HALBACH STRUCTURES FOR PERMANENT MAGNETS BEARINGS

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Abstract—This paper is the third part of a series dealing with permanent magnet passive magnetic bearings. It presents analytical expressions of the axial force and stiffness in radial passive magnetic bearings made of ring permanent magnets with perpendicular polarizations: the inner ring polarization is perpendicular to the outer ring one. The main goal of this paper is to present a simple analytical model which can be easily implemented in Matlab or Mathematica so as to carry out parametric studies. This paper first compares the axial force and stiffness in bearings with axial, radial and perpendicular polarizations. Then, bearings made of stacked ring magnets with alternate polarizations are studied for the three kinds of polarizations, axial, radial and perpendicular. The latter correspond to Halbach structures. These calculations are useful for identifying the structures required for having great axial forces and the ones allowing to get great axial stiffnesses.

1. INTRODUCTION

Radial permanent magnet magnetic bearings are made of ring magnets that can have various polarization directions [1]. Devices with magnets axially [2] or radially [3] magnetized have already been thoroughly studied. This paper presents structures with perpendicular polarizations (the inner ring polarization is perpendicular to the outer

ring one) and stacked structures with alternate polarization or Halbach pattern. Studying radial magnetic bearings requires the calculation of the force and stiffness exerted between the inner and outer permanent magnets. Authors generally use either numerical approaches or 2D analytical calculations for determining the magnetic fields produced by ring permanent magnets [4, 5]. Moreover, two models of the magnetic sources are available and used: the Amperian approach, which is often used for coils [6, 7], and the Coulombian approach, which is often used for permanent magnets. However, both approaches are valid for each kind of source but the authors have demonstrated that depending on the polarization direction of the source only one of them generally yields an analytical formulation [8–10]. Indeed, the calculations are simplified when the adapted model is chosen. Furthermore, recent works presented 3D analytical expressions of the magnetic field created by ring or tile permanent magnets which used elliptic integrals [11, 12] or special functions [13, 14] for permanent magnets axially or radially polarized. These formulations are more suitable for parametric studies and optimization. The 3D analytical expression of the force exerted between two cuboidal permanent magnets [15] allowed the dimensioning of magnetic couplings [16]. This paper deals with the calculation of the forces exerted between ring magnets for several kind of applications [17–25]. This paper first compares the axial force and stiffness in bearings with axial, radial and perpendicular polarizations. The structures considered have only one ring on both inner and outer parts. Then, bearings made of stacked ring magnets with alternate polarizations [26] are studied for the three kinds of polarizations, axial, radial and perpendicular. The latter are arranged so as to have Halbach patterns [27–29]. The studied stacks are constituted by three and then five ring magnets. The comparison of the performances shows the advantages of the Halbach structures.

2. FORCE EXERTED BETWEEN TWO RING PERMANENT MAGNETS WITH PERPENDICULAR POLARIZATIONS

The geometry considered is shown in Fig. 1. A cross-section view is shown in Fig. 2. The following parameters are used:
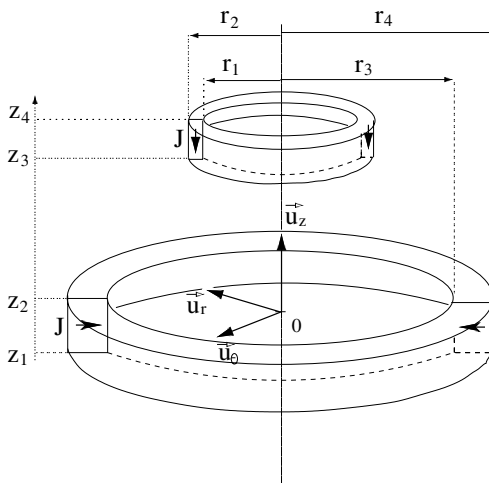
r_1, r_2: inner and outer radius of the inner ring permanent magnet [m].
r_3, r_4: inner and outer radius of the outer ring permanent magnet [m].
z_1, z_2: lower and upper axial abscissa of the inner ring [m].
z_3, z_4: inner and outer axial abscissa of the outer ring [m].
The two ring permanent magnets are assumed to be radially centered and their polarizations are perpendicular.

Figure 1. 3D representation of two ring permanent magnets with perpendicular polarizations.
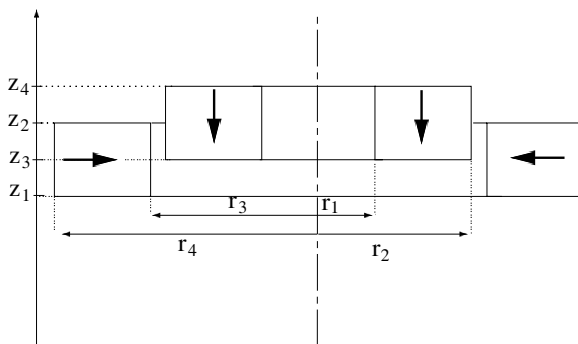
Figure 2. Cross-section view of two ring permanent magnets with perpendicular polarizations.

The axial force exerted between two ring permanent magnets with perpendicular polarizations can be determined by using the Coulombian model of a magnet. Consequently, each ring permanent magnet is represented by faces charged with fictitious magnetic pole surface densities. For the outer ring permanent magnet whose polarization is radial, the faces are cylindrical: the outer face is charged with the fictitious magnetic pole surface density $-\sigma^*$ and the inner one is charged with the fictitious magnetic pole surface density $+\sigma^*$.

For the inner ring permanent magnet whose polarization is axial,

the faces are plane: the upper face is charged with the fictitious magnetic pole surface density $-\sigma^*$ and the lower one is charged with the fictitious magnetic pole surface density $+\sigma^*$. It is noted that all the illustrative calculations have been carried out with $\sigma^* = \vec{J} \cdot \vec{n} = 1\,\text{T}$, where $\vec{J}$ is the magnetic polarization vector and $\vec{n}$ is the unit normal vector. Moreover, the magnetic pole volume density exists for ring permanent magnets whose polarizations are radial in order to have a charge balance in the ring magnet. However, this contribution can be neglected for simplifying the calculations. It is emphasized here that a simple analytical model can be easily implemented in Mathematica or Matlab.

The calculation of the axial force exerted by the outer ring permanent magnet on the inner one requires the exact calculation of the axial field produced by the outer ring permanent magnet. By using the Coulombian model of a magnet, this axial field can be expressed as follows:

$$H_z(r, z) = \frac{J}{4\pi\mu_0} \iint_S \frac{(z - \tilde{z})}{R(r_3, \tilde{\theta}, \tilde{z})} r_3 d\tilde{\theta} d\tilde{z} - \frac{J}{4\pi\mu_0} \iint_S \frac{(z - \tilde{z})}{R(r_4, \tilde{\theta}, \tilde{z})} r_4 d\tilde{\theta} d\tilde{z} \quad (1)$$

with

$$R(r_i, \tilde{\theta}, \tilde{z}) = \left(r^2 + r_i^2 - 2rr_i \cos(\tilde{\theta}) + (z - \tilde{z})^2 \right)^{\frac{3}{2}} \quad (2)$$

Then, the axial force can be determined by using the following equation:

$$F_z = \frac{J^2}{4\pi\mu_0} \int_{r_1}^{r_2} \int_0^{2\pi} H_z(r, z_3) r dr d\theta - \frac{J^2}{4\pi\mu_0} \int_{r_1}^{r_2} \int_0^{2\pi} H_z(r, z_4) r dr d\theta \quad (3)$$

The previous expression can be reduced in the following form:

$$F_z = \frac{J^2}{4\pi\mu_0} \sum_{i,k=1}^{2} \sum_{j,l=3}^{4} (-1)^{i+j+k+l} \left(A_{i,j,k,l} \right)$$

$$+ \frac{J^2}{4\pi\mu_0} \sum_{i,k=1}^{2} \sum_{j,l=3}^{4} (-1)^{i+j+k+l} \left(S_{i,j,k,l} \right) \quad (4)$$

with

$$A_{i,j,k,l} = -8\pi r_i \epsilon \mathbf{E} \left[-\frac{4r_i r_j}{\epsilon} \right]$$

$$S_{i,j,k,l} = -2\pi r_j^2 \int_0^{2\pi} \cos(\theta) \ln \left[\beta + \alpha \right] d\theta \quad (5)$$

where $\mathbf{E}[m]$ gives the complete elliptic integral which is expressed as follows:

$$\mathbf{E}\left[m\right] = \int_0^{\frac{\pi}{2}} \sqrt{1 - m\sin(\theta)^2} d\theta \tag{6}$$

The parameters ϵ, α and β depend on the ring permanent magnet dimensions and are defined as follows:

$$\begin{aligned}
\epsilon &= (r_i - r_j)^2 + (z_k - z_l)^2 \\
\alpha &= \sqrt{r_i^2 + r_j^2 - 2r_i r_j \cos(\theta) + (z_k - z_l)^2} \\
\beta &= r_i - r_j \cos(\theta)
\end{aligned} \tag{7}$$

3. STIFFNESS EXERTED BETWEEN TWO RING PERMANENT MAGNETS WITH PERPENDICULAR POLARIZATIONS

The section presents an analytical model of the axial stiffness exerted between two ring permanent magnets with perpendicular polarizations. The axial stiffness derives from the axial force by using the following equation:

$$K_z = -\frac{d}{dz} F_z \tag{8}$$

where F_z is determined with $R(r_i, \tilde{\theta}, \tilde{z})$ and Eq. (4). After mathematical manipulations, the previous expression can be reduced in the following form:

$$K_z = \frac{J^2}{4\pi\mu_0} \sum_{i,k=1}^{2} \sum_{j,l=3}^{4} (-1)^{i+j+k+l} \left(k_{i,j,k,l}\right) \tag{9}$$

with

$$k_{i,j,k,l} = -\int_0^{2\pi} \frac{r_j(z_k - z_l)(\alpha + r_i)}{\alpha\left(\alpha + \beta\right)} d\theta$$

It is emphasized here that the axial stiffness expression has a low computational cost (less than 0.1 s for calculating the axial stiffness between two ring permanent magnets).

4. ACCURACY OF THE SIMPLIFIED ANALYTICAL MODELS

Our simplified model does not take into account the magnetic pole volume density. The main reason lies in the fact that we want to obtain a simple analytical model whose computational cost is low (about 0.1 s
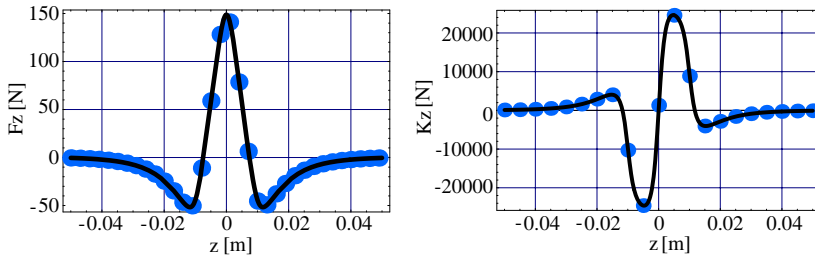
Figure 3. Axial force and stiffness versus axial displacement for two ring permanent magnets with perpendicular polarizations; $r_1 = 0.01$ m, $r_2 = 0.02$ m, $r_3 = 0.022$ m, $r_4 = 0.032$ m, $z_2 - z_1 = z_4 - z_3 = 0.01$ m, $J = 1$ T, (Line: this work, points: the magnetic pole volume density is taken into account).

for calculating the axial force between two ring permanent magnets). Having a low computational cost is suitable for carrying out parametric studies. We have represented in Fig. 3 the axial force and stiffness versus axial displacement with the following dimensions: $r_1 = 0.01$ m, $r_2 = 0.02$ m, $r_3 = 0.022$ m, $r_4 = 0.032$ m, $z_2 - z_1 = z_4 - z_3 = 0.01$ m, $J = 1$ T.

The Fig. 3 shows that our 3D analytical expression is accurate for the dimensions we use throughout this paper. Consequently, this implies that we can use the simplified expression so as to study configurations made of one or several stacked ring permanent magnets for the dimensions given in this paper. We think that this element of information is important for carrying out parametric optimizations in which the computational cost must be low.

## 5. HALBACH AND ALTERNATE PERMANENT MAGNET STRUCTURES

5.1. Elementary Structures

This section presents a comparison between three elementary configurations using one ring permanent magnet on each part of the passive magnetic bearing. The first configuration considered is composed of two ring permanent magnets with axial polarizations (Fig. 4). The axial force and stiffness, shown in Fig. 4, have been determined by using the analytical model presented in the Part 1 of this work [2].

The maximal axial force exerted by the outer ring on the inner one is 125 N and the maximal axial stiffness is $|K_z| = 39848$ N/m.
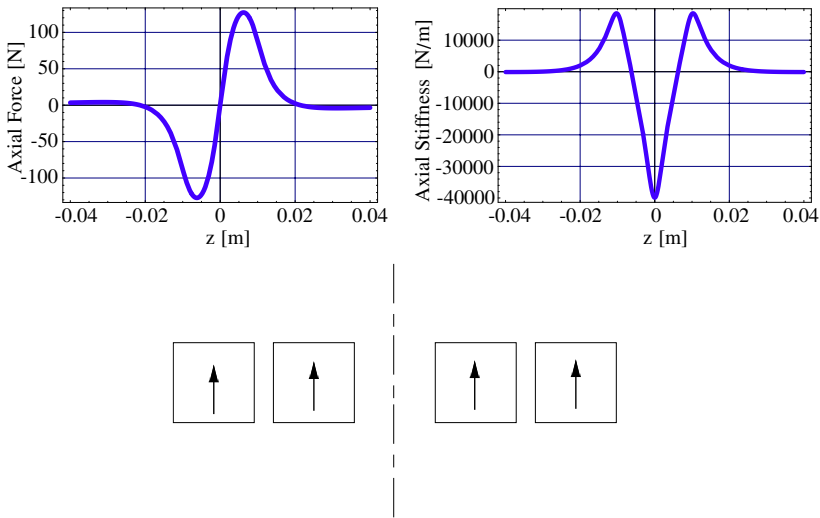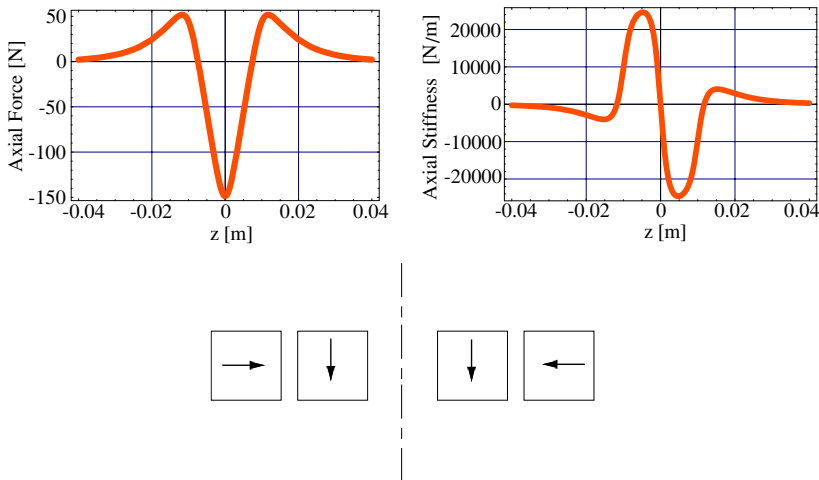
Figure 4. Axial force and axial stiffness versus axial displacement for two ring permanent magnets with axial polarizations; $r_1 = 0.01$ m, $r_2 = 0.02$ m, $r_3 = 0.022$ m, $r_4 = 0.032$ m, $z_2 - z_1 = z_4 - z_3 = 0.01$ m, $J = 1$ T.
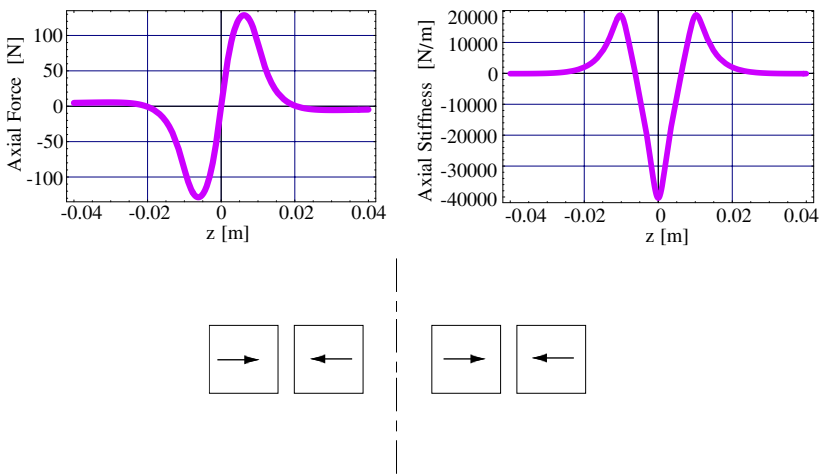
Figure 5. Axial force and axial stiffness versus axial displacement for two ring permanent magnets with radial polarizations; $r_1 = 0.01$ m, $r_2 = 0.02$ m, $r_3 = 0.022$ m, $r_4 = 0.032$ m, $z_2 - z_1 = z_4 - z_3 = 0.01$ m, $J = 1$ T.

The same calculations have also been carried out for a passive magnetic bearing made of two ring permanent magnets radially magnetized of same dimensions (Fig. 5). In this configuration, the maximal axial force exerted by the outer ring on the inner one is 126.3 N and the maximal axial stiffness is $|K_z| = 40282\,\text{N/m}$. It has to be noted that the behavior of both bearings is exactly the same. However, the values obtained with the radial polarizations are slightly higher than with the axial ones, the increase being approximately 5%. The third configuration considered is constituted of two ring permanent magnets with perpendicular polarizations. The analytical expressions of the axial force and stiffness exerted between these two ring permanent magnets have been determined in the previous sections. Fig. 6 presents the axial force and stiffness versus the axial displacement of the inner ring permanent magnet. These simulations show that the maximal axial force is 148 N and the maximal axial stiffness is $|K_z| = 24460\,\text{N/m}$.

So, the previous calculations show that the most interesting configuration for having the greatest axial force is the one using ring permanent magnets with perpendicular polarizations. The most interesting configuration for having the greatest axial stiffness is the one using ring permanent magnets with radial polarizations.

Figure 6. Axial force and axial stiffness versus axial displacement for two ring permanent magnets with perpendicular polarizations; $r_1 = 0.01\,\text{m}$, $r_2 = 0.02\,\text{m}$, $r_3 = 0.022\,\text{m}$, $r_4 = 0.032\,\text{m}$, $z_2 - z_1 = z_4 - z_3 = 0.01\,\text{m}$, $J = 1\,\text{T}$.
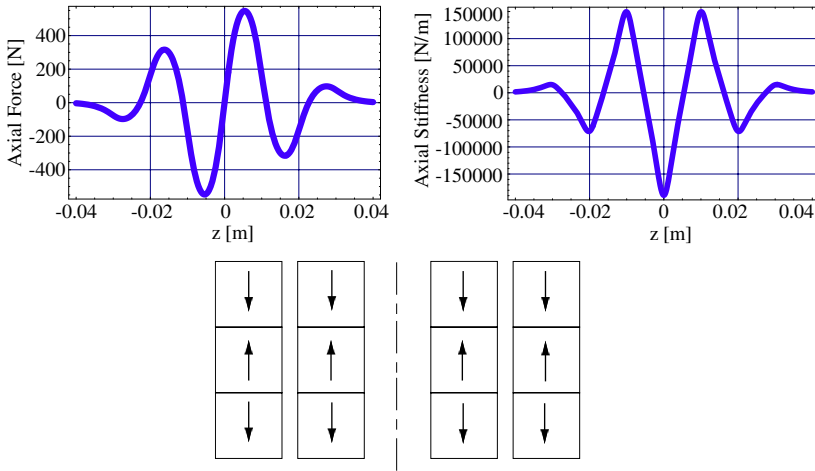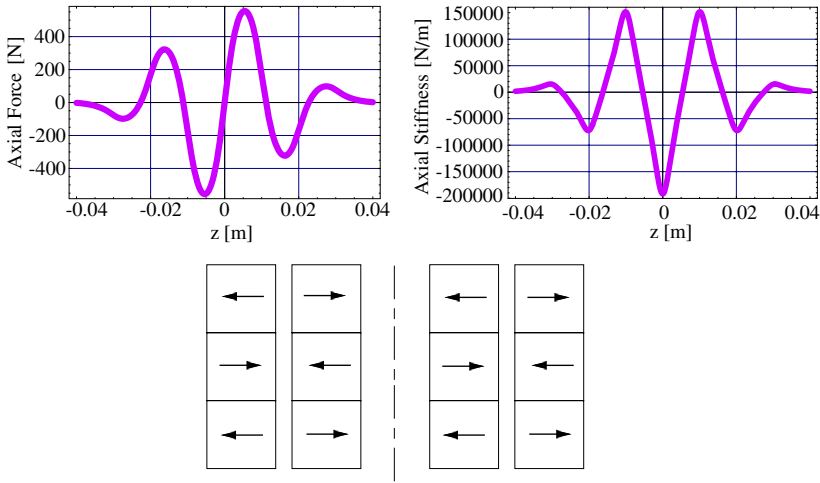
Figure 7. Cross-section view for a stack of three ring permanent magnets with alternate axial polarizations; $r_1 = 0.01$ m, $r_2 = 0.02$ m, $r_3 = 0.022$ m, $r_4 = 0.032$ m, $J = 1$ T, height of each ring permanent magnet $= 0.01$ m.

## 5.2. Stacked Structures with Three Ring Permanent Magnets

This section presents a comparison between alternate and Halbach permanent magnet structures. The first configuration considered is composed of three ring permanent magnets with alternate axial polarizations. Fig. 7 represents the axial force and stiffness versus the axial shift between the two parts of the bearing. The maximal axial force exerted by the outer stack on the inner one is 547 N and the maximal axial stiffness is $|K_z| = 189318$ N/m.

The previous calculations have also been carried out for the case of a passive magnetic bearing made of alternate permanent magnets radially magnetized (Fig. 8). In this configuration, the maximal axial force exerted by the outer stack on the inner one is 556 N and the maximal axial stiffness is $|K_z| = 191901$ N/m. Again, the device with radial polarizations has performances slightly higher, more exactly 4% higher.

The third configuration considered is made of a stack of ring permanent magnets with perpendicular polarizations (Fig. 9). Fig. 9 presents the axial force and stiffness versus the axial displacement of the inner stack. These calculations show that the maximal axial force is 733 N and the maximal axial stiffness is $|K_z| = 162428$ N/m. The

Figure 8. Cross-section view for a stack of three ring permanent magnets with alternate radial polarizations; $r_1 = 0.01$ m, $r_2 = 0.02$ m, $r_3 = 0.022$ m, $r_4 = 0.032$ m, $J = 1$ T, height of each ring permanent magnet $= 0.01$ m.
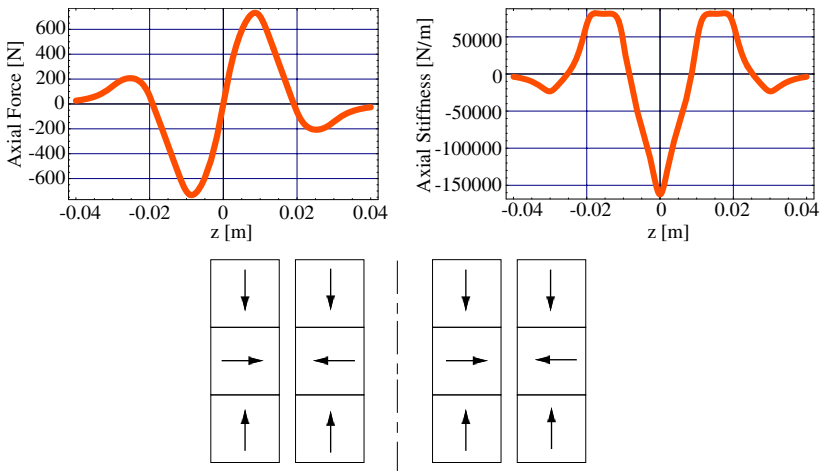
Figure 9. Cross-section view for a stack of three ring permanent magnets with perpendicular polarizations; $r_1 = 0.01$ m, $r_2 = 0.02$ m, $r_3 = 0.022$ m, $r_4 = 0.032$ m, $J = 1$ T, height of each ring permanent magnet $= 0.01$ m.

force is increased threefold and the stiffness twofold with regard to structures with alternate radial polarizations.

So, the previous calculations show that the most interesting configuration for having the greatest axial force is the one using ring permanent magnets with perpendicular polarizations.

5.3. Stacked Structures with Five Ring Permanent Magnets

The previous observations remain valid for configurations using more ring permanent magnets. Fig. 10 presents the axial force and stiffness exerted by the outer part made of five ring permanent magnets with perpendicular polarizations on the inner part having the same magnet configuration versus the axial shift between the two parts. This configuration is compared to a passive magnetic bearing made of two parts of ring permanent magnets with alternate permanent magnets whose polarizations are radial (Fig. 11).

The calculations shown in Figs. 10 and 11 demonstrate that the
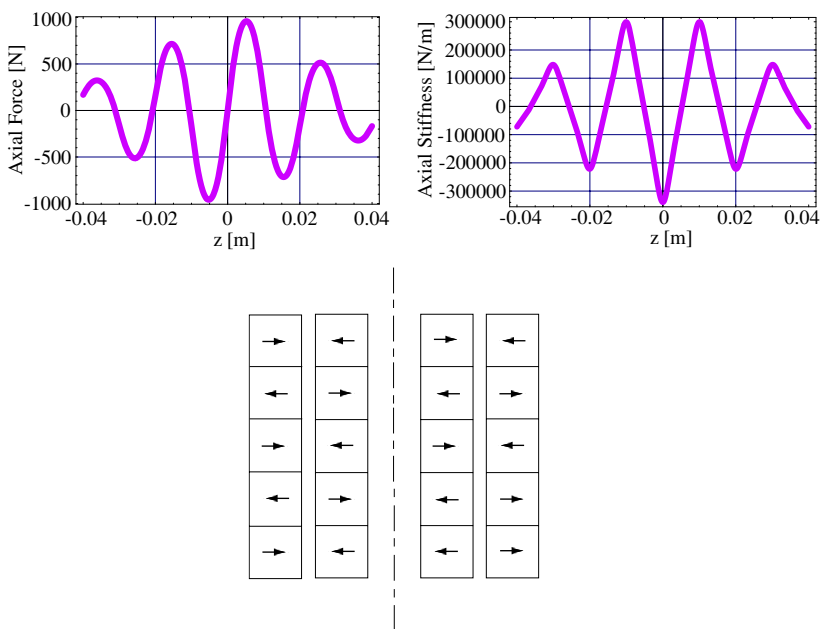
Figure 10. Cross-section view for a stack of five ring permanent magnets with radial polarizations; $r_1 = 0.01$ m, $r_2 = 0.02$ m, $r_3 = 0.022$ m, $r_4 = 0.032$ m, $J = 1$ T, height of each ring permanent magnet $= 0.01$ m.
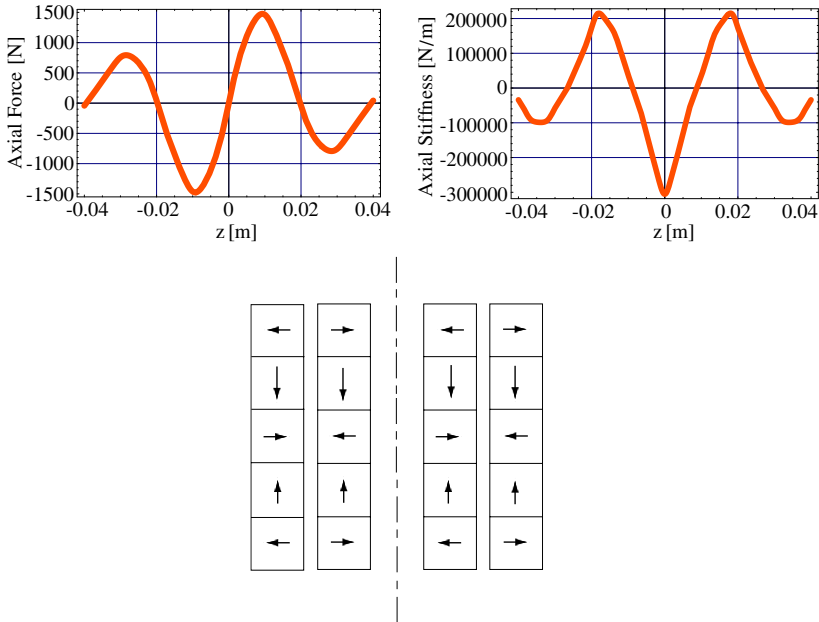
Figure 11. Cross-section view for a stack of five ring permanent magnets with perpendicular polarizations; $r_1 = 0.01$ m, $r_2 = 0.02$ m, $r_3 = 0.022$ m, $r_4 = 0.032$ m, $J = 1$ T, height of each ring permanent magnet $= 0.01$ m.

maximal axial force exerted in a passive magnetic bearing made of five ring permanent magnets with alternate magnetizations is 960 N whereas the maximal axial force exerted in a passive magnetic bearing with a Halbach configuration is 1475 N. Moreover, the maximal axial stiffness for the case of the configuration shown in Fig. 10 is $|K_z = 339738$ N/m whereas the maximal axial stiffness in Fig. 11 is $|K_z| = 305301$ N/m. The force is increased fourfold and the stiffness two fold in the Halbach structure. This shows that the Halbach configuration which is constituted of both radial and perpendicular elementary structures benefits the advantages of both. This leads to the most efficient device for a given magnet volume.

6. CONCLUSION

This paper presents analytical expressions of the axial force and stiffness exerted between two ring permanent magnets with perpendicular polarizations. The comparison of the performances of

this structure with the ones of structures with axially or radially polarized ring magnets shows that perpendicular structure yield the highest axial force whereas radial ones yield the highest stiffness. Then, Halbach and alternate permanent magnet configurations are compared. As a result, for a given magnet volume, Halbach structures are far more efficient than the alternate ones, the increase in force and stiffness being at least twofold. The presented approach is efficient for optimizing quickly such devices.

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