A HYBRID OPTIMIZED ALGORITHM BASED ON EGO AND TAGUCHI'S METHOD FOR SOLVING EXPENSIVE EVALUATION PROBLEMS OF ANTENNA DESIGN

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Abstract—In this paper, we propose a hybrid optimization approach that combines the Efficient Global Optimization (EGO) algorithm with Taguchi's method. This hybrid optimized algorithm is suited for problems with expensive cost functions. As a Bayesian analysis optimization algorithm, EGO algorithm begins with fitting the Kriging model with n sample points and finds the $(n + 1)$th point where the expected improvement is maximized to update the model. We employ Taguchi's method in EGO to obtain the $(n + 1)$th point in this paper. A numerical simulation demonstrates that our algorithm has advantage over the original EGO. Finally, we apply this hybrid optimized algorithm to optimize an ultra-wide band (UWB) transverse electromagnetic (TEM) horn antenna and a linear antenna array. Compared to Taguchi's method and the Integer Coded Differential Evolution Strategy, our algorithm converges to the global optimal value more efficiently.

1. INTRODUCTION

Many different optimization algorithms have been applied to antenna design [1–5]. Most of them require a large number of evaluations of the objective function. We propose a hybrid optimization approach that is suited for solving this problem. As a Bayesian analysis optimization algorithm, Efficient Global Optimization (EGO) converges to the

global optimal value efficiently. The reason is that only one best point is evaluated using the objective function during each iteration, and EGO can automatically balance local and global search. In order to find the best point during each iteration, the branch and bound method [6] and genetic algorithm [7] are used in EGO.

In this paper, we employ Taguchi's method [8] in EGO to obtain the best point. Taguchi's method is based on the concept of the orthogonal array (OA), which can effectively reduce the number of tests required in a design process [9]. Compared with the branch and bound method and genetic algorithm, Taguchi's method is easier to implement and more efficient in reaching the optimum solution.

This paper is organized as follows. In Section 2, the EGO algorithm is described in detail, and Taguchi's method is introduced briefly. A test function is used to demonstrate the global optimization performance of the hybrid optimized algorithm in Section 3. Section 4 presents two examples to prove that the hybrid optimized algorithm works efficiently. The conclusion is given in Section 5.

2. EFFICIENT GLOBAL OPTIMIZATION

We sample n points by the Latin Hypercube Sampling [10] and evaluate their fitness using the objective function. The fitness $\mathbf{y}$ is a n-dimensional vector, and each component is a function of the k independent variables $\mathbf{x}$. For the initial n samples, the vectors $\mathbf{y}$ and $\mathbf{x}$ are

$$\mathbf{y} = [y^{(1)}, \ y^{(2)}, \ y^{(3)}, \ \ldots, \ y^{(n)}]_{n \times 1} \tag{1}$$

$$\mathbf{x} = [x_1^{(i)}, \ x_2^{(i)}, \ x_3^{(i)}, \ldots, x_k^{(i)}]_{k \times 1} \quad i = 1, 2, \ldots, n \tag{2}$$

The initial number of n is usually chosen as $11k - 1$ in order to ensure a robust search of the model space.

2.1. Kriging Model

The Kriging model is used in EGO, which can be written as [11]

$$y(\mathbf{x}^{(i)}) = \mu + \varepsilon(\mathbf{x}^{(i)}) i = 1, 2, \ldots, n \tag{3}$$

Here, μ is a constant term, and $\varepsilon(\mathbf{x}^{(i)})$ represents a deviation for Gaussian stochastic process. The correlation between $\varepsilon(\mathbf{x}^{(i)})$ and $\varepsilon(\mathbf{x}^{(j)})$ is strong when the two corresponding points $\mathbf{x}^{(i)}$ and $\mathbf{x}^{(j)}$ are close. In EGO, a special weighted distance function between the points $\mathbf{x}^{(i)}$ and $\mathbf{x}^{(j)}$ is defined as

$$d(\mathbf{x}^{(i)}, \mathbf{x}^{(j)}) = \sum_{h=1}^{k} \theta_h \left| x_h^{(i)} - x_h^{(j)} \right|^{p_h} \tag{4}$$

where $\theta_h(0 \leq \theta_h \leq \infty)$ are the unknown correlation parameters representing the activity of the variable x_h. The $p_h(1 \leq p_h \leq 2)$ is related to the smoothness of the function in coordinate direction h. The function is smooth with $p_h = 2$ and less smooth when p_h is near 1. The correlation between the point $\varepsilon(\mathbf{x}^{(i)})$ and $\varepsilon(\mathbf{x}^{(j)})$ is defined as

$$corr\left[\varepsilon(\mathbf{x}^{(i)}), \varepsilon(\mathbf{x}^{(j)})\right] = \exp\left[-d(\mathbf{x}^{(i)}, \mathbf{x}^{(j)})\right] \tag{5}$$

The Kriging predictor, which is the best linear unbiased predictor (BLUP) of $y(\mathbf{x}^*)$ can be written as [12]

$$\widehat{y}(\mathbf{x}^*) = \widehat{y}^{(*)} = \widehat{\mu} + \mathbf{r^T R^{-1}}(\mathbf{y} - \mathbf{1}\widehat{\mu}) \tag{6}$$

where $\mathbf{x}^*$ is an arbitrary point in the function space. $\mathbf{R}$ is a $n \times n$ matrix whose (i, j) entry is $corr\left[\varepsilon(\mathbf{x}^{(i)}), \varepsilon(\mathbf{x}^{(j)})\right]$, and $\mathbf{I}$ denotes an n-dimensional unit vector. $\mathbf{r}$ is the vector whose ith element is

$$\mathbf{r}_i = corr\left[\varepsilon(\mathbf{x}^*), \varepsilon(\mathbf{x}^{(i)})\right] \tag{7}$$

The mean of this model is

$$\widehat{\mu} = \frac{(\mathbf{I}^T \mathbf{R}^{-1} \mathbf{y})}{(\mathbf{I}^T \mathbf{R}^{-1} \mathbf{I})} \tag{8}$$

The unknown parameters, θ_h and p_h, can be determined by maximizing the following likelihood function

$$L(\mu, \sigma^2) = \frac{1}{(2\pi)^{\frac{n}{2}}(\sigma^2)^{\frac{n}{2}}|\det(\mathbf{R})|^{\frac{1}{2}}} \exp\left[-\frac{(\mathbf{y} - \mathbf{I}\mu)^T \mathbf{R}^{-1}(\mathbf{y} - \mathbf{I}\mu)}{2\sigma^2}\right] \tag{9}$$

where

$$\widehat{\sigma}^2 = \frac{(\mathbf{y} - \mathbf{I}\widehat{\mu})^T \mathbf{R}^{-1}(\mathbf{y} - \mathbf{I}\widehat{\mu})}{n} \tag{10}$$

The Nelder-Mead algorithm [13] is used to obtain θ_h and p_h. With the parameters, we can obtain the Kriging model using (6)–(8).

2.2. Update the Model

The key point of the EGO algorithm is how to select the next data point for the evaluation of the objective function to update the Kriging model.

The mean squared error of the model predictor can be estimated as follow:

$$s^2(\mathbf{x}^*) = \sigma^2\left[1 - (\mathbf{r}^T \mathbf{R}^{-1} \mathbf{r}) + \frac{(\mathbf{I} - \mathbf{I}^T \mathbf{R}^{-1} \mathbf{r})^2}{(\mathbf{I}^T \mathbf{R}^{-1} \mathbf{I})}\right] \tag{11}$$

Notice that when $\mathbf{x}^* = \mathbf{x}^{(i)}$ with $i \leq n$, we have $\mathbf{r}^T\mathbf{R}^{-1}\mathbf{r} = 1$, $\mathbf{I}^T\mathbf{R}^{-1}\mathbf{r} = 1$, and $s^2(\mathbf{x}^{(i)}) = 0$. This means that the uncertainty of a known point is zero.

Let $f_{\min} = \min[y^{(1)}, y^{(2)}, y^{(3)}, \ldots, y^{(n)}]$, the improvement at the point $\mathbf{x}^*$ is

$$I(\mathbf{x}^*) = \max(f_{\min} - \widehat{y}, 0) \tag{12}$$

The expected improvement can be expressed as

$$E\left[I(\mathbf{x}^*)\right] = \begin{cases} (f_{\min} - \widehat{y})\phi\left[\frac{f_{\min}-\widehat{y}}{s}\right] + s\varphi\left[\frac{f_{\min}-\widehat{y}}{s}\right] & s > 0 \\ 0 & s = 0 \end{cases} \tag{13}$$

where $\phi(x)$ and $\varphi(x)$ are the normal probability distribution function and normal probability density function, respectively.

We evaluate $E\left[I(\mathbf{x}^*)\right]$ for a large number of points and find the $(n+1)$th point $\mathbf{x}$ where the expected improvement is maximized, and evaluate the fitness of the $\mathbf{x}$. If the convergence criterion is not met, the Kriging model is updated with this new sample point, and the searching for the next point is repeated until the convergence criterion is satisfied.

We use Taguchi's method to find the new $\mathbf{x}$. Compared to the branch and bound method and the genetic algorithm it is easier to implement and more efficient to obtain the optimum solution.

2.3. Taguchi's Method

Taguchi's method begins with selecting the orthogonal array (N, k, s, t) which is a $N \times k$ matrix with s levels and t strength. The strength t usually is 2 [14]. The values of N and k are determined as

$$N = s^p \tag{14}$$

$$k = \frac{N-1}{s-1} \tag{15}$$

where p is a positive integer starting with 2. The number of levels typically is 3. And in the first iteration, the values of parameters for level 2 are selected at the center of the optimization range. The values of levels 1 and 3 are defined as

$$Level1/Level3 = Level2 + / - LD_1 \tag{16}$$

where

$$LD_1 = \frac{\max - \min}{numble\ of\ levels + 1} \tag{17}$$

The input parameters are designed using the orthogonal array. A response table can be built with the parameters. Based on the response

table, the optimal level for these parameters is used as the values of the level 2 for the next iteration. But for the $(i + 1)$th iteration

$$LD_{i+1} = rr \times LD_i \tag{18}$$

where rr is reduced rate the value of which can be set between 0.7 and1. In this paper rr is set to 0.9.

When the optimal level is identified, a confirmation experiment is performed. The following equation may be used as a termination criterion for the optimization procedure

$$\frac{LD_i}{LD_1} < convergent\ value \tag{19}$$

The convergent value is set as 0.0001. After the optimization parameters are set, the step of Taguchi's method is constant. Here, the objective function of Taguchi's method is the expected improvement. Compared to a large number of electromagnetics evaluations this evaluation is ignore. So there is almost no effect when optimization parameters are varied.

2.4. Termination Criteria

The EGO algorithm has two convergence criterions. One is that the absolute value of the expected improvement $E\left[I(\mathbf{x}^*)\right]$ at the new point

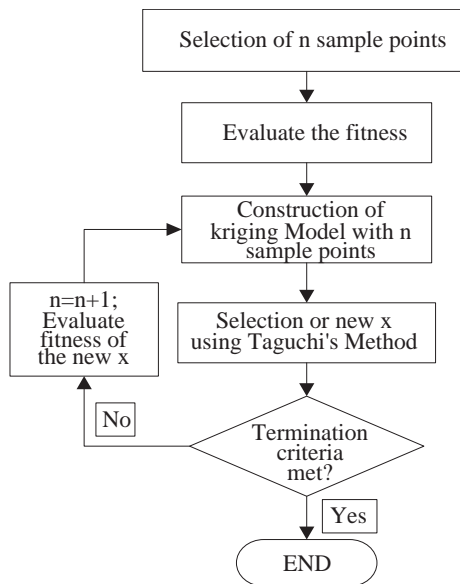**Figure 1.** The flowchart of EGO.

x is less than 1% of the minimum value of the function at the known points. We take 0.1% instead of 1% to obtain better accuracy. One could choose a factor less than 0.1%; however, the smaller number, the more expensive function calls. The other stopping rule is the total number of iterations of the algorithm. This prevents the algorithm from running extensively.

The flowchart of the hybrid optimized algorithm is shown in Figure 1.

3. A TEST FUNCTION

We take log 10 of The Goldstein-Price function as the test function:

$$f(x_1, x_2) = \log_{10}\big(\big[1 + (x_1 + x_2 + 1)^2 (19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1 x_2 + 3x_2^2)\big]$$
$$\times \big[30 + (2x_1 - 3x_2)^2 (18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1 x_2 + 27x_2^2)\big]\big)$$
$$(-2 \leq x_1, x_2 \leq 2) \tag{20}$$

It has two independent variables and one global minimum that is equal to 0.477 at $(0, 1)$. A average value of ten results of the hybrid optimized algorithm is 0.488. The comparison of four methods is shown in Table 1. Compared with the original EGO, the hybrid optimized algorithm works more efficiently and has a smaller convergence value.

Table 1. Comparison of four methods.

Methods	The hybrid optimized algorithm	EGO [6]	Zilinskas' algorithm [15]	Mockus' Bayesian algorithm [15]
Stopping Criteria	0.001	0.01	0.001	0.001
Numbers of calculating objective function	20	32	153	362

4. NUMERICAL EXAMPLES

4.1. Application on the Design of the Ultra-wide Band (UWB) TEM Horn Antenna

As a high-power and ultra-wide band (UWB) antenna transverse electromagnetic (TEM) horn antenna can effectively radiate time-

domain short pulse signal. It has a very broad application in military area, such as radar systems, high-power microwave weapons and so on. The geometry of a TEM horn antenna is shown in Figure 2. The parameters of the TEM horn antenna are shown in Table 2 [16].

The structural parameters apical angle (α) and included angle (β) are set to be the optimized variables. The optimization range is

$$5° \leq \alpha/2 \leq 60°, \quad 5° \leq \beta/2 \leq 60°$$

The fitness function is set as,

$$f = \max\{VSWR_{0.6\text{--}3\,\text{GHz}}\} \tag{21}$$

the value is as smaller as possible. In order to fit the Kriging model well, the fitness function is transforming with the log transformation, $\ln(f)$.

The hybrid optimized algorithm stops after a total of 66 iterations and converges at the point $\mathbf{x}$, (13.866, 45.5214). The result is shown in Figure 3.

The comparison of the hybrid optimized algorithm with Taguchi's method and the Integer Coded Differential Evolution Strategy [16] is shown in Table 3 and Figure 4.
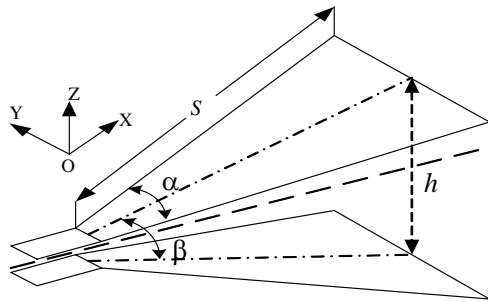

Figure 2. TEM horn antenna.

Table 2. Parameters of the TEM horn antenna.

Quantity	Value
hypotenuse length of the trapezoidal plate	0.35 m
parallel-plate length	0.03 m
parallel-plate width	0.05 m
parallel-plate height	0.01 m
thickness of metal plates	0.01 m

It can be seen that all of the three algorithms obtain good fitness values. However, the numbers of calculating fitness function of the hybrid optimized algorithm is only 87 which is about 10.98 percent of that in Taguchi's method and about 4.35 percent of that in the Integer Coded Differential Evolution Strategy. This proves the hybrid optimized algorithm works efficiently as mentioned above.

4.2. Optimize Element Amplitudes of a Linear Antenna Array

Figure 5 shows a 2N-element symmetrical array placed on the x-axis. The phase φ_n is taken as zero, and the spacing between adjacent
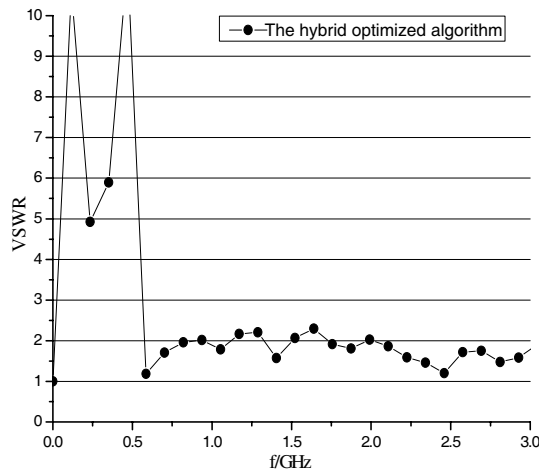

Figure 3. The results of the hybrid optimized algorithm.

Table 3. Comparison of the optimal results.

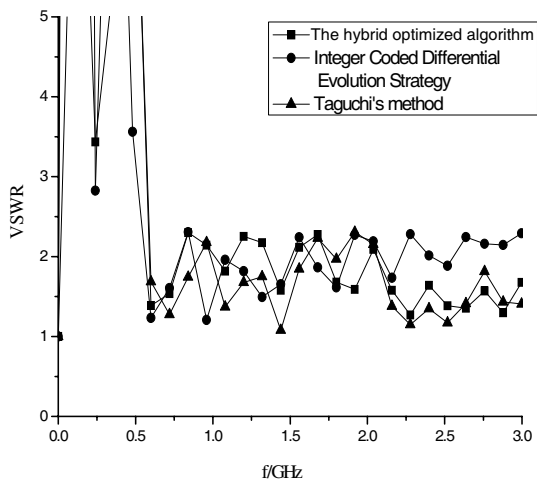|	$\alpha/2$	$\beta/2$	VSWR	Numbers of Calculating Fitness Function
The hybrid optimized algorithm	13.866	45.5214	2.2985	87
Taguchi's method	10.49	39.66	2.30743	792
Integer Coded Differential Evolution Strategy	17.94	42.62	2.27935	2000

Figure 4. Comparison of VSWR.

elements is taken as $\lambda/2$. The first element is placed at $x_=\lambda/4$, so the array factor (AF) can be written as:

$$AF(\theta) = 2\sum_{n=1}^{N} I_n \cos[(n - 0.5)\pi \sin\theta] \tag{22}$$

where $I_n(n = 1, 2, \ldots, N)$ are the excitation amplitudes which will be optimized in the range $[0, 1]$. The fitness function for maximum side lobe level (SLL) minimization is expressed as:

$$\text{Minimize} \begin{array}{l} fitt = \max\{20\log|AF(\theta)|\} \\ \theta = [14, 90] \end{array} \tag{23}$$
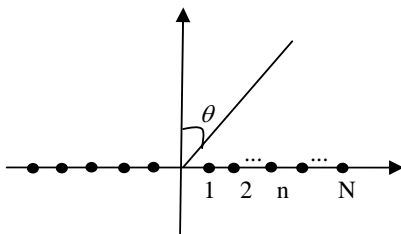

Figure 5. 2N-element symmetrical array placed on the x-axis.

A linear array with 10 elements is optimized using the hybrid optimized algorithm. The maximum SLL is $-24.30\,\text{dB}$ which is not

better than result of Taguchi's method in [14]. While the hybrid optimized algorithm stops after 14 iterations which means it calculates object function only 68 times. The optimum amplitude values of two methods are given in Table 4. Figure 6 shows the radiation patterns.
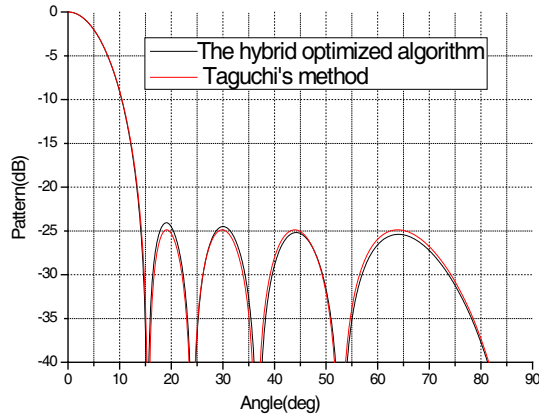

Figure 6. The radiation patterns.

Table 4. The optimum amplitude values.

n	The hybrid optimized algorithm	Taguchi's method
1	0.9473	1.0000
2	0.8573	0.8999
3	0.6837	0.7228
4	0.4949	0.5077
5	0.3876	0.3994

5. CONCLUSION

In this paper, we propose a hybrid optimized algorithm based on EGO algorithm and the Taguchi's method. A numerical simulation example demonstrates that our algorithm works more efficiently than the original EGO. Then we applied it to optimize the ultra-wide band (UWB) transverse electromagnetic (TEM) horn antenna and a linear antenna array. The results prove that the hybrid optimized

algorithm has advantages over Taguchi's method and the Integer Coded Differential Evolution Strategy and that it is suited for problems with expensive cost functions. It will be applied to more antenna design problems. And we will optimize, build and measure a new antenna in future.

ACKNOWLEDGMENT

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