Progress In Electromagnetics Research C, Vol. 18, 185–195, 2011

ON THE USE OF GEGENBAUER PROTOTYPES IN THE SYNTHESIS OF WAVEGUIDE FILTERS

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Abstract—Filter prototypes derived from Gegenbauer polynomials can represent a useful trade-off between amplitude and phase behavior. This paper discusses the main features of this prototype through a comparison with the more classical Chebyshev and Butterworth solutions; it shows, in the case of an X-band waveguide realization, how its intermediate characteristics, with respect to both amplitude and phase responses, can be very useful in satisfying particular filter performance requirements without increasing filter order.

1. INTRODUCTION

Modern modulation techniques require more and more demanding phase characteristics of the filters employed, which often cannot be met by classical Chebyshev prototypes. As a consequence, an effort has been recently made to develop new prototypes with a prescribed group delay. In a large part of these solutions [1], the equalization is obtained by creating multipath between input and output, in such a way that signals flowing within different paths combine together and give the desired phase response in the pass band. The intrinsic deterioration of the out of band filter performance, due to the presence of multipaths, is usually acceptable; this is because the use of high order filters is made necessary only by the need of increasing the roll-off in the neighborhood of the pass-band, while the attenuation is typically much larger than that required in the out-band. On the other hand, non-minimum phase filters are more expensive than classical single-path solutions because of their higher complexity. The main drawback

Received 25 October 2010, Accepted 7 December 2010, Scheduled 13 December 2010
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concerns tuning, which is more involved; this is due to the presence of additional coupling elements [2, 3]. In addition, in high power applications, despite the fact that often tight specifications on group delay must be accomplished, multipaths are not well suited. This is because of the presence of the small cross-coupling elements, which may originate breakdown. In minimum phase filters, it is well known that phase and amplitude behaviors are not independent of each other. They are, in fact, linked together via a Hilbert's transform [4], which states that the better the amplitude response, the worst the phase behavior. The physical explanation is quite evident when considering the time domain response of a single resonator. Multiple reflections combine in phase only at the center frequency, where the length of the cavity is exactly a multiple of $\lambda_g/2$. As much as the frequency moves away from the centre band, as the electrical path covered by each partial wave deviates from a multiple of $\lambda_g/2$, thus producing an undesired group delay. Of course, the effect is stronger close to band edges and as filter selectivity increases. For this reason, prototypes such as Bessel and Butterworth are often preferred to classical Chebyshev in those applications where phase linearity and high power capability are required. Therefore, it can be useful to have a variety of minimum phase filter prototypes, other than Chebyshev, Butterworth and Bessel [5]; this could provide the best trade-off between amplitude and group delay response. The idea of using prototypes deriving from Gegenbauer polynomials simply originates from the consideration that they have a intermediate behavior between Chebyshev prototypes and Butterworth prototypes.

2. GEGENBAUER POLYNOMIALS

Gegenbauer polynomials $C_n^\alpha(\omega)$, depending on the degree n and order α, belong to the class of orthogonal polynomials. They are defined by the explicit expression [6]:

$$C_n^\alpha(\omega) = d_n \sum_{m=0}^{N} c_m g_m(x) \tag{1}$$

where:

$$d_n = \frac{1}{\Gamma(\alpha)} \qquad c_m = (-1)^m \frac{\Gamma(\alpha + n - m)}{m!\,(n-2m)!}$$

$$g_m(x) = (2x)^{n-2m} \qquad N = \left[\frac{n}{2}\right] \tag{2}$$

for $\alpha > -\frac{1}{2}$.

They also could be defined by the Rodriguez formula:

$$C_n^\alpha(\omega) = \frac{(-1)^n \Gamma(n+2\alpha)\Gamma\left(\alpha+\frac{1}{2}\right)}{2^n n! \Gamma\left(n+\alpha+\frac{1}{2}\right)\Gamma(2\alpha)} \left(1-\omega^2\right)^{-\alpha+\frac{1}{2}} \frac{d^n}{d\omega^n}\left[\left(1-\omega^2\right)^{n+\alpha-\frac{1}{2}}\right] \tag{3}$$

or by the recurrence formula:

$$(n+1)C_{n+1}^\alpha(\omega) = 2(n+\alpha)\omega C_n^\alpha(\omega) - (n+2\alpha-1)C_{n-1}^\alpha(\omega) \tag{4}$$

Furthermore, $C_n^\alpha(\omega)$ is even in ω for even n, and odd in ω for odd n. It also has n single zero locations in $\omega \in (-1, 1)$, as shown in Fig. 1.

For $\alpha = 0$ the relationship between Gegenbauer polynomials and Chebyshev polynomials of first type is

$$T_n(\omega) = \frac{n}{2} C_n^0(\omega) \tag{5}$$

instead for $\alpha = 1$ Gegenbauer polynomials are equal to Chebyshev polynomials of second type:

$$U_n(\omega) = C_n^1(\omega). \tag{6}$$

3. PROTOTYPE SYNTHESIS

The transmission of a Gegenbauer low-pass filter prototype is given by

$$|s_{21}(j\omega)|^2 = \frac{1}{1+\epsilon^2\left[\frac{C_n^\alpha(j\omega)}{C_n^\alpha(1)}\right]^2} \tag{7}$$

where $C_n^\alpha(j\omega)$ is a Gegenbaeur polynomial of order α and degree n, where α is a real number. As usual, ϵ represents the maximum ripple in

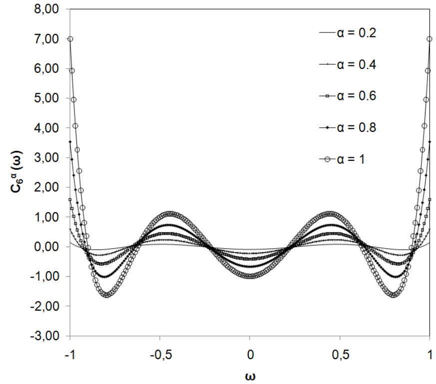

Figure 1. Gegenbauer polynomials, $n = 6$.

band-pass, which is the interval $[0, 1]$. The derivation of the parameters of a lumped low-pass prototype is divided into the following steps.

(i) From the expression (7) of $|s_{21}|^2$ in the complex S-plane,

$$|s_{21}(s)|^2 = s_{21}(s)\, s_{21}(-s) \tag{8}$$

$s_{21}(s)$ is derived in such a way that is a bounded real function or, in other words, its poles lie on the left part of the complex Gaussian plane.

(ii) Then, from the bounded real function $s_{11}(s)$, a positive real input impedance is built by the well known formula

$$Z(s) = \frac{1 + s_{11}(s)}{1 - s_{11}(s)} \tag{9}$$

Since $s_{11}(s)$ is a bounded real function, $Z(s)$ is a positive real function, and it is physically realizable.

(iii) Now, by following Cauer's synthesis method [7], a ladder network as in Fig. 2 having $Z(s)$ as input impedance can be obtained.

Tables A1–A3, reported in the Appendix A, give the values of the immittances g_k for the prototypes up to degree 10 and order 0.4, 5, 20 respectively.

4. PROPERTIES OF THE GEGENBAUER FILTERS

Through direct inspection of Figs. 3 and 4, respectively, showing the reflection in pass-band and the attenuation in stop-band of some prototypes of degree $n = 6$, it is shown that the Gegenbauer responses span between Butterworth to Chebyshev, as the order α goes from infinity to 0.

For all prototypes, the Return Loss at the band edge ($\omega = 1$) has been set equal to 20 dB: unlike the Chebyshev case, where the response is equiripple, in the other cases the return loss in the pass-band is higher. This corresponds to a deterioration of the attenuation in the stop-band, as can be realized by examining Fig. 4. This shows

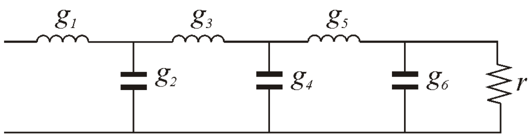

Figure 2. Lumped ladder network ($n = 6$) prototype.

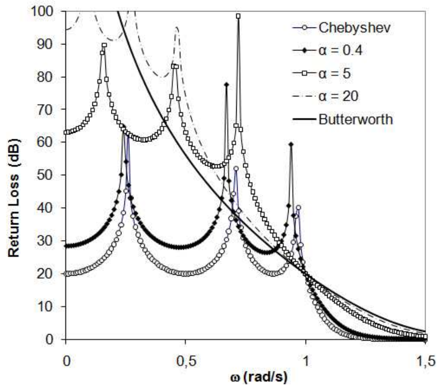

Figure 3. Comparison between the Return Loss responses of Butterworth, Chebyshev and Gegenbauer prototypes of degree 6. The latter spans between Butterworth and Chebyshev as α runs from infinity to 0.

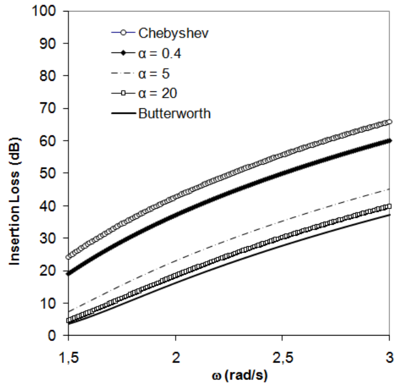

Figure 4. Comparison between the stop-band Insertion Loss of Butterworth, Chebyshev and Gegenbauer prototypes of degree 6. The attenuation decreases as the order increases.

that the better the return loss in pass-band the worse the attenuation in stop-band. Through inspection of the immittances g_k's of the tables reported in Appendix A, it is also shown that reflections, due to each immittance, increase as the order α decreases. Considering that pass-band matching depends on the combination of the partial waves bouncing at each immittance, it is realized that the number of reflections necessary to get the stationary situation increase as g_k's do. Since both losses and maximum field increase with the number of partial reflections, it shows that a proper choice of the order α gives to the designer an extra degree of freedom; this applies to selecting the best trade-off between pass-band return loss, stop-band attenuation, losses and power handling capability. It is not surprising that a similar feature is also seen with respect to the group delay; this also depends on the number of partial waves bouncing back and forth (i.e., resonance) between the immittances, as can be immediately observed by examining Figs. 5 and 6.

Even in this case, group delays are related to the order α of Gegenbauer polynomials. The larger the order, the smaller the group delay. In addition, even the variation of the group delay significantly reduces as the order α.

It must be noted that the main feature of the proposed prototype is the extra flexibility; this given by the choice of the order α, which

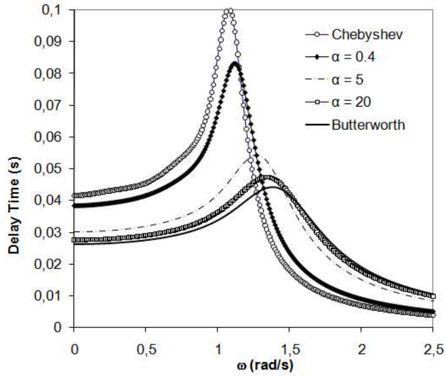

Figure 5. Comparison between the group delays of Butterworth, Chebyshev and Gegenbauer prototypes of degree 6. The latter is characterized by an intermediate behaviour between Butterworth and Chebyshev responses as α runs from infiity to 0.

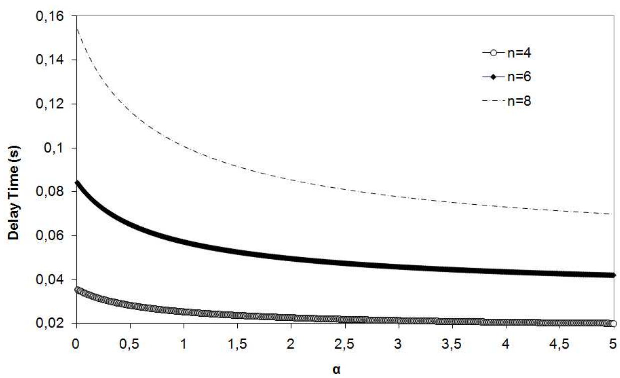

Figure 6. Group delay behaviour respect to the order α fixed the filter degrees at 4, 6 and 8, calculated at the band edge frequency ($\omega = 1$).

allows the designer to get the best trade-off between Chebyshev and Butterworth.

For instance, for a given degree n, if the specifications on group delay are not satisfied, it is possible to continuously increase the order α in order to reduce the group delay and stopping exactly when the specifications are accomplished. Of course, since the stop-band attenuation slightly deteriorates, it could be required an increasing of the filter degree. However, it must be stressed that every adjustment is continuous and limited to what really necessary. Finally, as typically occurs in filter design, the response obtained by the synthesized circuit is 'scaled' by a factor K and the $r:1$ transformer at the end of the network is inserted in order to restore the desired response.

5. X-BAND WAVEGUIDE REALIZATION

As an interesting application of the present prototype, we have considered an X band filter whose midband frequency is 9.220 GHz, a bandwidth of 160 MHz having 20 dB of minimum return loss in passband, an attenuation at 9.415 GHz greater than 45 dB and a maximum delay time variation lower than 5.5 ns. The specifications on amplitude are accomplished by a Chebyshev prototype of order 6 [8]. Unfortunately, this solution does not satisfy the group delay requirements. On the other hand, both specifications can be met by using a Butterworth polynomial of a much higher degree. We have designed two alternative filters satisfying the same pass-band requirements by using Gegenbauer prototypes of degree 6 and orders 0.4 and 5 respectively. All filters of inductive iris type, as the one drawn in Fig. 7, were designed according to our home-made software. In Table 1, their geometrical dimensions are reported.

Table 1. Dimensions of the designed filters (W_i, width of the i)-th window, C_i, length of the i)-th cavity. All dimensions are in mm, $a = 22.86, b = 10.16$, window thickness = 1, edge roundness = 3.

	Chebyshev	Gegenbauer 0.4	Gegenbauer 5
W_1	10.089	10.566	11.605
C_1	20.957	20.306	19.528
W_2	5.726	6.005	6.833
C_2	22.230	22.150	21.866
W_3	5.127	5.229	5.61
C_3	22.364	22.33	22.21
W_4	5.054	5.123	5.38
C_4	22.362	22.329	22.209
W_5	5.127	5.228	5.610
C_5	22.228	22.15	21.87
W_6	5.724	6.002	6.833
C_6	20.604	20.313	19.527
W_7	10.087	10.563	11.605

Transmission, Return Loss and time-delay of the three filters simulated by HFSS [9] are shown in Figs. 8, 9 and 10. As can be seen, the attenuation in stop-band of both Gegenbauer solutions

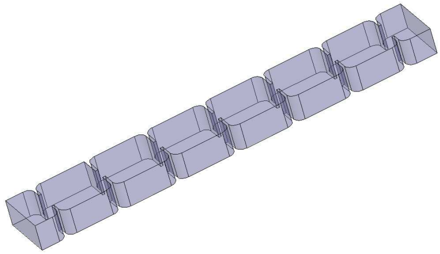

Figure 7. Sketch of the inductive iris filters designed. All filters are of degree 6.

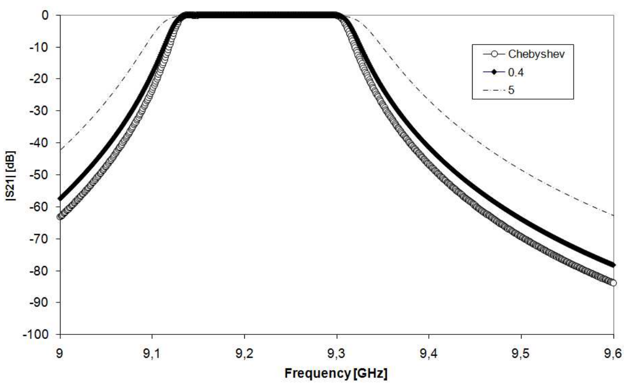

Figure 8. Comparison between the transmissions of the three filters designed. Of course, for a given degree, the best selectivity occurs in the Chebyshev case.

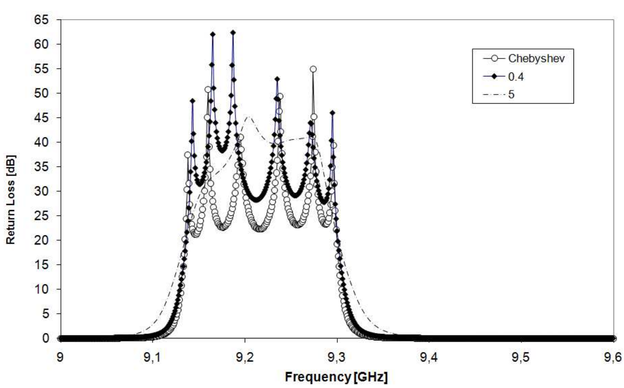

Figure 9. Comparison between the Return Loss of the three lters designed.

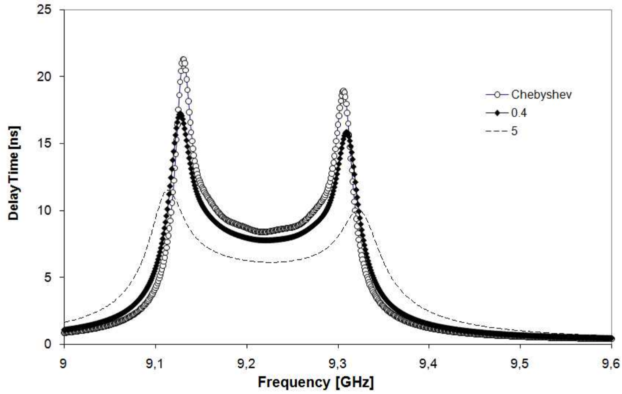

Figure 10. Comparison between the group delays in the pass-band of the Chebyshev and Gegenbauer filters designed. As predicted, the time-delay variation within the pass-band decreases as the order of the Gegenbauer polynomial increases.

are worse than the Chebyshev solution and degrade as the order increases. On the other hand, time-delay variation in pass band improves considerably (Chebyshev: 6.8 ns, Gegenbauer 0.4: 5.05 ns, Gegenbauer 5: 2.2 ns). It is immediately to see that the order 0.4 is an acceptable trade-off between selectivity and time delay response. This proves the usefulness of this class of prototype.

6. CONCLUSION

It has been shown that prototypes, based on Gegenbauer polynomials offer an additional degree of flexibility with respect to classical solutions. This feature can be exploited when designing filters that have to combine unusual specifications on group delay and amplitude response. With this prototype, it is then possible to tailor a solution which is intermediate between Butterworth and Chebyshev.

APPENDIX A. VALUES OF THE IMMITTANCES OF THE GEGENBAUER PROTOTYPE UP TO ORDER 8

Table A1. $\alpha = 0.4$ — Among the case reported in the tables, this is the one closer to the Chebyshev prototype.

n	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	r
1	0.2010								1
2	0.5750	0.5143							0.9457
3	0.7057	1.0764	0.7057						1
4	0.7518	1.2743	1.3917	0.6884					1.0451
5	0.7685	1.3568	1.6173	1.3568	0.7685				1
6	0.7735	1.3968	1.7069	1.5820	1.5071	0.7169			0.9627
7	0.7734	1.4182	1.7487	1.6757	1.7487	1.4182	0.7734		1
8	0.7709	1.4302	1.7701	1.7228	1.8443	1.6534	1.5311	0.7201	1.0347

Table A2. $\alpha = 5$.

n	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	r
1	0.2010								1
2	0.4916	0.4420							0.9909
3	0.5132	0.9715	0.5132						1
4	0.5008	1.1194	1.1241	0.4897					1.0021
5	0.4771	1.1449	1.3568	1.1449	0.4771				1
6	0.4529	1.1302	1.4358	1.4338	1.1318	0.4523			0.9993
7	0.4308	1.1024	1.4543	1.5556	1.4543	1.1024	0.4308		1
8	0.4113	1.0713	1.4478	1.6044	1.6053	1.4470	1.0719	0.4111	1.0003

Table A3. $\alpha = 20$ — Among the cases reported in the tables, this is the closest to the Butterworth prototype.

n	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	r
1	0.2010								1
2	0.4543	0.4521							0.9976
3	0.4785	0.9422	0.4785						1
4	0.4511	1.0649	1.0652	0.4509					1.0002
5	0.4159	1.0612	1.2924	1.0612	0.4159				1
6	0.3829	1.0184	1.3563	1.3563	1.0184	0.3829			1
7	0.3540	0.9656	1.3519	1.4803	1.3519	0.9656	0.3540		1
8	0.3292	0.9131	1.3192	1.5197	1.5197	1.3192	0.9131	0.3292	1

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