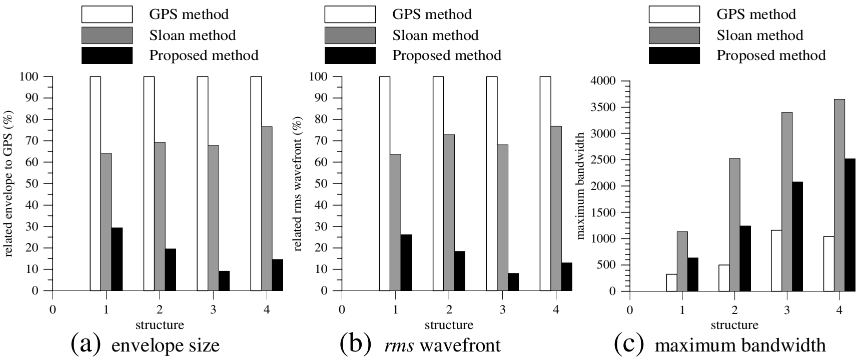

Figure 3. Compared characteristics of stiffness matrices obtained for renumbered meshes of structures given in Table 1.

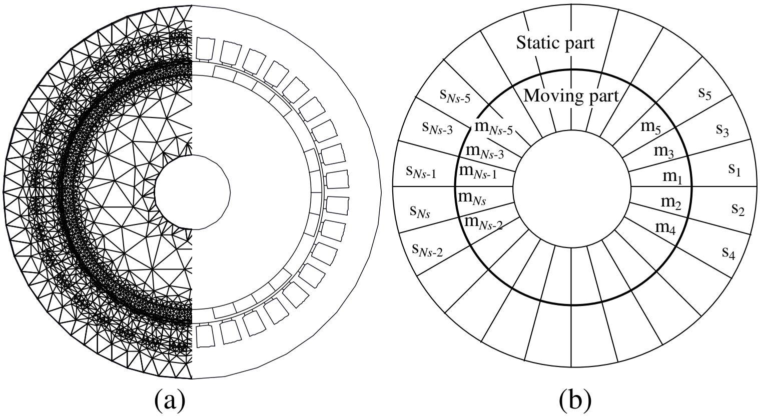

Figure 4. (a) Six pole permanent magnet machine (b) portioning in sectors.

then consider Sloan reduction as good. So, the profiles obtained with the proposed method equal 9.1% to 29.4% of those obtained with GPS results. Thus, the profile is more reduced with the proposed method (see Figure 3(a)). We can consider the proposed method is better than the two traditional methods.

The second factor of comparison is the rms wavefront defined by (8). In Figure 3(b), we can see that this value is most reduced with the proposed method, than with the other methods.

For band methods, the GPS method, which is a bandwidth reduction method, presents an extreme reduced value of maximum bandwidth (see Figure 4). The proposed method can be considered as a profile reduction method. We can note that the bandwidth obtained with the proposed method is smaller than obtained with Sloan method.

4. MOVEMENT CONSIDERING WITH THE MOVING BAND METHOD

4.1. The Moving Band

The moving band is one of the first techniques to take into account the movement in electromagnetic structures in Finite Elements Analysis [16]. It consists to create, in the case of cylindrical structures, a band in airgap. A local remeshing is done when the elements deformation is important. This method presents some of numerical problems [17]. One of these problems is that the bandwidth and

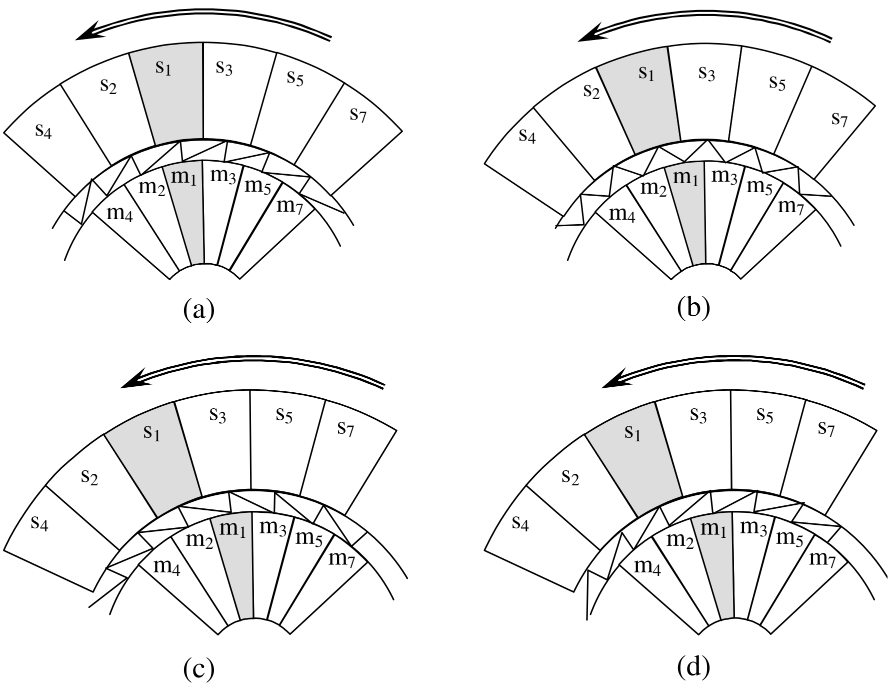

Figure 5. Applying the moving band for the sectored structure. (a) initial position, (b) beginning movement, (c) movement with a step angular, (d) Reconnecting the elements band.

profile values increase with the movement. Instead, it is necessary to apply a renumbering method with each remeshing of band. These renumbering processes take a supplementary time coast in resolution. The preferred methods for finite elements analysis are GPS for bandwidth minimization for band methods and Sloan method for profile reduction for envelope methods, which are the faster methods in these applications [3]. Other problems are in boundary conditions when we consider a portion of the structure (see [18, 19]). This problem can be solved by considering the complete structure (in case of rotating machines). Problems of distortion of elements can be solved by choosing an appropriate step [17, 20] of moving, refine moving band meshing [21], and to solve with direct methods such Gauss method [22] for precision and no divergence.

For our application, we have take into account all these solutions, and we have considered a complete structure with fine meshing moving band. For storage of stiffness matrix, we choose envelope method which is available with Cholesky method (this method is faster than

Elimination Gauss method [5]).

We consider in following application (Figure 4(a)) for a six pole surface permanent magnet structure (structure 4 in Table 1). In the case of movement, we consider two parts for the cylindrical structure: a moving part and a static part (Figure 4(b)). Sectors defined by the proposed method are building directly with sectors of static and moving parts.

Two matrices are created and contain the sectors of each part of structure: matrix M for moving part and S for static part. These are 1440 sectors by part (0.25 angular degree by sector. Each sector constitutes a column of associate matrix (example, s_3 is the third column of matrix S). The formed columns contain like data the nodes' numbers contained in their sectors.

The permutation vector is built like this:

$$Prm = \{s_1, m_1, s_2, m_2, s_3, m_3, \dots\} \tag{11}$$

4.2. Adapted Method for Renumbering While Movement

When the elements of the moving band are reconnected (Figure 5(d)), the "static" sectors si are displaced relatively of "moving" sectors mi.

The proposed method is then adapted by connecting in special way these sectors. The sector s_1 is then connected with sector m_2, sector s_2 is connected with sector m_4 ....

The new permutation vector is then given, for a first step of rotation:

$$Prm = \{s1, m2, s2, m4, s3, m1, \dots\} \tag{12}$$

At each step, after applying a local remeshing in moving band, this vector is built and applied to renumbering the nodes of the mesh. We have also applied the GPS method and Sloan algorithm for renumbering the same structure in same conditions of rotation.

4.3. Results and Comparison

With applying this method for modification of vector permutation for each step reconnecting (we consider all possible cases, then there are 1440 reconnections and 1440 renumbering elements of moving band, we obtain a relatively constant profile (a maximal variation about 0.54% of average value) for the proposed method (see Figure 6(a)). This result is the same for the root mean square wavefront, which is relatively constant on the 1440 steps. We can see that the proposed method is stable. The Cholesky time resolution is then constant for each step. Total resolution time can then be easily estimated.

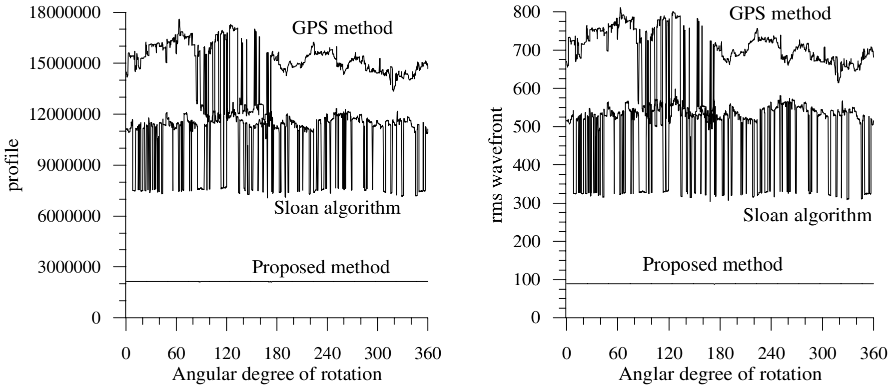

Figure 6. Applied methods on moving band with electrical machine.

Table 2. Times required by the system for a complete rotation (1440 steps) with mesh renumbering.

	Sloan	GPS	Proposed
Geometrical operations (s)	15.03	15.03	15.03
Graph building time (s)	36.47	71.71	—
Permut. vector building (s)	247.17	13.36	0.81
Vectors s and m built (s)	—	—	0.03
Total time (s)	298.67	100.1	15.87

GPS and Sloan methods present a large variation in results with movement, and gives profile and rms wavefront greater than proposed method results (these values are divided at least by 3 for obtain the proposed method results). We can note that Sloan provides results better than GPS values. We can conclude then the proposed method is more adaptive for moving band for cylindrical structures such electrical rotating machines for solving with envelope methods.

4.4. Time Execution Comparison

A fortran90 program is compiled with a Pentium IV CPU 3.0 GHz with RAM 1Go. A complete rotation of rotor is done with reconnecting elements of the band each degree geometrical. At each reconnection, a renumbering is done, and the stiffness matrix is rebuilt with envelope storage. In Table 2, we can see times required for a complete rotation (360 angular degrees) of a structure with a step equal to 0.25 angular degree.

In geometrical operations, we means: recalculate coordinates of moving part nodes, applying reconnections in moving band, applying vector permutation obtained with the given renumbering method for renumbering elements and redefine their coordinates. These operations are the same for all methods and takes 15.03 s. For the proposed method, this time is the major part and represents 94.5% of the total time. The time required for renumbering is composed by sectors partitioning of the cylindrical structure, which is executed only once time (0.03 s), and the vector permutation built at each step of rotation (0.81 s for 1440 steps, ie 0.56 ms by step and 2.35 10–8 s by node and step).

Classical methods referenced in this work (GPS and Sloan methods) are matrix graph renumbering. Thus, at each step, we need a procedure to transform a mesh to graph. At each step, the mesh is modified by the moving band. It is then necessarily to build the graph associate to stiffness matrix at each step. For Sloan method, we have used the GRAPH subroutine given by Sloan [7]. For GPS program, we have developed our subroutine and used it.

For GPS method, the geometrical operations and permutations vector building have approximately the same cost. The proposed method is 16.5 time faster for building permutation vector than GPS method.

Graph building is required, and its cost is very important (71,6% of total cost). For actual CPU processors, these values are acceptable. Unlike proposed and GPS method, the cost of building the permutation vector for the Sloan method represents the major part of the simulation (82.7%) and represents more than 300 time proposed method cost and more than 18 time GPS method time.

For Cholesky factorization, with envelope methods, we can estimate with f (rms frontwidth) ratio that proposed method can reduce considerably the cost. Indeed, value of f in average is divided in average by 7 for GPS results and by 5 for Sloan results. The factorization time depends on the square of f. The Cholesky time cost is then consequently reduced in the case of the proposed method.

5. CHOLESKY RESOLUTION WITH MOTION

In Table 3, we can see influence of the mesh renumbering for three structures in the case of the three methods for the first step of compute and solve the algebraic equation in linear magnetostatic case.

Sloan method gives for the three structures in solving better results than GPS in order of half. But the proposed method permits a more important reduction of cost in resolution. Moreover, Proposed method gives rms frontwidth and profile constant during the rotation

Table 3. Times required for the first step Cholesky resolution for the three methods. (Fact. for factorization LLt, sol. for solving the system factorized and Tot. for total time for Cholesky Resolution).

Studied structure	Proposed (s)			GPS (s)			Sloan (s)		
	Fact	sol	Tot.	Fact	sol	Tot.	Fact	sol	Tot.
Struct. 2			**0.18**	03.79	0.04	**03.83**	02.09	0.03	**02.12**
Struct. 3	0.36	0.01	**0.37**	47.08	0.20	**47.28**	21.78	0.15	**21.93**
Struct. 4	0.97	0.05	**1.02**	57.25	0.28	**57.53**	31.53	0.21	**31.74**

Table 4. Times required in seconds by the structures for Cholesky resolution in the case of one, ten and hundred steps of rotation.

Nb steps	One step			10 steps			100 steps		
Method	Gps	Slo	Pro.	Gps	Slo	Pro	Gps	Slo	Pro.
Struct. 2	03.83	02.12	**0.18**	037.7	20.9	**1.9**	380.9	212.9	**19.2**
Struct. 3	47.28	21.93	**0.37**	432.7	222.5	**4.1**	$>$ 1H		**41.5**
Struct. 4	57.53	31.74	**1.02**	541.5	321.4	**11.1**	$>$ 1H	$\approx$ 1H	**114.8**

(Figure 6). Thus, we can estimate correctly the total simulation time contrarily to the two other methods.

In Table 4, we can see that the proposed method provides a very large time reduction compared with GPS and Sloan methods. These methods have a times costs in resolution which are about 50 times those of proposed method for GPS method and about 30 times those of proposed method for Sloan method. Geometrical operations represent only 1% at the maximum of the total time resolution for the proposed method.

6. CONCLUSION

A powerful renumbering mesh method for hollow circular structures is presented. Its algorithm is very easy to code and compute, and it is adaptive for movement method considerations with finite elements method. Performances of method, compared with those of Sloan and GPS methods, are better. It reduces considerably the profile and the root mean square wavefront. The time coast is also considerably reduced.

In movement consideration with moving band in circular structures, the method is more adaptative than the compared methods. Cost of graph associated to mesh is removed and the permutation vector is obtained in very reduced time compared to GPS, which is one of the faster graph reordering methods at today. Indeed, for each moving step, the permutation vector is obtained with special circulation of vectors. This requires a minimum coast of renumbering.

A complete rotation with 1440 steps is given in total time which is 6.3 time reduced than GPS algorithm and 18.8 time reduced than Sloan results. If we introduce the Cholesky resolution, these times are reduced by about 50 time for GPS method and 30 time for Sloan method.

Moreover, the profile and root mean square and maximum bandwidth obtained are relatively constant during the movement, as opposed to the compared method. The Sloan average profile is reduced with a factor around 5.

However, this method is adaptive only cylindrical structures in 2-D modeling, and then interesting field analysis with finite elements methods in rotating electrical machines.

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