

## **ANALYZING THE MULTILAYER OPTICAL PLANAR WAVEGUIDES WITH DOUBLE-NEGATIVE METAMATERIAL**

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**Abstract**—In this study, a general method for analyzing the multilayer optical planar waveguides with photonic metamaterial is presented. The propagation characteristics of TE waves guided by the film with both the permittivity and permeability less than zero are investigated theoretically. The formulae for the electric fields of TE modes in this structure have been proposed. Typical numerical results for dispersion characteristics are shown. The analytical and numerical results show excellent agreement.

### **1. INTRODUCTION**

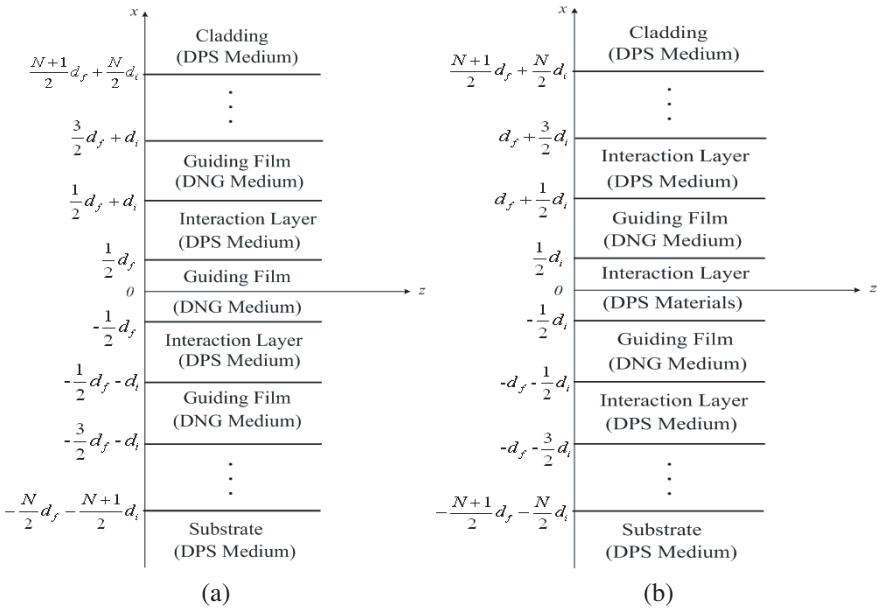
In recent years, there has been a dramatic proliferation of research concerned with the photonic metamaterial [1–10]. Some applications of metamaterial have been studied. The simulations and measurements of metamaterial, such as the self-collimation of light in metamaterial [11, 12], band-pass filters [13], and antennas [14–17], have

been presented. The wave propagating in a double-negative (DNG) medium that has simultaneous negative dielectric permittivity and magnetic permeability have been excellently discussed in [18–21]. For optical waveguides, the analyses of waves guided by thin films with photonic metamaterial are special interest [22–25]. These papers have dealt with the three-layer optical planar waveguide structures where a DNG film was bounded by two DPS media, i.e., a medium having positive dielectric permittivity and positive magnetic permeability.

In conventional materials, the multilayer optical planar waveguides play an important role in guided-wave optics. A number of potential application for such multilayer systems to all-optical signal processing have been identified, e.g., logic gates, switching, wavelength division multiplexer, etc. [26–28]. Studies of TE wave propagating on the multilayer systems in linear/nonlinear have attracted a great deal of interest in the past two decade [29–36]. Wu et al. developed accurate calculations for multilayer systems with DPS materials, as multilayer planar waveguide with nonlinear cladding [37], multilayer planar waveguide with nonlinear cladding and substrate [38], multilayer planar waveguide with a localized arbitrary nonlinear guiding film [39], multilayer planar waveguide with all nonlinear guiding film [40], and multilayer planar waveguide with all nonlinear layers [41]. The complete set of modes of all possible solutions for the TE wave in the three-layer planar waveguide with photonic metamaterial was analyzed and disused [42]. Therefore, it may be reasonable to expect that the presence of multilayer system with DNG guiding films with their more exotic dispersion relations could lead to interesting optical properties. In this work, we have proposed a general modal formalism for modeling TE waves propagating in this metamaterial multilayer structure. The transverse electric field distributions and dispersion relations in this structure have been obtained. This result can be used to predict the propagation characteristics in optical planar waveguide. In the future, the novel phenomenon can be applied in the design of optical devices. The analytical and numerical results show excellent agreement.

## 2. ANALYSIS

In this section, we have used the modal theory [43] to derive the general formulae that can be used to analytically calculate the multilayer optical planar waveguides with photonic metamaterial, as shown in Fig. 1. The multilayer structure is composed of guiding film with DNG materials ( $N + 1$  layers), interaction layers with DPS materials ( $N$  layers), cladding with DPS materials, and substrate with DPS materials. The total number of layers is  $2N + 3$ . The constants  $d_i$



**Figure 1.** The structure of multilayer planar waveguides with photonic metamaterial: (a) for  $N = \text{even}$  and (b) for  $N = \text{odd}$ .

and  $d_f$  are the widths of the interaction layer and the guiding film, respectively. The cladding and substrate layers are assumed to extend to infinity in the  $+x$ - and  $-x$ -directions, respectively. The major significance of this assumption is that there are no reflections in the  $x$ -direction to be concerned with, except for those occurring at the interfaces. For the simplicity, we have considered the transverse electric plane waves propagating along the  $z$ -direction. The wave equation can be reduced to

$$\nabla^2 \Phi_{yj} = \frac{\varepsilon_j \mu_j}{c^2} \frac{\partial^2 \Phi_{yj}}{\partial t^2}, \quad j = i, f, c, s \tag{1}$$

with solutions of the form

$$\Phi_{yj}(x, z, t) = E_j(x) \exp[i(\omega t - n_e k_0 z)], \quad j = i, f, c, s \tag{2}$$

The subscripts  $i, f, c,$  and  $s$  in Eq. (1) are used to denote the interaction layer, guiding film, cladding, and substrate, respectively. For TE waves, the electric field components  $\Phi_x$  and  $\Phi_z$  are zero. It can also be noted that in Eq. (2), that the transverse electric field  $E(x)$  has no  $y$ - and  $z$ -dependence because the planar layers are assumed to be infinite in these directions, precluding the possibility of reflections and resultant

standing waves. In Eq. (2),  $n_e$  is the effective refractive index,  $\omega$  is the angular frequency, and  $k_0$  is the wave number in the free space. By substituting Eq. (2) into the wave equation, the transverse electric fields in each layer has the general form

$$E_c(x) = A_c \exp(-k_0 q_c x), \quad \text{in the cladding} \quad (3a)$$

$$E_f(x) = A_{f(m)} \cos[k_0 Q(x - x_f(m))], \quad \text{in the guiding film} \quad (3b)$$

$$E_i(x) = A_{i(m)} \cosh[k_0 q_i(x - x_i(m))], \quad \text{in the interaction layer} \quad (3c)$$

$$E_s(x) = A_s \exp(k_0 q_s x), \quad \text{in the substrate} \quad (3d)$$

where subscripts  $m = 1, 2, \dots, N$ . The  $A_c$ ,  $A_{f(m)}$ ,  $A_{i(m)}$ ,  $A_s$ ,  $q_c$ ,  $Q$ ,  $q_i$ ,  $q_s$ ,  $x_f(m)$ , and  $x_i(m)$  are all constants. These constants can be determined by matching the boundary conditions. For the linear case, the constant  $A_c$  is arbitrary. The amplitude parameters  $A_{f(m)}$ ,  $A_{i(m)}$ , and  $A_s$  are proportional to  $A_c$ . The detail derivations of these constants  $A_{f(m)}$ ,  $A_{i(m)}$ ,  $A_s$ ,  $q_c$ ,  $Q$ ,  $q_i$ ,  $q_s$ ,  $x_f(m)$  are shown in the appendix.

The  $q_j$  and  $Q$  can be expressed as

$$q_j = \sqrt{n_e^2 - \varepsilon_j \mu_j}, \quad j = i, c, s \quad (4a)$$

$$Q = \sqrt{\varepsilon_f \mu_f - n_e^2} \quad (4b)$$

For simplicity, we considered  $\varepsilon_i = \varepsilon_c = \varepsilon_s$  and  $\mu_i = \mu_c = \mu_s$  so that  $q_i = q_c = q_s = q$ . It can be noted that in Eqs. (4a) and (4b) that  $Q$  and  $q_j$  are given in terms of a single unknown, the effective index  $n_e$ . By matching the boundary conditions, we obtained the following equations (appendix):

$$\tan[k_0 Q_f d_f] = \frac{\mu_f \mu_c Q q (1 + \tanh \Psi)}{\mu_c^2 Q_f^2 - \mu_f^2 q^2 \tanh \Psi} \quad (5a)$$

$$\tan[k_0 Q_f d_f] = \frac{\mu_f \mu_c Q q (1 + \tanh \Phi)}{\mu_c^2 Q_f^2 - \mu_f^2 q^2 \tanh \Phi} \quad (5b)$$

where  $\Psi$  and  $\Phi$  can be expressed as

$$\Psi = k_0 q \left[ \frac{d_f}{2} + \left( \frac{N-1}{2} \right) d_f + \frac{N}{2} d_i - x_i(1) \right], \quad \text{for } n = \text{even} \quad (6a)$$

$$\Phi = k_0 q \left[ \frac{d_i}{2} + \left( \frac{N-1}{2} \right) d_i + \left( \frac{N-1}{2} \right) d_f - x_i(1) \right], \quad \text{for } n = \text{odd} \quad (6b)$$

$$x_i(1) = \left( \frac{N-1}{2} \right) d_f + \left( \frac{N}{2} - 1 \right) d_i - \frac{1}{k_0 q} \tanh^{-1} \left\{ \frac{\mu_s Q}{\mu_f q} \tan \left\{ k_0 Q \left[ \left( \frac{1-N}{2} \right) d_f + \left( 1 - \frac{N}{2} \right) d_i + x_f(2) \right] \right\} \right\} \quad (6c)$$

The dispersion equations can be solved numerically. When the constant  $n_e$  is determined, the constants  $A_c$ ,  $A_f(m)$ ,  $A_i(m)$ ,  $A_s$ ,  $q_c$ ,  $Q$ ,  $q_i$ ,  $q_s$ ,  $x_f(m)$ , and  $x_i(m)$  are also determined (appendix). A diagram indicating the computation step is shown in Fig. 2.

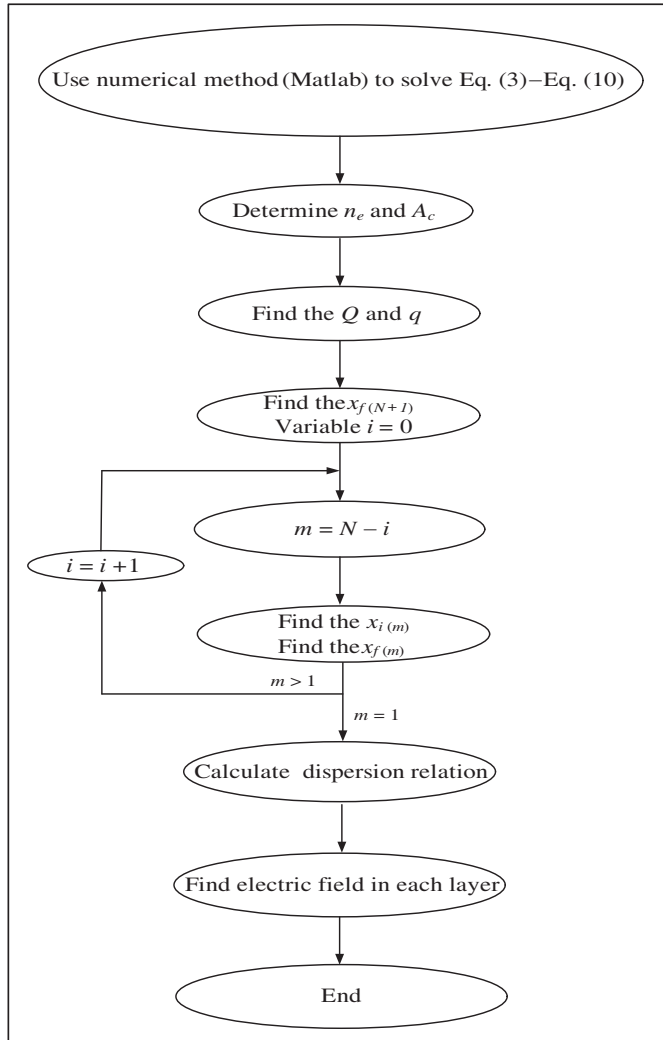
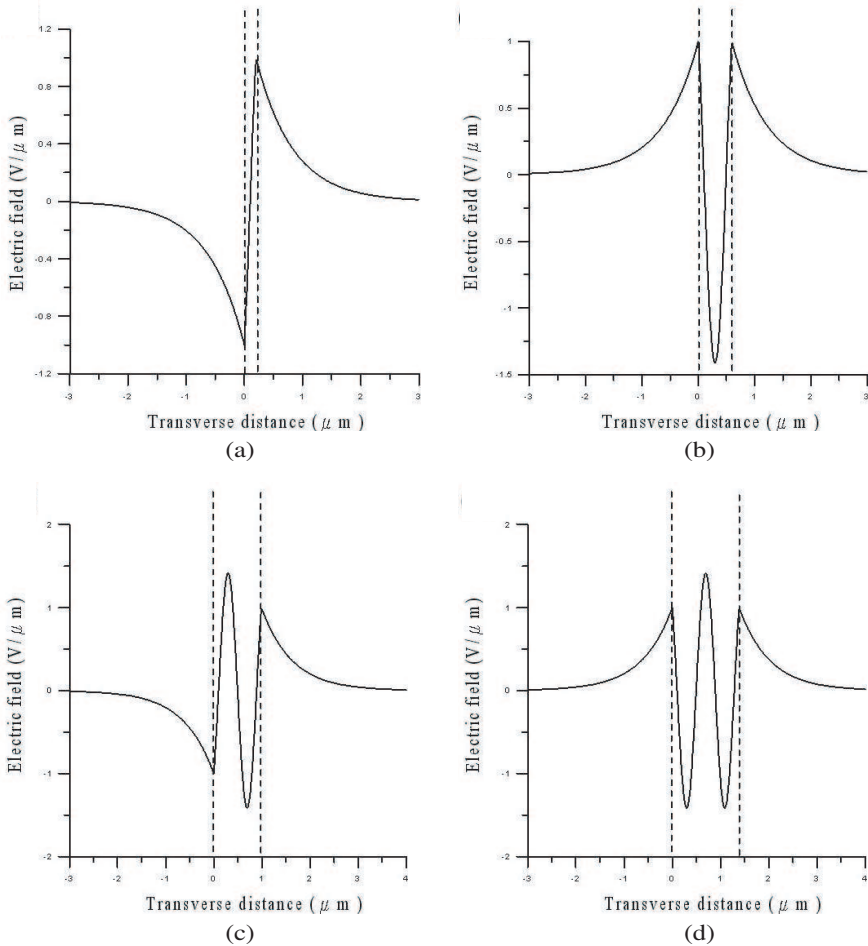


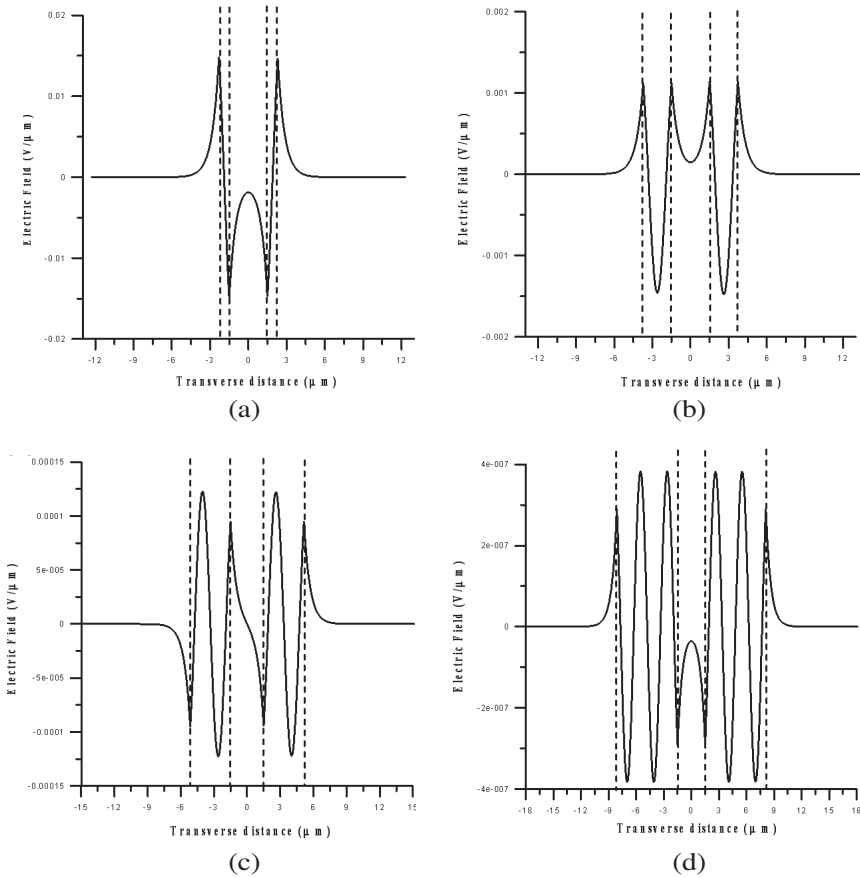
Figure 2. Diagram of the computation steps.

### 3. NUMERICAL RESULTS

In this section, we used the analytical formulae derived in the preceding section to calculate the transverse electric field function  $E(x)$ . The numerical examples are shown, as follows. When  $N = 0$  and 1, the general formulas can be simplified to two special cases, as three-layer and five-layer planar waveguides, respectively. The numerical results are shown in Figs. 3 and 4. The numerical data in three-layer



**Figure 3.** The electric field distributions of a three-layer optical planar waveguide ( $N = 0$ ): (a)  $n_e = 1.074$ ,  $d_f = 0.198 \mu m$ ; (b)  $n_e = 1.074$ ,  $d_f = 0.593 \mu m$ ; (c)  $n_e = 1.074$ ,  $d_f = 0.988 \mu m$ ; and (d)  $n_e = 1.074$ ,  $d_f = 1.383 \mu m$ .



**Figure 4.** The electric field distributions of a five-layer optical planar waveguide ( $N = 1$ ): (a)  $n_e = 1.558$ ,  $d_f = 0.8 \mu\text{m}$ ; (b)  $n_e = 1.558$ ,  $d_f = 2.23 \mu\text{m}$ ; (c)  $n_e = 1.558$ ,  $d_f = 3.64 \mu\text{m}$ ; and (d)  $n_e = 1.558$ ,  $d_f = 6.6 \mu\text{m}$ .

structure have been calculated with the values: the wavelength in free space  $\lambda = 1.55 \mu\text{m}$ ,  $\epsilon_f = -\epsilon_c = -\epsilon_s$ ,  $\mu_f = -5\mu_c = -5\mu_s$ . Fig. 3 shows some eigen-modes of a three-layer waveguide. In Fig. 3(a), the electric field distribution is plotted with the parameters  $n_e = 1.074$ ,  $d_f = 0.198 \mu\text{m}$ . In Fig. 3(b), the electric field distribution is plotted with the parameters  $n_e = 1.074$ ,  $d_f = 0.593 \mu\text{m}$ . In Fig. 3(c), the electric field distribution is plotted with the parameters  $n_e = 1.074$ ,  $d_f = 0.988 \mu\text{m}$ . In Fig. 3(d), the electric field distribution is plotted with the parameters  $n_e = 1.074$ ,  $d_f = 1.383 \mu\text{m}$ . The numerical data in

five-layer structure have been calculated with the values:  $d_i = 3 \mu\text{m}$ ,  $\varepsilon_f = -2.4649$ ,  $\varepsilon_c = \varepsilon_s = \varepsilon_i = 2.4025$ ,  $\mu_f = -\mu_c = -\mu_s = -\mu_i$ . This structure can not support fundamental guided mode. As the parameter  $d_f$  increases, the modes will appear in surface modes and guided modes. The guided modes and surface modes are depended on a discrete eigenvalue of effective refractive index  $n_e$ . The studies of three-layer planar waveguides have been presented in [10–17, 34]. In this special case, the electric field and optical properties are the same these literatures.

Here we show another special case of this general analytical method. Fig. 4 shows some eigenmodes of a five-layer waveguide. In Fig. 4(a), the electric field distribution is plotted with the parameters  $n_e = 1.558$ ,  $d_f = 0.8 \mu\text{m}$ . In Fig. 4(b), the electric field distribution is plotted with the parameters  $n_e = 1.558$ ,  $d_f = 2.23 \mu\text{m}$ . In Fig. 4(c), the electric field distribution is plotted with the parameters  $n_e = 1.558$ ,  $d_f = 3.64 \mu\text{m}$ . In Fig. 4(d), the electric field distribution is plotted with the parameters  $n_e = 1.558$ ,  $d_f = 6.6 \mu\text{m}$ . The guiding film also supports the guided modes and surface modes. The power could be carried by the guided mode and the surface mode in longitudinal direction. The phase of leaky wave in interaction layers have depended on the high-order guided modes.

#### 4. CONCLUSION

In this paper, we have proposed a general method to analyze wave propagation in the multilayer optical planar waveguide with photonic metamaterial. The general method can also be degenerated into other special cases for analyzing multilayer photonic metamaterial optical waveguide. This method can be used to predict the propagation characteristics in three-layers, five-layers, or more. The transverse electric field distributions and dispersion relations have been obtained. A thorough understanding of the wave equations and how they are solved helps us understand the optical waveguides themselves. It is useful to design all-optical devices with photonic metamaterial. For a special case in this work, the general method can be degenerated into the three-layer optical planar waveguide with DNG guiding film. The results are entirely consistent with those reported in previous studies. The five-layer optical planar waveguide with DNG guiding film has been proposed. The present study enhances the previous studies' findings by providing a much more detailed examination of wave propagation in the media whose optical response is DNG metamaterial. Rigorous analyses and numerical results show that our formulations are correct.



### ACKNOWLEDGMENT

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### APPENDIX A. DETAILED DERIVATIONS OF THE CONSTANT $A_{f(m)}$ , $A_{i(m)}$ , $A_s$ , $x_{f(m)}$ , AND $x_{i(m)}$

Part I:  $N = \text{even}$

$$x_f(N + 1) = -\frac{d_f}{2} - \frac{Nd_f}{2} - \frac{Nd_i}{2} + \frac{1}{k_0Q} \tan^{-1} \left( \frac{\mu_f q}{\mu_s Q} \right) \quad (\text{A1})$$

$$x_i(N - p) = -\frac{d_f}{2} - \left[ \frac{N}{2} - (p + 1) \right] d_f - \left( \frac{N}{2} - p \right) d_i - \frac{1}{k_0q} \tanh^{-1} \left\{ \frac{\mu_s Q}{\mu_f q} \tan \left\{ k_0 Q \left[ \frac{d_f}{2} + \left[ \frac{N}{2} - (p + 1) \right] d_f + \left( \frac{N}{2} - p \right) d_i + x_f(N - p + 1) \right] \right\} \right\} \quad (\text{A2})$$

$$x_f(N - p) = -\frac{d_f}{2} - \left[ \frac{N}{2} - (p + 1) \right] d_f - \left[ \frac{N}{2} - (p + 1) \right] d_i + \frac{1}{k_0Q} \tan^{-1} \left\{ \frac{\mu_f q}{\mu_i Q} \tanh \left\{ k_0 q \left[ \frac{d_f}{2} + \left[ \frac{N}{2} - (p + 1) \right] d_f + \left[ \frac{N}{2} - (p + 1) \right] d_i + x_i(N - p) \right] \right\} \right\} \quad (\text{A3})$$

for  $0 \leq p \leq \frac{N}{2} - 1$

$$x_i(N - p) = -\frac{d_f}{2} - \left[ \frac{N}{2} - (p + 1) \right] d_f - \left( \frac{N}{2} - p \right) d_i + \frac{1}{k_0q} \tanh^{-1} \left\{ \frac{\mu_s Q}{\mu_f q} \tan \left\{ k_0 Q \left[ -\frac{d_f}{2} - \left[ \frac{N}{2} - (p + 1) \right] d_f - \left( \frac{N}{2} - p \right) d_i - x_f(N - p + 1) \right] \right\} \right\} \quad (\text{A4})$$

$$x_f(N - p) = -\frac{d_f}{2} - \left[ \frac{N}{2} - (p + 1) \right] d_f - \left[ \frac{N}{2} - (p + 1) \right] d_i + \frac{1}{k_0Q} \tan^{-1} \left\{ \frac{\mu_f q}{\mu_i Q} \tanh \left\{ k_0 q \left[ \frac{d_f}{2} + \left[ \frac{N}{2} - (p + 1) \right] d_f + \left[ \frac{N}{2} - (p + 1) \right] d_i + x_i(N - p) \right] \right\} \right\} \quad (\text{A5})$$

for  $\frac{N}{2} \leq p \leq N - 1$

$$A_f(1) = A_c \frac{\exp \left[ -k_0 q \left( \frac{d_f}{2} + \frac{Nd_f}{2} + \frac{Nd_i}{2} \right) \right]}{\cos \left\{ k_0 Q \left[ \frac{d_f}{2} + \frac{Nd_f}{2} + \frac{Nd_i}{2} - x_f(1) \right] \right\}} \quad (\text{A6})$$

$$A_i(1) = A_f(1) \frac{\cos \left\{ k_0 Q \left[ \frac{d_f}{2} + \left( \frac{N}{2} - 1 \right) d_f + \frac{Nd_i}{2} - x_f(1) \right] \right\}}{\cosh \left\{ k_0 q \left[ \frac{d_f}{2} + \left( \frac{N}{2} - 1 \right) d_f + \frac{Nd_i}{2} - x_i(1) \right] \right\}} \quad (\text{A7})$$

$$A_f(N-p) = A_i(N-p-1) \frac{\cosh \left\{ k_0 q \left[ -\frac{d_f}{2} - \left( \frac{N}{2} - (p+2) \right) d_f - \left( \frac{N}{2} - (p+1) \right) d_i - x_i(N-p-1) \right] \right\}}{\cos \left\{ k_0 Q \left[ -\frac{d_f}{2} - \left( \frac{N}{2} - (p+2) \right) d_f - \left( \frac{N}{2} - (p+1) \right) d_i - x_f(N-p) \right] \right\}} \quad (\text{A8})$$

$$A_i(N-p) = A_f(N-p) \frac{\cos \left\{ k_0 Q \left[ -\frac{d_f}{2} - \left( \frac{N}{2} - (p+1) \right) d_f - \left( \frac{N}{2} - (p+1) \right) d_i - x_f(N-p) \right] \right\}}{\cosh \left\{ k_0 q \left[ -\frac{d_f}{2} - \left( \frac{N}{2} - (p+1) \right) d_f - \left( \frac{N}{2} - (p+1) \right) d_i - x_i(N-p) \right] \right\}} \quad (\text{A9})$$

for  $\frac{N}{2} \leq p \leq N - 2$

$$A_f \left( \frac{N}{2} + 1 \right) = A_i \left( \frac{N}{2} \right) \frac{\cosh \left\{ k_0 q \left[ \frac{d_f}{2} - x_i \left( \frac{N}{2} \right) \right] \right\}}{\cos \left\{ k_0 Q \left[ \frac{d_f}{2} - x_f \left( \frac{N}{2} + 1 \right) \right] \right\}} \quad (\text{A10})$$

$$A_i \left( \frac{N}{2} + 1 \right) = A_f \left( \frac{N}{2} + 1 \right) \frac{\cos \left\{ k_0 Q \left[ \frac{d_f}{2} + x_f \left( \frac{N}{2} + 1 \right) \right] \right\}}{\cosh \left\{ k_0 q \left[ \frac{d_f}{2} + x_i \left( \frac{N}{2} + 1 \right) \right] \right\}} \quad (\text{A11})$$

$$A_f(N-p) = A_i(N-p-1) \frac{\cosh \left\{ k_0 q \left[ \frac{d_f}{2} + \left( \frac{N}{2} - (p+2) \right) d_f + \left( \frac{N}{2} - (p+1) \right) d_i + x_i(N-p-1) \right] \right\}}{\cos \left\{ k_0 Q \left[ \frac{d_f}{2} + \left( \frac{N}{2} - (p+2) \right) d_f + \left( \frac{N}{2} - (p+1) \right) d_i + x_f(N-p) \right] \right\}} \quad (\text{A12})$$

$$A_i(N-p) = A_f(N-p) \frac{\cos \left\{ k_0 Q \left\{ \frac{d_f}{2} + \left[ \frac{N}{2} - (p+1) \right] d_f + \left[ \frac{N}{2} - (p+1) \right] d_i + x_f(N-p) \right\} \right\}}{\cosh \left\{ k_0 q \left\{ \frac{d_f}{2} + \left[ \frac{N}{2} - (p+1) \right] d_f + \left[ \frac{N}{2} - (p+1) \right] d_i + x_i(N-p) \right\} \right\}} \quad (\text{A13})$$

for  $0 \leq p \leq \frac{N}{2} - 2$ ,

$$A_f(N+1) = A_i(N) \frac{\cosh\left\{k_0q \left[\frac{d_f}{2} + \left(\frac{N}{2} - 1\right)d_f + \frac{N}{2}d_i + x_i(N)\right]\right\}}{\cos\left\{k_0Q \left[\frac{d_f}{2} + \left(\frac{N}{2} - 1\right)d_f + \frac{N}{2}d_i + x_f(N+1)\right]\right\}} \quad (\text{A14})$$

$$A_s = A_f(N+1) \frac{\cos\left\{k_0Q \left[\frac{d_f}{2} + \frac{N}{2}d_f + \frac{N}{2}d_i + x_f(N+1)\right]\right\}}{\exp\left[-k_0q \left(\frac{d_f}{2} + \frac{N}{2}d_f + \frac{N}{2}d\right)\right]} \quad (\text{A15})$$

Part II :  $N = \text{odd}$

$$x_f(N+1) = -\frac{d_i}{2} - \left(\frac{N}{2} - \frac{1}{2}\right)d_i - \left(\frac{N}{2} + \frac{1}{2}\right)d_f + \frac{1}{k_0Q} \tan^{-1}\left(\frac{\mu_fq}{\mu_iQ}\right) \quad (\text{A16})$$

$$x_i(N-p) = -\frac{d_i}{2} - \left(\frac{N}{2} - p - \frac{1}{2}\right)d_i - \left(\frac{N}{2} - p - \frac{1}{2}\right)d_f - \frac{1}{k_0q} \tanh^{-1}\left\{\frac{\mu_sQ}{\mu_fq} \tan\left\{k_0Q \left[\frac{1}{2}d_i + \left(\frac{N}{2} - p - \frac{1}{2}\right)d_i + \left(\frac{N}{2} - p - \frac{1}{2}\right)d_f + x_f(N-p+1)\right]\right\}\right\} \quad (\text{A17})$$

$$x_s(N-p) = -\frac{d_i}{2} - \left(\frac{N}{2} - p - \frac{3}{2}\right)d_i - \left(\frac{N}{2} - p - \frac{1}{2}\right)d_f - \frac{1}{k_0Q} \tan^{-1}\left\{\frac{\mu_fq}{\mu_iQ} \tanh\left\{k_0q \left[\frac{d_i}{2} + \left(\frac{N}{2} - p - \frac{3}{2}\right)d_i + \left(\frac{N}{2} - p - \frac{1}{2}\right)d_f + x_i(N-p)\right]\right\}\right\} \quad (\text{A18})$$

for  $0 \leq p \leq \frac{N-3}{2}$ ,

$$x_i\left(\frac{N}{2} + \frac{1}{2}\right) = -\frac{d_i}{2} - \frac{1}{k_0q} \tanh^{-1}\left\{\frac{\mu_sQ}{\mu_fq} \tan\left[k_0Q \left(\frac{d_i}{2} + x_f\left(\frac{N}{2} + \frac{3}{2}\right)\right)\right]\right\} \quad (\text{A19})$$

$$x_f\left(\frac{N}{2} + \frac{1}{2}\right) = \frac{d_i}{2} - \frac{1}{k_0Q} \tan^{-1}\left\{\frac{\mu_fq}{\mu_iQ} \tanh\left[k_0Q \left(\frac{d_i}{2} - x_i\left(\frac{N}{2} + \frac{1}{2}\right)\right)\right]\right\} \quad (\text{A20})$$

$$x_i(N-p) = -\frac{d_i}{2} - \left(\frac{N}{2} - p - \frac{1}{2}\right)d_i - \left(\frac{N}{2} - p - \frac{1}{2}\right)d_f + \frac{1}{k_0q} \tanh^{-1}\left\{\frac{\mu_sQ}{\mu_fq} \tan\left\{k_0Q \left[-\frac{d_i}{2} - \left(\frac{N}{2} - p - \frac{1}{2}\right)d_i - \left(\frac{N}{2} - p - \frac{1}{2}\right)d_f - x_f(N-p+1)\right]\right\}\right\} \quad (\text{A21})$$

$$\begin{aligned}
x_f(N-p) &= -\frac{d_i}{2} - \left(\frac{N}{2} - p - \frac{3}{2}\right) d_i - \left(\frac{N}{2} - p - \frac{1}{2}\right) d_f + \frac{1}{k_0 Q} \\
&\quad \tan^{-1} \left\{ \frac{\mu_f q}{\mu_i Q} \tanh \left\{ k_0 q \left[ -\frac{d_i}{2} - \left(\frac{N}{2} - p - \frac{3}{2}\right) d_i \right. \right. \right. \\
&\quad \left. \left. \left. - \left(\frac{N}{2} - p - \frac{1}{2}\right) d_f - x_i(N-p) \right] \right\} \right\} \quad (A22)
\end{aligned}$$

for  $\frac{N+1}{2} \leq p \leq N-1$ ,

$$A_f(1) = A_c \frac{\exp \left\{ -k_0 q \left[ \frac{d_i}{2} + \left(\frac{N}{2} - \frac{1}{2}\right) d_i + \left(\frac{N}{2} + \frac{1}{2}\right) d_f \right] \right\}}{\cos \left\{ k_0 Q \left[ \frac{d_i}{2} + \left(\frac{N}{2} - \frac{1}{2}\right) d_i + \left(\frac{N}{2} + \frac{1}{2}\right) d_f - x_f(1) \right] \right\}} \quad (A23)$$

$$A_i(1) = A_f(1) \frac{\cos \left\{ k_0 Q \left[ \frac{d_i}{2} + \left(\frac{N}{2} - \frac{1}{2}\right) d_i + \left(\frac{N}{2} - \frac{1}{2}\right) d_f - x_f(1) \right] \right\}}{\cosh \left\{ k_0 q \left[ \frac{d_i}{2} + \left(\frac{N}{2} - \frac{1}{2}\right) d_i + \left(\frac{N}{2} - \frac{1}{2}\right) d_f - x_i(1) \right] \right\}} \quad (A24)$$

$$\begin{aligned}
A_f(N-p) &= A_i(N-p-1) \frac{\cosh \left\{ k_0 q \left[ -\frac{d_i}{2} - \left(\frac{N}{2} - p - \frac{3}{2}\right) d_i \right. \right. \\
&\quad \left. \left. - \left(\frac{N}{2} - p - \frac{3}{2}\right) d_f - x_i(N-p-1) \right] \right\}}{\cos \left\{ k_0 Q \left[ -\frac{d_i}{2} - \left(\frac{N}{2} - p - \frac{3}{2}\right) d_i \right. \right. \\
&\quad \left. \left. - \left(\frac{N}{2} - p - \frac{3}{2}\right) d_f - x_f(N-p) \right] \right\}} \quad (A25)
\end{aligned}$$

$$\begin{aligned}
A_i(N-p) &= A_f(N-p) \frac{\cos \left\{ k_0 Q \left[ -\frac{d_i}{2} - \left(\frac{N}{2} - p - \frac{3}{2}\right) d_i \right. \right. \\
&\quad \left. \left. - \left(\frac{N}{2} - p - \frac{1}{2}\right) d_f - x_f(N-p) \right] \right\}}{\cosh \left\{ k_0 q \left[ -\frac{d_i}{2} - \left(\frac{N}{2} - p - \frac{3}{2}\right) d_i \right. \right. \\
&\quad \left. \left. - \left(\frac{N}{2} - p - \frac{1}{2}\right) d_f - x_i(N-p) \right] \right\}} \quad (A26)
\end{aligned}$$

for  $\frac{N-1}{2} \leq p \leq N-2$ ,

$$\begin{aligned}
A_f(N-p) &= A_i(N-p-1) \frac{\cosh \left\{ k_0 q \left[ \frac{d_i}{2} + \left(\frac{N}{2} - p - \frac{3}{2}\right) d_i \right. \right. \\
&\quad \left. \left. + \left(\frac{N}{2} - p - \frac{3}{2}\right) d_f + x_i(N-p-1) \right] \right\}}{\cos \left\{ k_0 Q \left[ \frac{d_i}{2} + \left(\frac{N}{2} - p - \frac{3}{2}\right) d_i \right. \right. \\
&\quad \left. \left. + \left(\frac{N}{2} - p - \frac{3}{2}\right) d_f + x_f(N-p) \right] \right\}} \quad (A27)
\end{aligned}$$

$$\begin{aligned}
A_i(N-p) &= A_f(N-p) \frac{\cos \left\{ k_0 Q \left[ \frac{d_i}{2} + \left(\frac{N}{2} - p - \frac{3}{2}\right) d_i \right. \right. \\
&\quad \left. \left. + \left(\frac{N}{2} - p - \frac{1}{2}\right) d_f + x_f(N-p) \right] \right\}}{\cosh \left\{ k_0 q \left[ \frac{d_i}{2} + \left(\frac{N}{2} - p - \frac{3}{2}\right) d_i \right. \right. \\
&\quad \left. \left. + \left(\frac{N}{2} - p - \frac{1}{2}\right) d_f + x_i(N-p) \right] \right\}} \quad (A28)
\end{aligned}$$

for  $0 \leq p \leq \frac{N-3}{2}$ ,

$$A_f(N+1) = A_i(N) \frac{\cosh\left\{k_0q \left[\frac{d_i}{2} + \left(\frac{N}{2} - \frac{1}{2}\right)d_i + \left(\frac{N}{2} - \frac{1}{2}\right)d_f + x_i(N)\right]\right\}}{\cos\left\{k_0Q \left[\frac{d_i}{2} + \left(\frac{N}{2} - \frac{1}{2}\right)d_i + \left(\frac{N}{2} - \frac{1}{2}\right)d_f + x_f(N+1)\right]\right\}} \quad (\text{A29})$$

$$A_s = A_f(N+1) \frac{\cos\left\{k_0Q \left[\frac{d_i}{2} + \left(\frac{N}{2} - \frac{1}{2}\right)d_i + \left(\frac{N}{2} + \frac{1}{2}\right)d_f + x_f(N+1)\right]\right\}}{\exp\left\{-k_0q \left[\left(\frac{d_i}{2} + \left(\frac{N}{2} - \frac{1}{2}\right)d_i + \left(\frac{N}{2} + \frac{1}{2}\right)d_f\right)\right]\right\}} \quad (\text{A30})$$

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