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ANALYSIS OF FOCAL REGION FIELDS OF PEMC GREGORIAN SYSTEM EMBEDDED IN HOMOGENEOUS CHIRAL MEDIUM

M. Q. Mehmood and M. J. Mughal

Faculty of Electronic Engineering
GIK Institute of Engineering Sciences and Technology
Topi, Swabi, N.W.F.P. 23640, Pakistan

Abstract—This paper presents the high frequency electromagnetic field expressions for Perfect electromagnetic conductor (PEMC) Gregorian system. In this Gregorian system both the reflectors are PEMC and are embedded in the homogenous chiral medium. Depending upon the values of chirality parameter ($k\beta$) two cases are analyzed. In the first case, chiral medium supports positive phase velocity (PPV) for both the left circularly polarized (LCP) and right circularly polarized (RCP) modes. In the second case, chiral medium supporting PPV for one mode and negative phase velocity (NPV) for the other mode is taken into account. Since Geometrical optics (GO) fails at the focal point, so Maslov's method is used to find the field expressions at the point. Field plots for different values of admittance (M) of the PEMC and the chirality parameter ($k\beta$) are given in the paper.

1. INTRODUCTION

PEMC is a non-reciprocal generalization of both the perfect electric conductor (PEC) and perfect magnetic conductor (PMC). Because PEMC does not allow electromagnetic energy to enter, so it can serve as boundary material. Possibilities for the realization of a PEMC boundary has been suggested by [1]. The boundary conditions for the PEC and PMC are given by following equations [2, 3]

$$\mathbf{n}\times\mathbf{E}=0, \qquad \mathbf{n}\cdot\mathbf{B}=0 \quad (PEC) \tag{1a}$$

$$\mathbf{n}\times\mathbf{H}=0, \qquad \mathbf{n}\cdot\mathbf{D}=0 \quad (PEMC) \tag{1b}$$

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Corresponding author: Muhammad Qasim Mehmood (qasim145ps@gmail.com).

where $\mathbf{n}$ denotes the unit vector normal to the boundary surface. The PEMC boundary conditions are of more general form

$$\mathbf{n} \times (\mathbf{H} + M\mathbf{E}) = 0, \qquad \mathbf{n} \cdot (\mathbf{D} - M\mathbf{B}) = 0 \quad (PEMC) \tag{2}$$

where M denotes the admittance of the PEMC boundary. PMC corresponds to $M = 0$, while PEC is obtained as the limit $M \to \pm\infty$ [4].

Our interest is to find high frequency field expressions for the PEMC Gregorian system when it is placed in chiral medium. Chiral medium is a composite of uniformly distributed and randomly oriented chiral objects [5]. This medium is homogeneous and supports both LCP and RCP modes. It may supports NPV propagation for both modes, or NPV for one mode and PPV for the other mode [6]. The Constitutive parameters for chiral medium is as following [7]

$$\mathbf{D} = \epsilon\left(\mathbf{E} + \beta\nabla \times \mathbf{E}\right) \tag{3}$$

$$\mathbf{B} = \epsilon\left(\mathbf{H} + \beta\nabla \times \mathbf{H}\right) \tag{4}$$

where ϵ, μ and β is permittivity, permeability and chirality constant of the medium. ϵ and μ has usual dimensions and β has the dimension of length. Using Maxwell's equations result we get the following equations

$$\left(\nabla^2 + k^2 n_1^2\right) \mathbf{Q}_L = 0 \tag{5}$$

$$\left(\nabla^2 + k^2 n_2^2\right) \mathbf{Q}_R = 0 \tag{6}$$

where $\mathbf{Q}_L$ and $\mathbf{Q}_R$ represents the LCP and RCP waves respectively. $n_1 = \frac{1}{1-k\beta}$ and $n_2 = \frac{1}{1+k\beta}$ are equivalent refractive indices of the medium seen by LCP and RCP waves respectively, and $k = \omega\sqrt{\epsilon\mu}$.

Different reflectors are placed in chiral medium due to its unique characteristics over an ordinary medium like polarization control, impedance matching and cross coupling of electric and magnetic fields. By changing the chiral media parameters ϵ, μ and $k\beta$ the desirable values of the wave impedance and propagation constants can be achieved by which reflections can be adjusted (decreased or increased). In this respect, the chiral medium can be controlled by variations of three parameters ϵ, μ, $k\beta$, whereas an achiral medium has only two variable parameters, ϵ, μ [8]. Moreover, we can use it as a chiral nihility media. Chiral nihility medium is one where both the relative permittivity ϵ and the relative permeability μ are very small. This type of medium favors the realization of a negative refraction and the propagation of backward waves when the chirality parameter is appropriately chosen [9, 10]. This backward propagation can give the advantage of invisibility. Due to these characteristics of chiral medium, we have embedded the PEMC Gregorian system in chiral medium in this problem. Maslov's method is used to study the fields at the focal regions [11, 12].

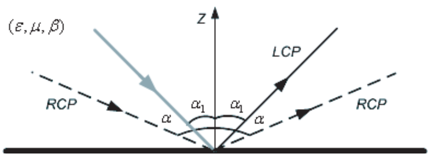

Figure 1. Reflection of RCP wave from PEMC plane.

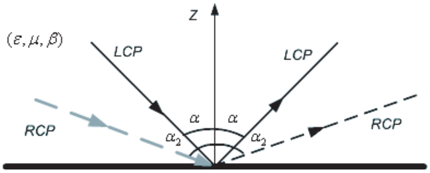

Figure 2. Reflection of LCP wave from PEMC plane.

2. PLANE WAVE REFLECTION FROM PEMC PLANE PLACED IN CHIRAL MEDIUM

Consider the reflection of plane wave from the PEMC plane placed in chiral medium. When RCP wave traveling with phase velocity ω/kn_2 and unit amplitude is incident on the PEMC plane making angle α with z-axis. Two waves of opposite handedness are reflected as shown in Figure 1. The RCP wave has amplitude $(\cos\alpha - \cos\alpha_1)/(\cos\alpha + \cos\alpha_1)$ and makes an angle α with z-axis and the LCP wave with amplitude $[(M\eta - j)/(M\eta + j)]\,[2\cos\alpha/(\cos\alpha + \cos\alpha_1)]$ traveling with phase velocity ω/kn_1 and makes an angle $\alpha_1 = [\sin^{-1}(n_2/n_1 \sin\alpha)]$ with z-axis. If we take $k\beta < 1$ then $n_1 > n_2$ and $\alpha_1 < \alpha$, i.e., LCP wave bends towards normal, because it is traveling slower than RCP wave. For $k\beta > 1$, α_1 is negative and the wave is reflected in the wrong way, it may be called negative reflection (shown as gray in Figure 1). This means that for $k\beta > 1$, LCP reflected wave sees the chiral medium as NPV medium. Similarly, when LCP wave with unit amplitude and angle α with z-axis, is incident on PEMC plane we get two reflected waves, the RCP wave with amplitude $[-(1 - jM\eta)/(1 + jM\eta)]\,[2\cos\alpha/(\cos\alpha + \cos\alpha_2)]$ traveling with phase velocity ω/kn_2 and makes an angle $\alpha_2 = [\sin^{-1}(n_1/n_2 \sin\alpha)]$ with z-axis and the LCP wave with amplitude $(\cos\alpha - \cos\alpha_2)/(\cos\alpha + \cos\alpha_2)$ traveling with phase velocity ω/kn_1 and makes an angle α with z-axis. If we take $k\beta < 1$, then $n_1 > n_2$ and $\alpha_2 > \alpha$. If $k\beta = 0$, then only normal reflection take place, and if $k\beta$ increases the difference between the angle α and α_1, α_2 increases. For $k\beta > 1$, we have negative reflection for RCP reflected wave (shown as gray in Figure 2) [13].

3. GEOMETRICAL OPTICS FIELDS OF TWO DIMENSIONAL PEMC GREGORIAN REFLECTOR PLACED IN CHIRAL MEDIUM

Gregorian system placed in homogeneous and reciprocal chiral medium is shown in Figure 3 (gray lines show the negative reflected waves).

When both RCP and LCP waves will hit on main PEMC parabolic reflector, it will cause four reflected waves designated as LL, RR, LR and RL [14]. These four waves are then incident on the PEMC elliptical subreflector and will cause eight reflected waves. These rays are designated as LLL, RRR, LLR, RRL, RLR, RLL, LRR and LRL. Only four of these rays (LLL, RRR, LLR, RRL) converge in the focal region while other four rays (RLR, RLL, LRR, LRL) diverge. Only converging rays are considered and shown in Figure 3. For, $k\beta > 1$, LCP wave travels with NPV and RCP wave with PPV. LLR wave diverges out and do not form a real focus while RRL wave forms a focal point which is much shifted towards the left as shown in Figure 3 by gray lines. The GO fields of these reflected waves has been calculated for PEC Gregorian system in [15]. For the case of PEMC, LLR and RRL waves will have different initial amplitudes as compared with PEC, while LLL and RRR waves have the same initial amplitudes. These initial amplitudes can be found as in [16]. Final expressions for the initial amplitudes of all the four converged rays are given as following.

$$A_{0LLL} = \left[\frac{\cos\alpha - \cos\alpha_2}{\cos\alpha + \cos\alpha_2}\right]\left[\frac{\cos\gamma - \cos\gamma_2}{\cos\gamma + \cos\gamma_2}\right] \tag{7a}$$

$$A_{0RRR} = \left[\frac{\cos\alpha - \cos\alpha_1}{\cos\alpha + \cos\alpha_1}\right]\left[\frac{\cos\gamma - \cos\gamma_1}{\cos\gamma + \cos\gamma_1}\right] \tag{7b}$$

$$A_{0RRL} = \left[\frac{\cos\alpha - \cos\alpha_1}{\cos\alpha + \cos\alpha_1}\right]\left[\frac{M\eta - j}{M\eta + j}\right]\left[\frac{2\cos\gamma}{\cos\gamma + \cos\gamma_1}\right] \tag{7c}$$

$$A_{0LLR} = \left[\frac{\cos\alpha - \cos\alpha_2}{\cos\alpha + \cos\alpha_2}\right]\left[-\frac{1 - jM\eta}{1 + jM\eta}\right]\left[\frac{2\cos\gamma}{\cos\gamma + \cos\gamma_2}\right] \tag{7d}$$

And the corresponding initial phases are

$$S_{0LLL} = -n_1\zeta_1 = n_1\left[2f\frac{\cos 2\alpha}{1 + \cos 2\alpha} - c\right] \tag{8a}$$

$$S_{0RRR} = -n_2\zeta_1 = n_2\left[2f\frac{\cos 2\alpha}{1 + \cos 2\alpha} - c\right] \tag{8b}$$

$$S_{0RRL} = -n_2\zeta_1 = n_2\left[2f\frac{\cos 2\alpha}{1 + \cos 2\alpha} - c\right] \tag{8c}$$

$$S_{0LLR} = -n_1\zeta_1 = n_1\left[2f\frac{\cos 2\alpha}{1 + \cos 2\alpha} - c\right] \tag{8d}$$

while the extra terms of the phase for all four rays are as following

$$\begin{aligned} S_{exLLL} = & -n_1\left[x\sin(2\alpha - 2\psi) + z\cos(2\alpha - 2\psi)\right] \\ & + n_1\left[\xi_2\sin(2\alpha - 2\psi) + \zeta_2\cos(2\alpha - 2\psi)\right] \end{aligned} \tag{9a}$$

$$S_{exRRR} = -n_2 \left[x \sin(2\alpha - 2\psi) + z \cos(2\alpha - 2\psi)\right] + n_2 \left[\xi_2 \sin(2\alpha - 2\psi) + \zeta_2 \cos(2\alpha - 2\psi)\right] \tag{9b}$$

$$S_{exRRL} = -n_1 \left[x \sin(\gamma_1 - \psi) + z \cos(\gamma_1 - \psi)\right] + n_1 \left[\xi_2 \sin(\gamma_1 - \psi) + \zeta_2 \cos(\gamma_1 - \psi)\right] \tag{9c}$$

$$S_{exLLR} = -n_2 \left[x \sin(\gamma_2 - \psi) + z \cos(\gamma_2 - \psi)\right] + n_2 \left[\xi_2 \sin(\gamma_2 - \psi) + \zeta_2 \cos(\gamma_2 - \psi)\right] \tag{9d}$$

and

$$t_1 = \sqrt{(\xi_2 - \xi_1)^2 + (\zeta_2 - \zeta_1)^2}, \quad t = \sqrt{(x - \xi_2)^2 + (z - \zeta_2)^2} \tag{10}$$

where (ξ_1, ζ_1) and (ξ_2, ζ_2) are the cartesian coordinates of the points on the parabolic and elliptical reflectors. Finite GO fields of these reflected rays around the focal point, not repeating the calculations, are as following.

$$U(r)_{LLL} = \sqrt{\frac{k}{2j\pi}} \left[\int_{A_1}^{A_2} + \int_{-A_1}^{-A_2}\right] A_{0LLL} \sqrt{R_1} \times \exp[-jk\{S_{0LLL} + n_1 t_1 + S_{exLLL}\}] d(2\alpha) \tag{11a}$$

$$U(r)_{RRR} = \sqrt{\frac{k}{2j\pi}} \left[\int_{A_1}^{A_2} + \int_{-A_1}^{-A_2}\right] A_{0RRR} \sqrt{R_1} \times \exp[-jk\{S_{0RRR} + n_2 t_1 + S_{exRRR}\}] d(2\alpha) \tag{11b}$$

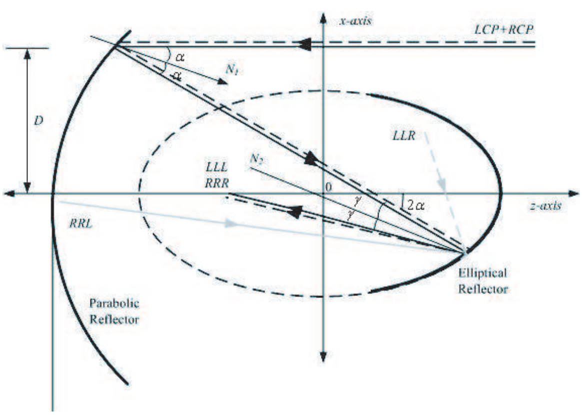

Figure 3. PEMC Gregorian reflector in chiral medium, $k\beta < 1$ and $k\beta > 1$.

$$U(r)_{RRL} = \sqrt{\frac{k}{2j\pi}} \left[\int_{A_1}^{A_2} + \int_{-A_1}^{-A_2} \right] A_{0RRL}(\xi) \frac{1}{\sqrt{n_1^2 - n_2^2 \sin^2 \gamma}}$$
$$\times \left[\frac{R_1 R_2 b n_2 \cos \gamma_1}{abn_2 + a\sqrt{(R_1R_2)(n_1^2 - n_2^2 \sin^2 \gamma)} - bn_2 R_1} \right]^{-1/2}$$
$$\times \exp[-jk\{S_{0RRL} + n_1 t_1 + S_{exRRL}\}] d(2\alpha) \tag{11c}$$

$$U(r)_{LLR} = \sqrt{\frac{k}{2j\pi}} \left[\int_{A_1}^{A_2} + \int_{-A_1}^{-A_2} \right] A_{0LLR}(\xi) \frac{1}{\sqrt{n_2^2 - n_1^2 \sin^2 \gamma}}$$
$$\times \left[\frac{R_1 R_2 b n_1 \cos \gamma_2}{abn_1 + a\sqrt{(R_1R_2)(n_2^2 - n_1^2 \sin^2 \gamma)} - bn_1 R_1} \right]^{-1/2}$$
$$\times \exp[-jk\{S_{0LLR} + n_2 t_1 + S_{exLLR}\}] d(2\alpha) \tag{11d}$$

while S_0, S_{ex} and t_1 are given in Eqs. (8a)–(8d), Eqs. (9a)–(9d) and Eq. (10) respectively.

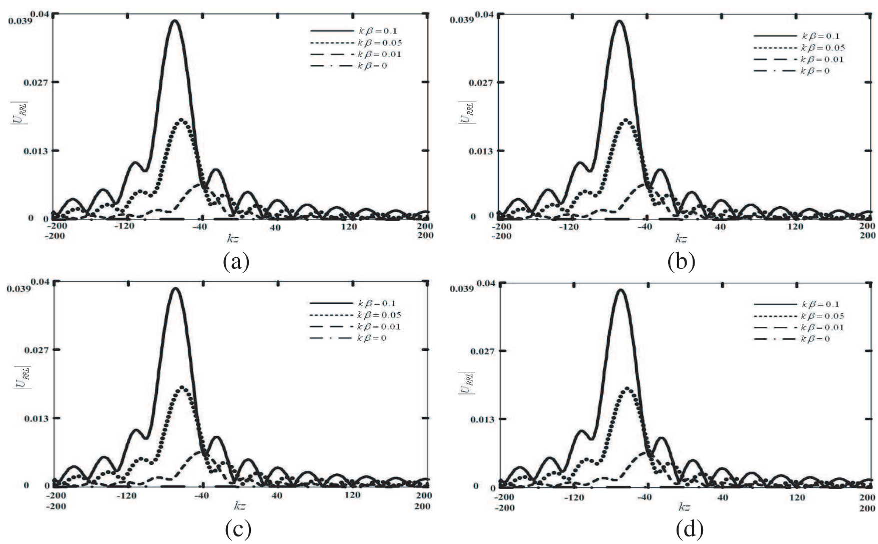

Figure 4. $|U_{RRL}|$ of PEMC Gregorian reflector at $kx = 0$ for $k\beta = 0$, 0.001, 0.005, 0.01 for: (a) $M_\eta = 0, \infty$ (b) $M_\eta = 1$ (c) $M_\eta = 5$ (d) $M_\eta = 10$.

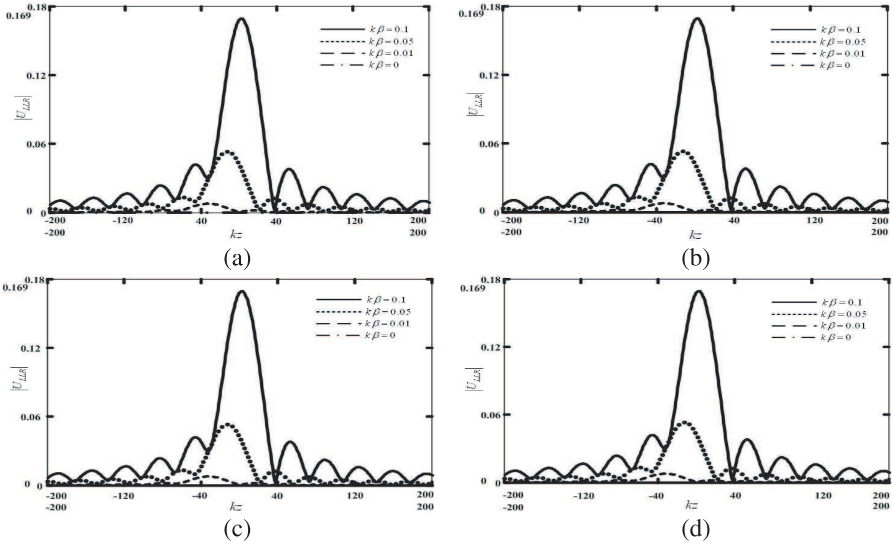

Figure 5. $|U_{LLR}|$ PEMC Gregorian reflector at $kx = 0$ for $k\beta = 0$, 0.001, 0.005, 0.01 for: (a) $M_\eta = 0, \infty$ (b) $M_\eta = 1$ (c) $M_\eta = 5$ (d) $M_\eta = 10$.

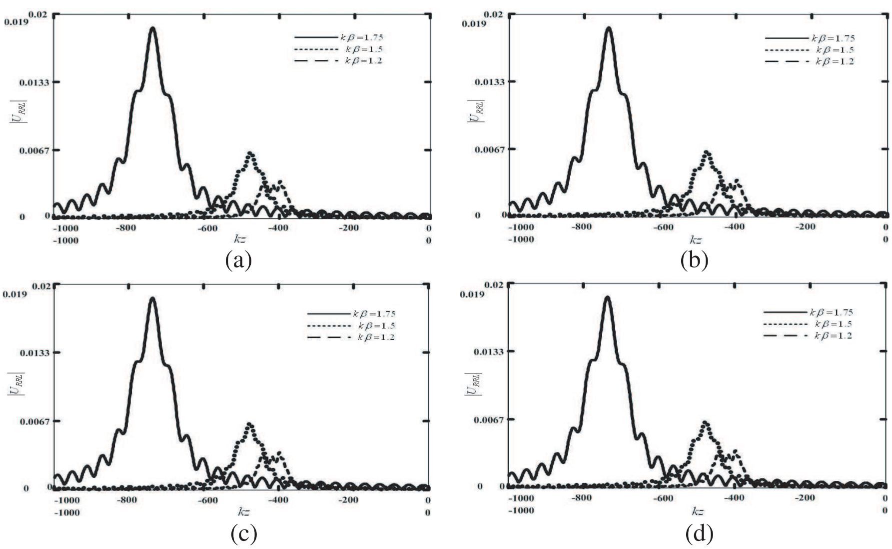

Figure 6. $|U_{RRL}|$ of PEMC Gregorian reflector at $kx = 0$ for $k\beta = 0$, 0.001, 0.005, 0.01 for: (a) $M_\eta = 0, \infty$ (b) $M_\eta = 1$ (c) $M_\eta = 5$ (d) $M_\eta = 10$.

4. RESULTS AND DISCUSSIONS

Variations in magnitude of the fields are shown along kz in Figure 4 to Figure 6. Values for different parameters of PEMC Gregorian reflector are: $kf = 170$, $ka = 40$, $kb = 60$, $kd = 50$, $kD = 150$. The line plots of U_{LLL} and U_{PRR} are exactly same as that of PEC Gregorian system. Plots of U_{RRL} and U_{LLR} for $k\beta = 0$, 0.001, 0.005, 0.01 and $M\eta = 0, \infty, 1, 5, 10$ are given in Figure 6 and Figure 5 respectively. These figures show that trends of the plot are same as in the case of PEC Gregorian system, i.e., increase in the value of $k\beta$ shifts RRL wave to the left and LLR wave to the right. For $k\beta > 1$, LLL and RRR rays are similar to that of PEC Gregorian system and RRL and LLR rays diverge out and do not form a real focus.

5. CONCLUSION

GO fields focused by a PEMC Gregorian system placed in chiral medium are studied in this paper. It is found that fields of LLL, RRR, RRL and LLR rays are same as that of PEC Gregorian system [15]. However, RRL and LLR rays have different amplitude than PEC case as given in Eq. (7c) and Eq. (7d). Both the cases of PPV and NPV are considered. It is seen that for PPV case, as the chirality parameter increases, gap between the focal points of LLR and RRL rays increases. For NPV case, the caustic points for LLL and RRR rays does not change, RRL has the focal point is shifted much towards the left while LLR ray diverges and do not form a focal point.

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