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COMPARATIVE PERFORMANCE OF GRAVITATIONAL SEARCH ALGORITHM AND MODIFIED PARTICLE SWARM OPTIMIZATION ALGORITHM FOR SYNTHESIS OF THINNED SCANNED CONCENTRIC RING ARRAY ANTENNA

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Abstract—Scanning a planar array in the x-z plane directs the beam peak to any direction off the broadside along the same plane. Reduction of sidelobe level in concentric ring array of isotropic antennas scanned in the x-z plane result in a wide first null beamwidth (FNBW). In this paper, the authors propose pattern synthesis methods to reduce the sidelobe levels with fixed FNBW by making the scanned array thinned based on two different global optimization algorithms, namely Gravitational Search Algorithm (GSA) and modified Particle Swarm Optimization (PSO) algorithm. The thinning percentage of the array is kept more than 45 percent and the first null beamwidth (FNBW) is kept equal to or less than that of a fully populated, uniformly excited and 0.5λ spaced concentric circular ring array of same scanning angle and same number of elements and rings.

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1. INTRODUCTION

Circular array has received considerable interest over other types of planar arrays because it is symmetric and provides a nearly invariant beam pattern for 360° azimuthal coverage. A circular ring array, also known as a concentric circular array (CCA) is a planar array that consists of one or more concentric rings, which has equally spaced elements on its circumference. Its main attraction is the cylindrical symmetry of its radiation pattern and compact structure. Depending upon implementations, the maximum gain can be directed to broadside or the array can also be scanned in the elevation plane by properly arranging the array elements and making the array factor a function of θ. Other implementations that require the maximum gain to be directed in $\theta = 90°$ or scan the beam in the azimuth plane obtained by proper arrangement of the array elements and making the array factor a function of φ. However, in its modest form the array suffers from a high sidelobe problem. One of the important configurations regarding CCA is the uniform concentric circular array (UCCA) where the inter-element spacing in each individual ring is kept almost half of the wavelength and all the elements in the array are uniformly excited. Generally low sidelobes in the array factor are obtained through optimum amplitude weights of the signals at each array element. Sidelobe reduction techniques in the concentric circular ring array appear in the literature.

The radiation pattern function of a concentric ring array has been expressed by Stearns and Stewart [1] as a truncated Fourier-Bessel series and the non-uniform distribution of the rings has been approximated to a smaller number of equally spaced ones. N. Goto and D. K. Cheng showed that for a Taylor weighted ring array the maximum allowable inter-element spacing should be about four-tenths of a wavelength, if high sidelobes are to be avoided [2]. L. Biller and G. Friedman used steepest descent iterative process to find out element weights and ring spacing to get lower sidelobe levels and control over beam width [3]. D. Huebner reduced the sidelobe levels for small concentric ring array by adjusting the ring radii using optimization technique [4]. B. P. Kumar and G. R. Branner also proposed optimum ring radii for getting lower sidelobes [5]. M. Dessouky, H. Sharshar and Y. Albagory showed that the existence of central element in case of concentric circular array of smaller innermost ring reduced the sidelobe levels significantly while minor increase in the beamwidth [6]. Sidelobe level can be reduced by thinning the array [7–9]. Sidelobe level can also be reduced by spacing the concentric ring non-uniformly, by varying the number of elements in each ring or by combining the both [7].

Gravitational Search Algorithm (GSA) [10] and modified Particle Swarm Optimization (PSO) [9, 12] algorithm have been introduced to make the array thin.

Synthesis of thinned array using Genetic Algorithm is reported in the article [13]. The paper [14] presents thinned concentric array design using modified PSO when the main beam is broadside.

In this paper, we propose to design a scanned thinned concentric array, which is different from [14] in three aspects: main beam is off the broadside, first null beamwidth is prefixed and the results of two different evolutionary algorithms have been compared.

Thinning the scanned array while keeping FNBW fixed and variable then reduces the side lobe levels. In case of fixed FNBW, it is kept less than or equal to that of a uniformly excited circular ring array consisting of the same number of rings, same number of elements and also scanned to the same angles. The comparative performance of GSA and modified PSO in terms of fitness value, computation time is also shown.

2. SYNTHESIS OF SCANNED THINNED ARRAY

Scanning a concentric ring array in the x-z plane steers the beam peaks to the scan directions in the same plane as well as changes the first null beamwidth (FNBW). Further reduction of the side lobe levels in scanned array again increases its FNBW. The desired array characteristics with lower sidelobes in scanned array can be obtained by thinning the array.

Thinning an array means turning off some of the elements from a uniformly spaced or periodic array to generate a pattern with low sidelobe levels. Typical applications for thinned array include satellite-receiving antennas that operate against a jamming environment [11], ground-based high frequency radars [11] and design of interferometer array for radio astronomy [11]. Here we assumed that the positions of the elements are fixed and all the elements have two states either 'on' or 'off', depending on whether the element is connected to the feed network or not. In the 'off' state, either the element is passively terminated to a matched load or an open circuited. If there is no coupling between the elements, it is equivalent to removing them from the array.

The far field pattern of a concentric circular planar array [6] shown in Figure 1 on the x-y plane with central element feeding and scanned

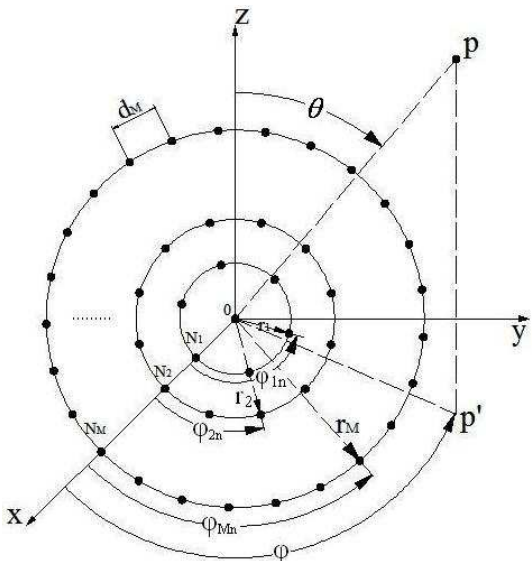

Figure 1. Multiple concentric circular ring array of isotropic antennas in XY plane.

to a specified angle can be defined as:

$$E(\theta,\varphi) = 1 + \sum_{m=1}^{M}\sum_{n=1}^{N_m} I_{mn}\, e^{jkr_m[\sin\theta\cos(\varphi-\varphi_{mn})-\sin\theta_0\cos(\varphi_0-\varphi_{mn})]} \qquad (1)$$

where, M = Number of concentric rings, N_m = Number of isotropic elements in m-th ring, I_{mn} = Excitation amplitude of the mn-th element, d_m = inter-element arc spacing of m-th circle, $r_m = N_m d_m/2\pi$, Radius of the m-th ring, $\varphi_{mn} = 2n\pi/N_m$, angular position of mn-th element, with $1 \le n \le N_m$, θ, φ = polar, azimuth angle; (θ_0, φ_0) = steering angle, λ = wave length, k = wave number = $2\pi/\lambda$; j = complex number.

Normalized absolute power pattern, $P(\theta,\varphi)$ in dB can be expressed as follows:

$$P(\theta,\varphi) = 10\log_{10}\left[\frac{|E(\theta,\varphi)|}{|E(\theta,\varphi)|_{\max}}\right]^2 = 20\log_{10}\left[\frac{|E(\theta,\varphi)|}{|E(\theta,\varphi)|_{\max}}\right] \qquad (2)$$

In this case, I_{mn} is 1 if the mn-th element is turned 'on' and 0 if it is 'off'. To make the scanned array thinned with desired array characteristics optimum set of I_{mn} is necessary. The fitness function

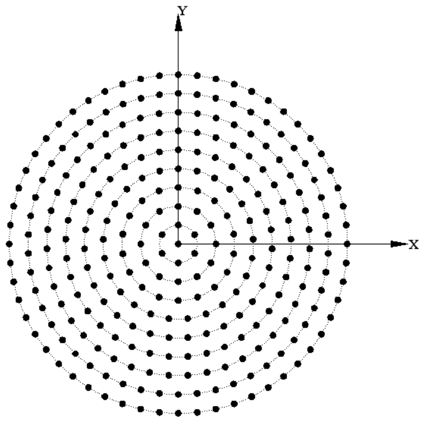

Figure 2. 9-ring concentric circular ring array of isotropic antennas.

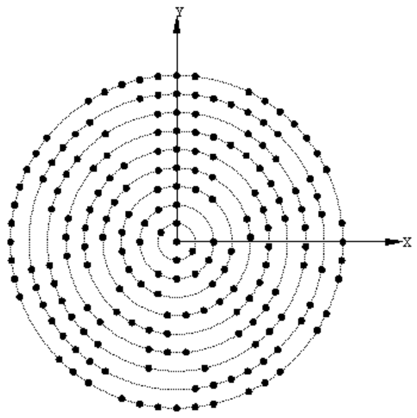

Figure 3. Thinned array of isotropic antennas with 9 concentric rings.

for this problem can be defined as:

$$\text{Fitness } 1 = k_1 \max \text{SLL} + k_2 (\text{FNBW}_o - \text{FNBW}_d) H(T) \quad (3)$$

$$\text{Fitness } 2 = \max \text{SLL} \quad (4)$$

Equation (3) and Equation (4) are reduced individually using GSA and modified PSO for optimal synthesis of the array, where max SLL is the value of maximum sidelobe level, FNBW_o, FNBW_d are obtained and desired value of first null beam width respectively, k_1, k_2 are weighting coefficients to control the relative importance given to each term of Equation (3). Equation (4) is for keeping FNBW variable.

$H(T)$ is Heaviside step functions defined as follows:

$$T = (\text{FNBW}_o - \text{FNBW}_d) \quad (5)$$

$$H(T) = \begin{cases} 0, & \text{if } T < 0, \\ 1, & \text{if } T \geq 0 \end{cases} \quad (6)$$

3. GRAVITATIONAL SEARCH ALGORITHM (GSA)

Gravitational Search Algorithm is a population based search algorithm based on the law of gravity and mass interaction. The algorithm considers agents as objects consisting of different masses. The entire agents move due to the gravitational attraction force acting between them and the progress of the algorithm directs the movements of all agents globally towards the agents with heavier masses. Each agent in GSA is specified by four parameters [10]: Position of the mass in d-th dimension, inertia mass, active gravitational mass and passive gravitational mass. The positions of the mass of an agent at specified dimensions represent a solution of the problem and the inertia mass of an agent reflect its resistance to make its movement slow. Both the gravitational mass and the inertial mass, which control the velocity of an agent in specified dimension, are computed by fitness evolution of the problem. The positions of the agents in specified dimensions (solutions) are updated with every iteration and the best fitness along with its corresponding agent is recorded. The termination condition of the algorithm is defined by a fixed amount of iterations, reaching which the algorithm automatically terminates. After termination of the algorithm, the recorded best fitness at final iteration becomes the global fitness for a particular problem and the positions of the mass at specified dimensions of the corresponding agent becomes the global solution of that problem.

The algorithm can be summarized as below:

Step 1: Initialization of the agents:

Initialize the positions of the N number of agents randomly within the given search interval as below:

$$X_i = (x_i^1, \ldots, x_i^d, \ldots, x_i^n), \quad \text{for } \ i = 1, 2, \ldots, N. \tag{7}$$

where, x_i^d represents the positions of the i-th agent in the d-th dimension and n is the space dimension.

Step 2: Fitness evolution and best fitness computation for each agents:

Perform the fitness evolution for all agents at each iteration and also compute the *best* and *worst* fitness at each iteration defined as below (for minimization problems):

$$best(t) = \min_{j \in \{1,\ldots,N\}} fit_j(t) \tag{8}$$

$$worst(t) = \max_{j \in \{1,\ldots,N\}} fit_j(t) \tag{9}$$

where, $fit_j(t)$ represents the fitness of the j-th agent at iteration t, $best(t)$ and $worst(t)$ represents the best and worst fitness at generation t.

Step 3: Compute gravitational constant G:

Compute gravitational constant G at iteration t using the following equation:

$$G(t) = G_0 e^{(-\alpha t/T)} \tag{10}$$

In this problem, G_0 is set to 100, α is set to 20 and T is the total number of iterations.

Step 4: Calculate the mass of the agents:

Calculate gravitational and inertia masses [10] for each agents at iteration t by the following equations:

$$M_{ai} = M_{pi} = M_{ii} = M_i, \quad i = 1, 2, \ldots, N. \tag{11}$$

$$m_i(t) = \frac{fit_i(t) - worst_i(t)}{best(t) - worst(t)} \tag{12}$$

$$M_i(t) = \frac{m_i(t)}{\sum_{j=1}^{N} m_j(t)} \tag{13}$$

where, M_{ai} is the active gravitational mass of the i-th agent [10], M_{pi} is the passive gravitational mass of the i-th agent [10], M_{ii} is the inertia mass of the i-th agent [10].

Step 5: Calculate accelerations of the agents:

Compute the acceleration of the i-th agents at iteration t as below:

$$a_i^d(t) = \frac{F_i^d(t)}{M_{ii}(t)} \tag{14}$$

where, $F_i^d(t)$ is the total force acting on i-th agent calculated as:

$$F_i^d(t) = \sum_{j \in \text{Kbest}, j \neq i} rand_j \, F_{ij}^d(t) \tag{15}$$

Kbest is the set of first K agents with the best fitness value and biggest mass. Kbest is computed in such a manner that it decreases linearly with time [10] and at last iteration the value of Kbest becomes 2% of the initial number of agents. $F_{ij}^d(t)$ is the force acting on agent 'i' from agent 'j' at d-th dimension and t-th iteration is computed as below:

$$F_{ij}^d(t) = G(t) \frac{M_{pi}(t) \times M_{aj}(t)}{R_{ij}(t) + \varepsilon} \left(x_j^d(t) - x_i^d(t) \right) \tag{16}$$

where, $R_{ij}(t)$ is the Euclidian distance between two agents 'i' and 'j' at iteration t and $G(t)$ is the computed gravitational constant at the same iteration. ε is a small constant.

Step 6: Update velocity and positions of the agents:

Compute velocity and the position of the agents at next iteration $(t+1)$ using the following equations:

$$v_i^d(t+1) = rand_i \times v_i^d(t) + a_i^d(t) \tag{17}$$

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1) \tag{18}$$

Step 7: Repeat from Steps 2–6 until iterations reaches their maximum limit. Return the best fitness computed at final iteration as a global fitness of the problem and the positions of the corresponding agent at specified dimensions as the global solution of that problem.

4. MODIFIED PARTICLE SWARM OPTIMIZATION ALGORITHM

Particle Swarm Optimization (PSO) is a population based stochastic optimization tool inspired by social behavior of bird flock, fish school etc. as developed by Kennedy and Eberhart in 1995 [12]. In PSO, a member in the swarm, called a *particle*, represents a potential solution, which is a point in the search space. The global optimum is regarded as the location of food. Each particle has a fitness value and a velocity to adjust its flying direction according to the best experiences of the swarm in search for the global optimum in the D-dimensional solution space. The steps involved in modified PSO are given below:

Step 1: Initialize positions and associate velocity to all particles (potential solutions) in the population randomly in the D-dimension space.

Step 2: Evaluate the fitness value of all particles.

Step 3: Compare the personal best (*pbest*) of every particle with its current fitness value. If the current fitness value is better, then assign the current fitness value to *pbest* and assign the current coordinates to *pbest* coordinates.

Step 4: Determine the current best fitness value in the whole population and its coordinates. If the current best fitness value is better than global best (*gbest*), then assign the current best fitness value to *gbest* and assign the current coordinates to *gbest* coordinates.

Step 5: Update velocity (V_{id}) and position (X_{id}) of the d-th dimension of the i-th particle using the following equations:

$$V_{id}^t = w(t) * V_{id}^{t-1} + c_1(t) * rand1_{id}^t * \left(pbest_{id}^{t-1} - X_{id}^{t-1}\right) + c_2(t) * \left(1 - rand1_{id}^t\right) * \left(gbest_d^{t-1} - X_{id}^{t-1}\right) \quad (19)$$

$$\text{If } V_{id}^t > V_{\max}^d \text{ or } V_{id}^t < V_{\min}^d, \text{ then } V_{id}^t = U(V_{\min}^d, V_{\max}^d) \quad (20)$$

$$X_{id}^t = rand2_{id}^t * X_{id}^{t-1} + (1 - rand2_{id}^t) * V_{id}^t \quad (21)$$

$c_1(t)$, $c_2(t)$ = time-varying acceleration coefficients with $c_1(t)$ decreasing linearly from 2.5 to 0.5 and $c_2(t)$ increasing linearly from 0.5 to 2.5 over the full range of the search, $w(t)$ = time-varying inertia weight changing randomly between $U(0.4, 0.9)$ with iterations, $rand1$, $rand2$ are uniform random numbers between 0 and 1, having different values in different dimension, t is the current generation number.

Equation (20) has been introduced to clamp the velocity along each dimension to uniformly distributed random value between

$V^d_{\min}$ and $V^d_{\max}$ if they try to cross the desired domain of interest. These clipping techniques are sometimes necessary to prevent particles from explosion. The maximum velocity is set to the upper limit of the dynamic range of the search ($V^d_{\max} = X^d_{\max}$) and the minimum velocity ($V^d_{\min}$) is set to ($X^d_{\min}$).
However, position-clipping technique is avoided in modified PSO algorithm. Moreover, the fitness function evaluations of errant particles (positions outside the domain of interest) are skipped to improve the speed of the algorithm.
Step 6: Repeat Steps 2–5 until a stop criterion is satisfied or a pre-specified number of iteration is completed, usually when there is no further update of best fitness value.

5. SIMULATION RESULTS

For a fully populated nine-ring concentric ring array of isotropic antennas [7] the radii of the rings are $r_m = m\lambda/2$ (m-th ring) and the interelement spacing of each ring are kept at $\lambda/2$. For this arrangement, the number of elements in the m-th ring is found out by rounding off the values of N_m expressed as $N_m = 2\pi r_m/d_m$ and the total numbers of isotropic elements becomes 279. A fully populated nine ring concentric ring array in the x-y plane having a total number of 279 isotropic elements with center element feeding is shown in Figure 2. The array with uniform excitation and without scanned to any direction gives the sidelobe levels of -17.4 dB [7] and FNBW of 14.8 degree. In this problem, the array is scanned in the x-z plan (θ_0, 0) and the scanning of the array becomes totally dependent on the values of θ_0. The values of θ_0 are taken 30 and 45 degrees for this problem. Fully populated array with uniform excitation and scanned to the direction $\theta_0 = 30$ degree, $\varphi_0 = 0$ degree gives sidelobe level of -17.4 dB and FNBW of 17.1 degree, whereas array with uniform excitation and scanned to the direction $\theta_0 = 45$ degree, $\varphi_0 = 0$ degree gives sidelobe levels of -17.4 dB and FNBW of 21.2 degree. The objective is to find out optimum set of amplitude distribution (on-off) of the array elements for the scanned array computed individually by GSA and modified PSO for getting lower sidelobe levels with fixed and variable FNBW. The array is then thinned in such a manner that the thinning percentage should always be more than 45% while keeping the desired array characteristics unchanged. The thinned array in the x-y plane is shown in Figure 3. Figure 4 shows normalized power patterns of uniformly excited broadside concentric ring array and scanned array to the direction $\theta_0 = 30$, $\varphi_0 = 0$ degrees. Figure 5 shows normalized power patterns of a uniformly excited broadside concentric ring array

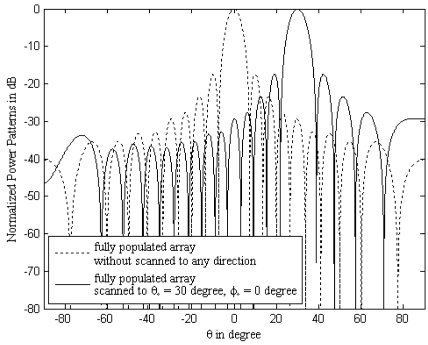

Figure 4. Normalized power patterns in dB in XZ plane for fully populated array without scanned to any direction and fully populated array scanned to the direction $\theta_0 = 30°$, $\varphi_0 = 0°$.

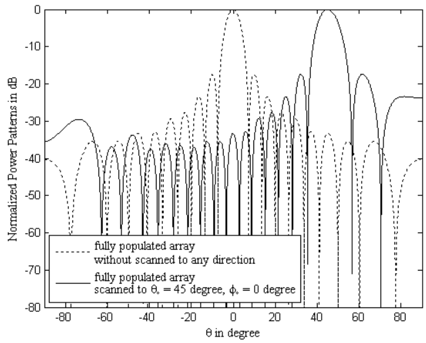

Figure 5. Normalized power patterns in dB in XZ plane for fully populated array without scanned to any direction and fully populated array scanned to the direction $\theta_0 = 45°$, $\varphi_0 = 0°$.

and scanned array to the direction $\theta_0 = 45$, $\varphi_0 = 0$ degrees.

Table 1 shows that the sidelobe levels in thinned array scanned in the x-z plane $(\theta_0, 0)$ for two different values of θ_0, 30 and 45 degree with fixed FNBW computed using GSA are −20.78 dB and −20.76 dB respectively, whereas sidelobe levels in the thinned array scanned to same angles with fixed FNBW computed using modified PSO are −20.38 dB and −20.50 dB respectively. For variable FNBW, sidelobe levels in the thinned array scanned to above mention angles computed using GSA are −29.97 dB and −31.29 dB respectively, whereas using modified PSO with same scanning angles the sidelobe levels are −24.83 dB and −24.92 dB respectively. Results clearly show that GSA can be able to reduce the sidelobe levels in a much better way than modified PSO. Table 1 also shows that the number of switched off elements in thinned array computed using GSA and scanned to 30 and 45 degree are 138 each for fixed FNBW and 129 each for variable FNBW. The numbers of switched off elements in the array thinned using modified PSO and scanned to same angles with fixed FNBW are 136 and 130 respectively. But for variable FNBW, the numbers of switched off elements are 127 and 126 respectively. Both the algorithms are able to fulfill the FNBW requirements while thinning the array scanned to same angles.

The percentages of thinning for the array scanned to above mention angles are 49.46% for fixed FNBW case and are 46.23% for variable FNBW case computed using GSA whereas using modified PSO

Table 1. SLL, FNBW, number of switched off elements and thinning percentage of uniform array and optimized array with and without fixed FNBW.

Types of array	Sidelobe Level (dB)	FNBW (degree)	Number of switched off elements	Thinning percentage (%)
Fully populated array (no scan)	−17.40	14.8	0	0
Fully populated array scan to 30°	−17.40	17.1	0	0
Fully populated array scan to 45°	−17.40	21.2	0	0
Thinned array of fixed FNBW scanned to 30° using GSA	−20.78	17.1	138	49.46
Thinned array of fixed FNBW scanned to 30° using modified PSO	−20.38	17.0	136	48.74
Thinned array of fixed FNBW scanned to 45° using GSA	−20.76	21.2	138	49.46
Thinned array of fixed FNBW scanned to 45° using modified PSO	−20.50	21.2	130	46.59
Thinned array of variable FNBW scanned to 30° using GSA	−29.97	23.2	129	46.23
Thinned array of variable FNBW scanned to 30° using modified PSO	−24.83	20.3	127	45.51
Thinned array of variable FNBW scanned to 45° using GSA	−31.29	29.3	129	46.23
Thinned array of variable FNBW scanned to 45° using modified PSO	−24.92	25.8	126	45.16

Table 2. Comparative performance of GSA and modified PSO.

Fitness functions	Types of array	GSA		Modified PSO	
		Best fitness	Time (hr : min)	Best fitness	Time (hr : min)
Fitness 1	Thinned array scanned to 30°	−20.7847	2:21	−20.3887	2:27
	Thinned array scanned to 45°	−20.7602	2:27	−20.5055	2.33
Fitness 2	Thinned array scanned to 30°	−29.9756	2:23	−24.8339	2:29
	Thinned array scanned to 45°	−31.2994	2:11	−24.9271	2:17

we get 48.74% and 46.54% thinned array with fixed FNBW and 45.51% and 45.16% with variable FNBW scanned to same angles. GSA thins the array more than modified PSO. From Table 2, we can see that the best fitness computed using fitness functions of Equation (3) and Equation (4) for the array scanned to $\theta_0 = 30$ and 45 degrees using GSA are better than the best fitness computed using modified PSO. Computation times are also less in case of GSA.

The excitation amplitude distributions for the thinned array of fixed FNBW scanned to above mention angles computed using GSA and modified PSO are shown in Table 3 and Table 4.

Table 5 and Table 6 shows the excitation amplitude distributions for the thinned array of variable FNBW scanned to above mention angles using GSA and modified PSO.

Both the algorithms are run for 400 iterations and number of agents in case of GSA is taken to be 50 and number of particles in case of modified PSO is taken to be 50. Figure 6 shows that the convergence rate of GSA is better than modified PSO for minimizing the cost while thinning the array scanned to $\theta_0 = 30$ degree, $\varphi_0 = 0$ degree keeping FNBW fixed. The normalized array factors for the thinned array scanned to $\theta_0 = 30$ degree, $\varphi_0 = 0$ degree with fixed FNBW computed individually using GSA and modified PSO are shown in Figure 7. Figure 8 again shows that the convergence rate of GSA is far better than modified PSO for reduction of the cost while thinning

Table 3. Excitation amplitude distribution (I_{mn}) of thinned array of fixed FNBW scanned to $\theta_0 = 30°$, $\varphi_0 = 0°$.

Algorithm	Ring number	Elements state in each ring (0 or 1)
GSA	1	101010
GSA	2	000011101010
GSA	3	110110100101101011
GSA	4	001001110111011011 0000001
GSA	5	100111101011001111000000001001011
GSA	6	011110111001001000010001101011 1010100
GSA	7	1001010101110100010011101110010111001001001
GSA	8	0111101010000000100101011011011001111011 0001000101
GSA	9	1111000110111000100001001101111101000110101100100100 11110
PSO	1	001010
PSO	2	110100101110
PSO	3	111001000101000000
PSO	4	001110100111000100111 1101
PSO	5	000110111110000000101100111001011
PSO	6	011000000101100010010001101100 00100100
PSO	7	0010100100101011111101100111101001111111111
PSO	8	0110001111100001101001101101001110111110110010 00000
PSO	9	1111010100001110101000011111110011000101110011000111 1011

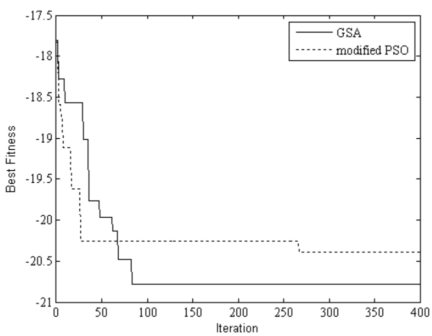

Figure 6. Convergence of GSA and modified PSO for minimization of cost while thinning the concentric ring array scanned to $\theta_0 = 30°$, $\varphi_0 = 0°$ with fixed FNBW.

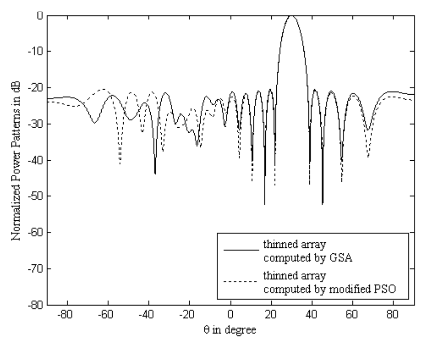

Figure 7. Normalized power patterns in dB in XZ plane for the thinned array scanned to $\theta_0 = 30°$, $\varphi_0 = 0°$ with fixed FNBW using GSA and modified PSO algorithms.

Table 4. Excitation amplitude distribution (I_{mn}) of thinned array of fixed FNBW scanned to $\theta_0 = 45°$, $\varphi_0 = 0°$.

Algorithm	Ring number	Elements state in each ring (0 or 1)
GSA	1	010101
	2	000110011110
	3	011101000110101111
	4	010000001110101100000l000
	5	000001111101110001010111101001l
	6	1100110110101010001110111110010011100
	7	110011111010001000000000100100100000010010010
	8	100101000101000010000100101101000110110000001110011
	9	110001100110111000110110111111110111111010111111l0001111
PSO	1	111101
	2	100011001111
	3	001011010101001010
	4	011011001001111100001000l
	5	1001000011010000111111110100000
	6	1111111101010111100011000111000010001
	7	0000001110011011001010000000110110101010001
	8	11000110101011100011101111001000010011110001110010
	9	1110100101101000111110101111111011111100101000111101111

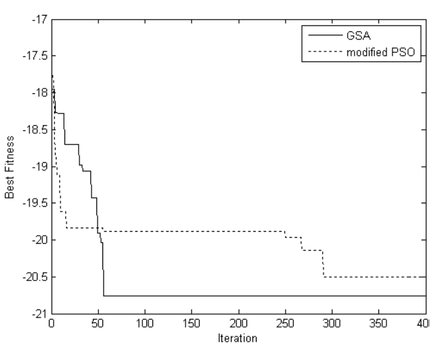

Figure 8. Convergence of GSA and modified PSO for minimization of cost while thinning the concentric ring array scanned to $\theta_0 = 45°$, $\varphi_0 = 0°$ with fixed FNBW.

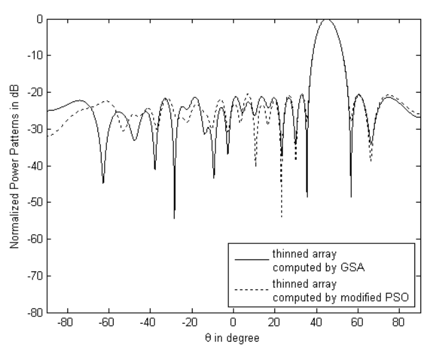

Figure 9. Normalized power patterns in dB in XZ plane for the thinned array scanned to $\theta_0 = 45°$, $\varphi_0 = 0°$ with fixed FNBW using GSA and modified PSO algorithms.

Table 5. Excitation amplitude distribution (I_{mn}) of thinned array of variable FNBW scanned to $\theta_0 = 30°$, $\varphi_0 = 0°$.

Ring number	GSA: Elements state in each ring (0 or 1)	PSO: Elements state in each ring (0 or 1)
1	111111	101001
2	100001101010	111010110001
3	011100111011111010	011110110101010111
4	011111110011101011001111	000001111110011111110111
5	001110101101010010011111001010	010011000111100001100111111010
6	011010110111100101001010111111011001	101000010110010111010111011110110000
7	000100110101010011011000001100010010100001 0	100101111001111111001100101101111111111000
8	011000110111111010000000101100100101001111111110001	000000110100100100011010101000010001111011001011 00
9	000100001011010110110010100100000111111011100001110 00101	001101001010011000010101010101001110011100101011 01110010

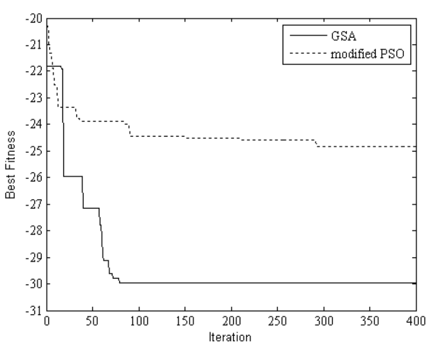

Figure 10. Convergence of GSA and modified PSO for minimization of cost while thinning the concentric ring array scanned to $\theta_0 = 30°$, $\varphi_0 = 0°$ without fixing FNBW.

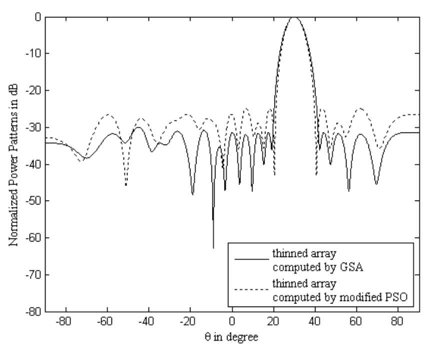

Figure 11. Normalized power patterns in dB in XZ plane for thinned the array scanned to $\theta_0 = 30°$, $\varphi_0 = 0°$ with variable FNBW using GSA and modified PSO algorithms.

Table 6. Excitation amplitude distribution (I_{mn}) of thinned array of variable FNBW scanned to $\theta_0 = 45°$, $\varphi_0 = 0°$.

Ring number	GSA: Elements state in each ring (0 or 1)	PSO: Elements state in each ring (0 or 1)
1	111100	101011
2	110111101111	101100110111
3	110010110011011001	111001101101101011
4	111111111101111100001011	001011111101011010101010111
5	101011101000011000101111111101	101000011111000101001001111110111
6	110101101011010000000101001111001101	111001111101010101100001110101000 1000
7	011100000101010101001010011111111110001000	010101101110011100100101011101011010101010 1
8	000100100011110010100010000000100011101110000 01011	011001000000010111111000011111001001011010100 000011
9	000001111111011011010001001100010111101001100 1000101 1010	001101100111111111111010010110000010011111010 1011000000 1001

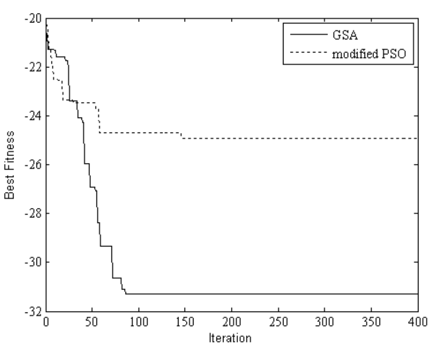

Figure 12. Convergence of GSA and modified PSO for minimization of cost while thinning the concentric ring array scanned to $\theta_0 = 45°$, $\varphi_0 = 0°$ without fixing FNBW.

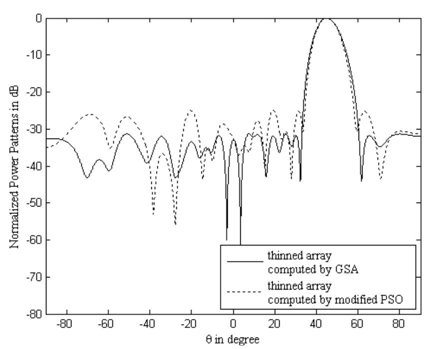

Figure 13. Normalized power patterns in dB in XZ plane for thinned the array scanned to $\theta_0 = 45°$, $\varphi_0 = 0°$ with variable FNBW using GSA and modified PSO algorithms.

the array scanned to $\theta_0 = 45$ degree, $\varphi_0 = 0$ degree keeping FNBW fixed.

Figure 9 shows the normalized array factors for the thinned array scanned to $\theta_0 = 45$ degree, $\varphi_0 = 0$ degree with fixed FNBW computed individually using GSA and modified PSO. Figure 10 shows the convergence of GSA and modified PSO for minimizing the cost while thinning the array scanned to $\theta_0 = 30$ degree, $\varphi_0 = 0$ degree for variable FNBW. The normalized array factors for the thinned array scanned to $\theta_0 = 30$ degree, $\varphi_0 = 0$ degree with variable FNBW computed individually using GSA and modified PSO are shown in Figure 11. Figure 12 again shows that the convergence rate of GSA is far better than modified PSO for reduction of the cost while thinning the array scanned to $\theta_0 = 45$ degree, $\varphi_0 = 0$ degree without fixing FNBW. Figure 13 shows the normalized absolute array factors for the array with variable FNBW scanned to $\theta_0 = 45$ degree, $\varphi_0 = 0$ degree computed individually using GSA and modified PSO.

6. CONCLUSIONS

The authors propose methods of thinning a large scanned concentric ring array of isotropic elements to reduce sidelobe level while retaining desired array characteristics. Here Gravitational Search Algorithm and modified Particle Swarm Optimization (PSO) have been effectively used as a global optimization algorithm to find out optimal set of on-off elements. The comparative performance of GSA is shown better in terms of computed final fitness values, computational time etc. than modified PSO algorithm. Both the algorithms can also be used for thinning other array configurations.

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