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PIECEWISE SURFACE IMPEDANCE BOUNDARY CONDITIONS BY COMBINING RYTOV'S PERTURBATION METHOD AND LEVEL SET TECHNIQUE

A. Bouzidi and T. Aguili

Syscom Laboratory, Engineer National School
B.P 37 Le Belvedere, Tunis 1002, Tunisia

Abstract—In this paper, we propose a computational method for constructing variable surface impedance, based on combining Rytov's perturbation method and level set technique. It is well-known that the choice of the most appropriate order of Rytov's expansion is important both for accuracy and implementation. By using level set method, we constructed a piecewise distribution of low- and high-order surface impedance boundary conditions on the surface of an arbitrarily shaped conductor. It is found that the proposed method is able to give good results both in terms of accuracy and implementation cost.

1. INTRODUCTION

The surface impedance was introduced to model conductors with a sufficiently strong skin effect. The main idea behind this concept is replacing the conducting volume by an approximate boundary condition applied to the interface conductor/dielectric. Therefore, the field distribution in the conductor can be omitted. We focus only on exterior field. Modeling conductors by SIBC have been shown to be of great importance because of their various applications in many areas such as electromagnetic scattering, geophysical problems, RCS computation and other sciences. The use of SIBC reduces the expected computational cost by eliminating conducting volume from the numerical implementation. Judicious choice of SIBC's order provides good compromise between accuracy and implementation cost, but for complicated geometry conductors it is difficult to have such an opportunity by using only one SIBC's order. Indeed, application of

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Corresponding author: Afif Bouzidi (afif.bouzidi@gmail.com).

high order SIBC on entire conductor's surface makes numerical solution very expensive, without necessarily providing significant improvement in accuracy. On the other hand, the improper use of low SIBC's orders degrades numerical accuracy. So, it will be interesting to apply a variable SIBC, where the choice of the approximation order depends on local geometric properties of the conductor's surface. By using level set technique, we tried to find where each approximation order should be applied on the conductor's surface to get good compromise between accuracy and implementation cost. The level set method is recognized as a numerical technique for tracking interfaces and shapes, in this paper we used level set representation of interfaces between domains with different approximation SIBC's order. So to identify the best suited region for each order we just need to identify the appropriate level set function.

2. RYTOV'S APPROACH

By using perturbation method, Rytov developed general method to construct low and high order SIBCs. He demonstrated that SIBC is equal to the infinite sum of asymptotic expansion in p [1], where p is the ratio between skin depth and characteristic dimension of the conductor's surface.

$$\tilde{E}_{vk} = (-1)^k \sum_{l=0}^{\infty} p^{l+1} Z_l(\tilde{H}_{v3-k}) \tag{1}$$

"~" denotes a non-dimensional values.

$$p = \frac{\delta}{R} \tag{2}$$

$$\delta = \sqrt{\frac{2}{\mu\omega\sigma}} \tag{3}$$

$$R = \min(R_s, R_o) \tag{4}$$

where μ, ω, σ, R_s, R_o and v_k are respectively the magnetic permeability, angular frequency, the electrical conductivity, the minimum radius of curvature, the minimum distance between the field source and the conductor, the principal curvature coordinates.

The condition of applicability of Rytov's expansion can be summarized as follows [2]:

$$p \ll 1 \tag{5}$$

The zero order expansion represents SIBC for perfect electric conductors (PEC), where there is no electromagnetic field penetration.

$$\tilde{E}_{vk} = 0 + O(p) \tag{6}$$

The first expansion order represents Leontovich SIBC, where the electromagnetic field variation parallel to the surface is assumed to be small compared to the variation perpendicular to the surface [1].

$$\tilde{E}_{vk} = (-1)^{3-k} p \frac{1+j}{\theta} \tilde{H}_{v3-k} + O(p^2) \tag{7}$$

$$\theta = \sqrt{1 + j\frac{\varepsilon\omega}{\sigma}} \tag{8}$$

where ε is the dielectrical permittivity of the conductor.

The second expansion order is Mitzner SIBC, take into consideration the radii of curvature [1].

$$\tilde{E}_{vk} = (-1)^{3-k} p \frac{1+j}{\theta} \left[\tilde{H}_{v3-k} + p \frac{1-j}{4\theta} \left(\tilde{d}_{3-k}^{-1} - \tilde{d}_k^{-1} \right) \tilde{H}_{\xi 3-k} \right] + O(p^3) \tag{9}$$

where $\tilde{d}_k$, $k = 1, 2$ are the local radii of curvature.

The third expansion order is Rytov SIBC, take into consideration, the variations of electromagnetic fields on the conductor's surface [1]

$$\begin{aligned} \tilde{E}_{vk} = & (-1)^{3-k} p \frac{1+j}{\theta} \Bigg[\tilde{H}_{v3-k} + p \frac{1-j}{4\theta} \left(\tilde{d}_{3-k}^{-1} - \tilde{d}_k^{-1} \right) \tilde{H}_{\xi 3-k} \\ & + \frac{p^2}{2j\theta} \left(\frac{3\tilde{d}_k^2 - \tilde{d}_{3-k}^2 - 2\tilde{d}_k \tilde{d}_{3-k}}{8\tilde{d}_k^2 \tilde{d}_{3-k}^2} \right) \tilde{H}_{v3-k} \\ & + \frac{p^2}{2j\theta} \left(-\frac{\partial^2 \tilde{H}_{v3-k}}{\partial v_k^2} + \frac{\partial^2 \tilde{H}_{v3-k}}{\partial v_{3-k}^2} + 2\frac{\partial^2 \tilde{H}_{vk}}{\partial v_k \partial v_{3-k}} \right) \Bigg] + O(p^4) \end{aligned} \tag{10}$$

The approximation errors of PEC, Leontovich, Mitzner, and Rytov SIBC are respectively p, p^2, p^3 and p^4.

3. LEVEL SET REPRESENTATION

Let consider Ω an open subset of $\mathbb{R}^3$ represents a conductor object and let Γ be a closed surface in Ω. We define ψ as a signed distance function by [3]:

$$\psi = \begin{cases} \text{distance}\,(r, \Gamma) & \text{if} \quad r \in \Omega \\ -\text{distance}\,(r, \Gamma) & \text{if} \quad r \notin \Omega \end{cases} \tag{11}$$

It is clear that Γ is the zero level set of the function ψ, Γ divides the domain Ω into two parts, then the level set function ψ is positive inside Ω and negative outside.

Once the level set function ψ is defined, we can use it to calculate the local geometric properties of the conductor's surface.

The normal vector is given by [4]:

$$\vec{n} = \frac{\nabla\psi}{\|\nabla\psi\|} \tag{12}$$

The mean curvature [4]:

$$\kappa = -\nabla\cdot\vec{n} = -\nabla\cdot\frac{\nabla\psi}{\|\nabla\psi\|} \tag{13}$$

The Gaussian curvature:

$$G = \vec{n}\cdot\mathrm{Adj}(\mathrm{He}(\psi))\vec{n} \tag{14}$$

where $\mathrm{He}(\psi)$ is the 3×3 Hessian matrix of the function ψ, $\mathrm{Adj}(\mathrm{He}(\psi))$ is the adjoint of the Hessian $\mathrm{He}(\psi)$.

Principal curvatures κ_1 and κ_2 can be expressed in terms of gaussian and mean curvature as follows:

$$\kappa_1 = \frac{\kappa - \sqrt{\kappa^2 - G}}{2} \tag{15}$$

$$\kappa_2 = \frac{\kappa + \sqrt{\kappa^2 - G}}{2} \tag{16}$$

We define the minimal radius of curvature at each point on the conductor's surface by:

$$R_\psi = \frac{1}{\max(|\kappa_1|,|\kappa_2|)} \tag{17}$$

$$R_\psi = \frac{2}{|\kappa| + \sqrt{\kappa^2 - G}} \tag{18}$$

where $|\kappa|$ is the absolute value of the mean curvature κ.

Any part of the object's surface Γ can be represented also by a level set function.

Let ζ be a closed curve defined on Γ and represents the boundary of a region Γ_Z in the conductor's surface Γ, we define φ as [5] :

$$\zeta = \varphi \cap \psi \tag{19}$$

ζ is the intersection of the zero level set function φ and the zero level set function ψ.

4. METHODOLOGY

For simplicity, let Z_0, Z_1, Z_2 and Z_3 denote respectively the PEC, Leontovich, Mitzner and Rytov SIBC.

By using many SIBC to model an arbitrarily shaped conductor, the total SIBC Z, is shown as a piecewise function, for example, assuming that Z equals Z_0 inside Γ_Z and equal Z_1 outside Γ_Z, it is easy to see that Z can be represented as:

$$Z = Z_0 H(\varphi) + Z_1(1 - H(\varphi)) \tag{20}$$

where H(φ) the Heaviside function, defined by:

$$H(\varphi) = \begin{cases} 1 & \text{if} \quad \varphi > 0 \\ 0 & \text{if} \quad \varphi < 0 \end{cases} \tag{21}$$

From (20), we deduce the total approximation error:

$$E = pH(\varphi) + p^2(1 - H(\varphi)) \tag{22}$$

$$p = \frac{\delta}{R} \geq \frac{\delta}{R_\psi} \tag{23}$$

It is clear that, if, $p \ll 1$ then $p_\psi = \frac{\delta}{R_\psi} \ll 1$.

So, we define E_ψ as:

$$E_\psi = p_\psi H(\varphi) + p_\psi^2(1 - H(\varphi)) \tag{24}$$

Assume now, we have two closed curves ζ_1 and ζ_2 on Γ, and we associate the two level set functions φ_1 and φ_2 with these curves. Then the conductor's surface Γ is divided into four parts:

$$\begin{aligned} \Gamma_1 &= \{r \in \Gamma,\ \varphi_1 > 0,\ \varphi_2 > 0\} \\ \Gamma_2 &= \{r \in \Gamma,\ \varphi_1 > 0,\ \varphi_2 < 0\} \\ \Gamma_3 &= \{r \in \Gamma,\ \varphi_1 < 0,\ \varphi_2 > 0\} \\ \Gamma_4 &= \{r \in \Gamma,\ \varphi_1 < 0,\ \varphi_2 < 0\} \end{aligned} \tag{25}$$

Using the Heaviside function again, we can express Z as:

$$\begin{aligned} Z \;=\; & Z_0 H(\varphi_1)H(\varphi_2) + Z_1 H(\varphi_1)(1 - H(\varphi_2)) \\ & + Z_2(1 - H(\varphi_1))H(\varphi_2) + Z_3(1 - H(\varphi_1))(1 - H(\varphi_2)) \end{aligned} \tag{26}$$

The associated approximation error E_ψ is:

$$\begin{aligned} E_\psi \;=\; & p_\psi H(\varphi_1)H(\varphi_2) + p_\psi^2 H(\varphi_1)(1 - H(\varphi_2)) \\ & + p_\psi^3(1 - H(\varphi_1))H(\varphi_2) + p_\psi^4(1 - H(\varphi_1))(1 - H(\varphi_2)) \end{aligned} \tag{27}$$

By generalizing, we see that n level set functions split the surface Γ in 2^n regions [6].

Let $bin(i-1) = (b_1^i, b_2^i, \ldots, b_m^i)$ the binary representation of $i - 1$

$$b_j^i \in \{0, 1\} \tag{28}$$

The total piecewise surface impedance, composed by Z_i, $i = 0, 1, 2, \ldots, 2_n - 1$ could be represented as:

$$Z = \sum_{i=1}^{2^n} Z_{i-1} \prod_{j=1}^{n} R_i(\varphi_j) \tag{29}$$

where

$$R_i(\varphi_j) = \begin{cases} H(\varphi_j) & \text{if} \quad b_j^i = 0 \\ 1 - H(\varphi_j) & \text{if} \quad b_j^i = 1 \end{cases} \tag{30}$$

The associated approximation error E_ψ is:

$$E_\psi = \sum_{i=1}^{2^n} p_\psi^i \prod_{j=1}^{n} R_i(\varphi_j) \tag{31}$$

Assume that we want to use m surface impedance $Z_{0 \leq i \leq m-1}$ for modeling a conductor object such that E_ψ don't exceed a fixed threshold S. We can use n level set functions to split the conductor's surface into m region, with n defined by:

$$2^{n-1} < m \leq 2^n \tag{32}$$

It is easy to reformulate this problem to an optimization one, as follow:

$$\varphi^*_{1 \leq i \leq n} = \underset{\varphi_{1 \leq i \leq n}}{\arg\min} J(\varphi_1, \varphi_2, \ldots, \varphi_n) \tag{33}$$

Find level set functions $\varphi^*_{1 \leq i \leq n}$, which minimize the cost functional J.

$$J(\varphi_1, \varphi_2, \ldots, \varphi_n) = (E_\psi - S)^2 = \left(\sum_{i=1}^{2^n} p_\psi^i \prod_{j=1}^{n} R_i(\varphi_j) - S \right)^2 \tag{34}$$

We will use the gradient type method to find $\varphi^*_{1 \leq i \leq n}$, we just need to compute the $\frac{\partial J}{\partial \varphi_i}$ for $1 \leq i \leq n$.

$$\frac{\partial J}{\partial \varphi_i} = 2\left(E_\psi - S\right) \frac{\partial E_\psi}{\partial \varphi_i} \tag{35}$$

where

$$\frac{\partial E_\psi}{\partial \varphi_i} = \sum_{i=1}^{2^n} p_\psi^i \left(\prod_{j=1, j \neq i}^{n} R_i(\varphi_j)) \right) D(\varphi_i) \tag{36}$$

and

$$D(\varphi_i) = \begin{cases} \delta(\varphi_i) & \text{if} \quad b_j^i = 0 \\ -\delta(\varphi_i) & \text{if} \quad b_j^i = 1 \end{cases} \tag{37}$$

$\delta(\varphi)$ denotes the Dirac function:

$$\delta(\varphi) = \begin{cases} 1 & \text{if} \quad \varphi = 0 \\ 0 & \text{if} \quad \varphi \neq 0 \end{cases} \tag{38}$$

So,

$$\frac{\partial J}{\partial \varphi_i} = 2\left(\sum_{i=1}^{2^n} p_\psi^i \prod_{j=1}^{n} R_i(\varphi_j) - S\right) \cdot \left(\sum_{i=1}^{2^n} p_\psi^i \left(\prod_{j=1, j\neq i}^{n} R_i(\varphi_j))\right) D(\varphi_i)\right) \quad (39)$$

We used the following gradient algorithm to construct a piecewise surface impedance for a given conductor shape:

1- Compute the minimum radius of curvature R.

2- For a given frequency f and conductivity σ, verify if $p \ll 1$.

3- Choose a threshold S.

4- Determine how many level set functions we need to use.

5- Choose initial level set functions $\varphi^0_{1\leq i\leq n}$.

6- for $k \geq 1$:

- Choose the step size α_i and update the level set functions as:

$$\varphi_i^k = \varphi_i^{k-1} - \alpha^i \frac{\partial J(\varphi_1^{k-1}, \varphi_2^{k-1}, \ldots, \varphi_n^{k-1})}{\partial \varphi_i} \quad (40)$$

- Go to the next iteration if not converged.

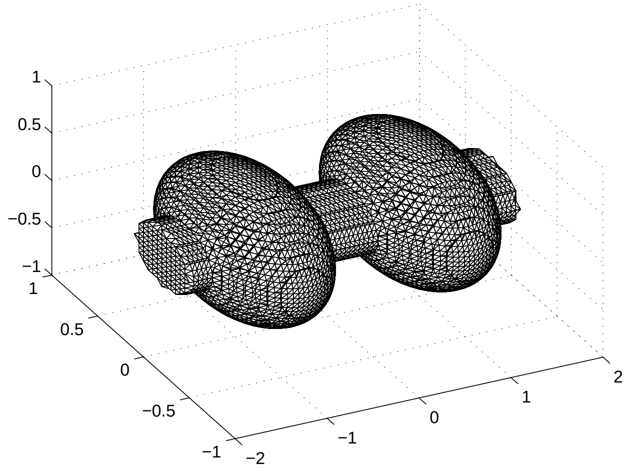

Figure 1. Conductor object.

5. NUMERICAL EXAMPLE

In order to show the feasibility of this method, we present an example of construction of piecewise surface impedance by the proposed scheme.We consider an aluminum smooth dumbbell ($\sigma = 3.5714 \times 10^7\,\mathrm{S \cdot m^{-1}}$) as shown in Figure 1, illuminated by an incident plane wave at 100 kHz.

We take the computation domain D as:

$$D = [-2, 2] \times [-2, 2] \times [-2, 2] \tag{41}$$

D is divided into square elements with uniform mesh size ($h = h_x = h_y = h_z = 0.05$). The proposed method has been developed and implemented within MATLAB environment and the level set Toolbox of Ian Mitchell [7]. The Figure 2 show the construction of piecewise surface impedance by using the first four SIBC (PEC, Leontovich, Mitzner and Rytov) where the approximation error don't exceed $10^{-4}\%$.

From Figure 2, it follows that, to get a small approximation order about $10^{-4}\%$, there is no need to use high SIBC every where on the conductor's surface. High SIBC's orders are localized near smooth corners where the curvature radius tends to zero, however low SIBC's orders are localized near flat surfaces.

We can also observe from the results, that the proposed method leads to a SIBC discontinuity between adjacent regions, which produce spurious edge effects when calculating the scattered fields

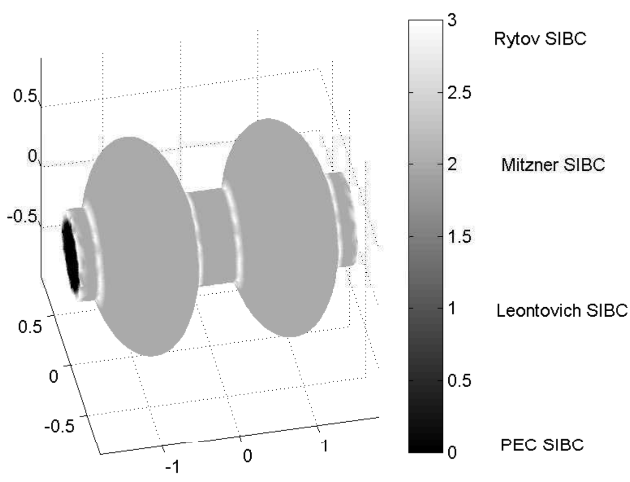

Figure 2. Piecewise surface impedance boundary conditions.

computationally. We can circumvent this drawback by introducing local buffer area between adjacent regions. Indeed, the interactions of sub-regions with buffer regions can suppress the singularities introduced by the abrupt termination of each sub-region and ensure stability and accuracy.

6. CONCLUSIONS

We have reported a numerical scheme combining Rytov's perturbation method and level set technique to construct a piecewise surface impedance for an arbitrarily shaped conductor. It is found that proposed method is able to automate the construction of variable SIBC with good compromise between accuracy and implementation cost. By using Level set technique, this method shows a great potential to be applied to electromagnetic structure design and optimisation.

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