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ELECTROMAGNETIC SCATTERING FROM A CHIRAL-COATED NIHILITY CYLINDER

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Abstract—Scattering of electromagnetic plane waves from an infinitely long nihility cylinder, coated with a chiral layer of uniform thickness, is presented. Cylindrical vector wave functions have been used to express the fields in different regions. The solution is determined by solving the wave equation for different regions and applying the appropriate boundary conditions at the discontinuities. Both TM and TE polarizations as incident plane wave have been considered in the analysis. Obtained Numerical results for the chiral-coated nihility cylinder are compared with a chiral-coated PEC cylinder.

1. INTRODUCTION

Electromagnetic scattering from chiral material has attracted many scientists and engineers in the last few decades [1–12]. A linearly polarized wave propagating in a chiral material undergoes a rotation of its polarization showing that the chiral media is an optically active media. Hence, TE and TM waves scattered by a chiral cylinder are coupled. Jaggard et al. studied behavior of electromagnetic waves in chiral media [1]. Engheta and Bassiri explained One- and two-dimensional dyadic Greens functions in chiral media [2]. Electromagnetic chirality and its applications has been discussed by Engheta and Jaggard [3]. Lakhtakia et al. discussed, Field equations, Huygens principle, integral equations, and theorems for radiation and scattering of electromagnetic waves in isotropic chiral media [4]. Kluskens and Newman presented the numerical results of computing the electromagnetic field scattered by a two-dimensional

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lossy and homogeneous chiral cylinder of arbitrary cross section [5]. Eigenfunction solution of the problem of electromagnetic scattering from circular chiral cylinders has been presented [6]. Electromagnetic Scattering from Chiral Cylinders of Arbitrary Cross Section has been studied by Kanhal [7]. Arvas and Alkanhal presented scattering behavior of electromagnetic from chiral cylinder of arbitrary cross-section [8]. MAl-Kanhal and Arvas presented simple integral equation and MoM solution to the problem of TE and TM scattering from a lossy homogeneous chiral cylinder of chiral cylinder of arbitrary cross-section [9].

Electromagnetic trinity from "negative permittivity" and "negative permeability" has been discussed by Lakhtakia [10, 11]. Nihility medium is a medium in which both relative permittivity and permeability are null-valued [10]. Due to this fact the medium does not allow the electromagnetic energy to propagate in it. Under nihility condition Maxwell equations reduce to form given below

$$\nabla \times \mathbf{E} = 0$$
$$\nabla \times \mathbf{H} = 0$$

The idea of nihility medium given by Lakhtakia is very attractive for researchers engaged in studying problem related to electromagnetics [12–22]. Scattering of electromagnetic waves from a nihility circular has been presented by Lakhtakia and Goddes [12]. Lakhtakia discussed the perfect lenses and nihility [13]. Orthorhombic materials and perfect lenses were analysed by Lakhtakia and Sherwin [14]. Ziolkowski [15] discussed the propagation in and scattering from a matched metamaterial having a zero index of refraction. Lakhtakia and Mackay found the Fresnel coefficients for a permittivity-permeability phase space encompassing vacuum, anti-vacuum, and nihility [16]. Tretyakov et al. presented the idea of the electromagnetic waves and energy in chiral nihility [17]. Cheng et al. discussed waves in planar waveguide containing chiral nihility metamaterial [18]. Naqvi [19] presented the characteristics of electromagnetic waves at planer slab of chiral nihility metamaterial backed by fractional dual/PEMC interface. Fractional Dual Solutions to Maxwell Equations in Chiral Nihility Medium have been given by Naqvi [20]. Ahmed and Naqvi [21] discussed directive EM radiation of a line source in the presence of a coated nihility cylinder. Scattering of electromagnetic waves from a nihility circular cylinder coated with a metamaterial has been discussed by Ahmed and Naqvi [22].

The purpose of this paper is to study scattering of electromagnetic waves from an infinite nihility cylinder of circular cross-section, coated with homogeneous, isotropic and linear chiral material. For simplicity, we have considered the coating layer of uniform thickness. Both the parallel and perpendicular polarization cases of the incident plane wave are discussed in the analysis. Eigenfunction expansion method has been used in the theoretical study. Geometry of the problem is divided in three regions, i.e., free space, chiral coating layer, and the core dielectric cylinder. Appropriate boundary conditions are applied at the interfaces. The monostatic and bistatic echo widths in the far-zone, are calculated by using the large argument approximation of Hankel function. The numerical results are compared with the published work for the special cases for the verification of the analytical formulation and the numerical code. Here $e^{+j\omega t}$ time dependence is used and has been suppressed throughout the analysis.

2. FORMULATION

The geometry of the scattering problem is shown in Fig. 1, which contains a dielectric circular cylinder which has been coated by a homogeneous layer of chiral media. Three regions have been specified, region $\rho > b$ is free space modeled by wave number $k_0 = \omega\sqrt{\mu_0\epsilon_0}$, region $a \leq \rho \leq b$ is the chiral layer characterized by $k_c = \omega\sqrt{\mu_c\epsilon_c}$ while the core cylinder is a dielectric cylinder with $k_d = \omega\sqrt{\mu_d\epsilon_d}$. Where μ and ϵ are the permeability and permittivity of the three regions and subscript '0', 'c' and 'd' represent free space, chiral medium and dielectric medium respectively. Due to different phase velocities for right-hand circularly polarized waves (RCP) and left-hand circularly polarized waves (LCP), the coating layer of chiral medium is characterized by with two different bulk wave numbers k_+ and k_- which are given by [23]

$$k_\pm = k_c\sqrt{1 + x^2} \pm x$$

where $x = \sqrt{\mu_c/\epsilon_c}\xi_c$, ξ_c is the chirality of the layer [10].

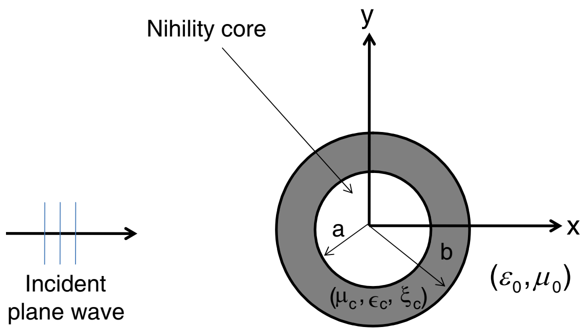

Figure 1. Chiral-coated infinite nihility circular cylinder.

The fields scattered in free space, i.e., in region I are expanded in terms of cylindrical vector wave functions as [24]

$$E^{\mathrm{I}} = E_0 \sum_{n=-\infty}^{\infty} j^{-n} \left[a_n N_n^{(4)}(k_0\rho) + b_n M_n^{(4)}(k_0\rho)\right] \tag{1}$$

$$H^{\mathrm{I}} = -\frac{1}{j\eta_0} E_0 \sum_{n=-\infty}^{\infty} j^{-n} \left[a_n M_n^{(4)}(k_0\rho) + b_n N_n^{(4)}(k_0\rho)\right] \tag{2}$$

Now, the fields inside the coating layer which is a chiral material with parameters $(\mu_c, \epsilon_c, \xi_c)$ in terms of left and right circularly polarized waves are expressed as

$$E^{\mathrm{II}} = E_0 \sum_{n=-\infty}^{\infty} j^{-n} \left[c_n E_{(+,n)}^{(1)} + d_n E_{(-,n)}^{(1)} + e_n E_{(+,n)}^{(2)} + f_n E_{(-,n)}^{(2)}\right] \tag{3}$$

$$H^{\mathrm{II}} = -\frac{1}{j\eta_0} E_0 \sum_{n=-\infty}^{\infty} j^{-n} \left[c_n E_{(+,n)}^{(1)} - d_n E_{(-,n)}^{(1)} + e_n E_{(+,n)}^{(2)} - f_n E_{(-,n)}^{(2)}\right] \tag{4}$$

And the fields transmitted into the center dielectric cylinder are given in the form

$$E^{\mathrm{III}} = E_0 \sum_{n=-\infty}^{\infty} j^{-n} \left[g_n E_{(+,n)}^{(1)} + h_n E_{(-,n)}^{(1)}\right] \tag{5}$$

$$H^{\mathrm{III}} = -\frac{1}{j\eta_0} E_0 \sum_{n=-\infty}^{\infty} j^{-n} \left[g_n E_{(+,n)}^{(1)} + h_n E_{(-,n)}^{(1)}\right] \tag{6}$$

where

$$E_{(+,n)}^{(p)} = M_n^{(p)}(k_+) + N_n^{(p)}(k_+) \tag{7}$$

$$E_{(-,n)}^{(p)} = M_n^{(p)}(k_-) - N_n^{(p)}(k_-) \tag{8}$$

In the above field, expressions a_n, b_n, c_n, d_n, e_n, f_n, g_n, and h_n are the unknown scattering coefficients which may be calculated by application of boundary conditions. The boundary conditions at the interfaces $\rho = b$ are listed below:

$$E_z^{\mathrm{inc}} + E_z^{\mathrm{I}} = E_z^{\mathrm{II}}, \qquad \rho = b, \quad 0 \le \phi \le 2\pi \tag{9}$$

$$H_\phi^{\mathrm{inc}} + H_\phi^{\mathrm{I}} = H_\phi^{\mathrm{II}}, \qquad \rho = b, \quad 0 \le \phi \le 2\pi \tag{10}$$

$$E_\phi^{\mathrm{I}} = E_\phi^{\mathrm{II}}, \qquad \rho = b, \quad 0 \le \phi \le 2\pi \tag{11}$$

$$H_z^{\mathrm{I}} = H_z^{\mathrm{II}}, \qquad \rho = b, \quad 0 \le \phi \le 2\pi \tag{12}$$

while the boundary conditions at the interface $\rho = a$ are

$$E_z^{\mathrm{II}} = E_z^{\mathrm{III}}, \qquad \rho = a, \quad 0 \le \phi \le 2\pi \tag{13}$$

$$H_\phi^{\rm II} = H_\phi^{\rm III}, \qquad \rho = a, \quad 0 \le \phi \le 2\pi \tag{14}$$
$$E_\phi^{\rm II} = E_\phi^{\rm III}, \qquad \rho = a, \quad 0 \le \phi \le 2\pi \tag{15}$$
$$H_Z^{\rm II} = H_z^{\rm III}, \qquad \rho = a, \quad 0 \le \phi \le 2\pi \tag{16}$$

The incident fields mentioned in Equations (9) and (10), on the coated geometry may be a TM-polarized or TE-polarized. In case of TM-polarization the incident fields may be expressed as

$$E^{\rm inc} = E_0 \sum_{n=-\infty}^{\infty} j^{-n} N_n^{(1)}(k_0\rho) \tag{17}$$
$$H^{\rm inc} = -\frac{1}{j\eta_0} E_0 \sum_{n=-\infty}^{\infty} j^{-n} M_n^{(1)}(k_0\rho) \tag{18}$$

while in TE-polarization case, the incident fields may be written as

$$E^{\rm inc} = E_0 \sum_{n=-\infty}^{\infty} j^{-n} M_n^{(1)}(k_0\rho) \tag{19}$$
$$H^{\rm inc} = -\frac{1}{j\eta_0} E_0 \sum_{n=-\infty}^{\infty} j^{-n} N_n^{(1)}(k_0\rho) \tag{20}$$

Solution of boundary conditions (9) to (16) using the incident (TM or TE case) and the scattered field expressions, gives us the unknown scattering coefficients. After finding a_n and b_n, the both the relative permittivity and permeability of the core cylinder are taken to be null-valued, $\epsilon_{2r} = \mu_{2r} = 0$, for the nihility case [2]. Applying this condition to the scattering coefficients a_n and b_n, the scattering characteristics of a chiral-coated nihility circular cylinder are obtained. The co- and cross-polarized components of the normalized far-zone back scattering cross-section are given as

$$\sigma_{\rm co}/\lambda_0 = \frac{2}{\pi}\left|\sum_{n=-\infty}^{\infty} a_n e^{jn\phi}\right|^2 \tag{21}$$
$$\sigma_{\rm cross}/\lambda_0 = \frac{2}{\pi}\left|\sum_{n=-\infty}^{\infty} b_n e^{jn\phi}\right|^2 \tag{22}$$

3. SIMULATIONS

This section presents some plots to study scattering behavior of a chiral-coated nihility cylinder. We have compared the results of chiral-coated nihility cylinder with those for a chiral-coated PEC cylinder.

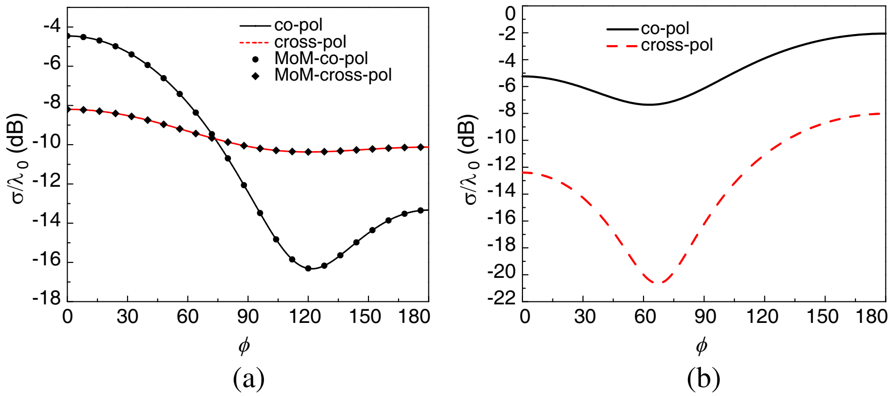

Figure 2. (a) Co- and cross-pol components of bistatic echo width of a circular chiral cylinder ($\mu_c = 4$, $\varepsilon_c = 1.5$, $\xi_c = 0.0005$, $a = 0.1\,\text{m}$, $\lambda_0 = 1\,\text{m}$). (b) Co- and cross-pol components of bistatic echo width of a circular chiral cylinder ($\mu_c = -4$, $\varepsilon_c = -1.5$, $\xi_c = 0.0005$, $a = 0.1\,\text{m}$, $\lambda_0 = 1\,\text{m}$).

Fig. 2 shows the far-zone bistatic echo width of an un-coated chiral circular cylinder. Fig. 2(a) presents the comparison of co- and cross-polarized components of the bistatic echo width thus obtained with the results obtained by MoM [7]. In this figure, we have taken $a = 0.1\,\text{m}$, $\mu_c = 4$, $\epsilon_c = 1.5$ and $\xi_c = 0.0005$, which shows the correctness of our formulation and the numerical code. In Fig. 2(b), the response of the chiral cylinder has been observed for the negative permittivity and permeability of the coating layer, i.e., $\mu_c = -4$, $\epsilon_c = -1.5$ and $\xi_c = 0.0005$ while $a = 0.1\,\text{m}$. Fig. 2(b) shows the effect of the negative permittivity and permeability on the forward and backward scattering widths of the chiral cylinder.

Figures 3 and 4 are reserved for the bistatic echo widths due to chiral-coated nihility circular cylinder. In these figures, the co- and cross-polarized components of bistatic echo width of chiral-coated nihility cylinder have been compared with those of chiral-coated PEC cylinder. Also both the TM and TE polarizations have been discussed in these plots and here we have taken $a = 0.5\,\text{m}$, $b = 1.0\,\text{m}$, and $\lambda_0 = 1\,\text{m}$. Figs. 3(a) and 3(b), presents the co- and cross-polarized components of bistatic echo width, respectively, when $\mu_c = 2$, $\epsilon_c = 3$ and $\xi_c = 0.002$. It is observed from Fig. 3(a) that, co-polarized component of chiral coated nihility and PEC cores are almost exactly the same in case of TE polarization. While these show different behavior for the case of TM polarization. And the situation is opposite in case of cross-polarization case as shown in Fig. 3(b), i.e., here in

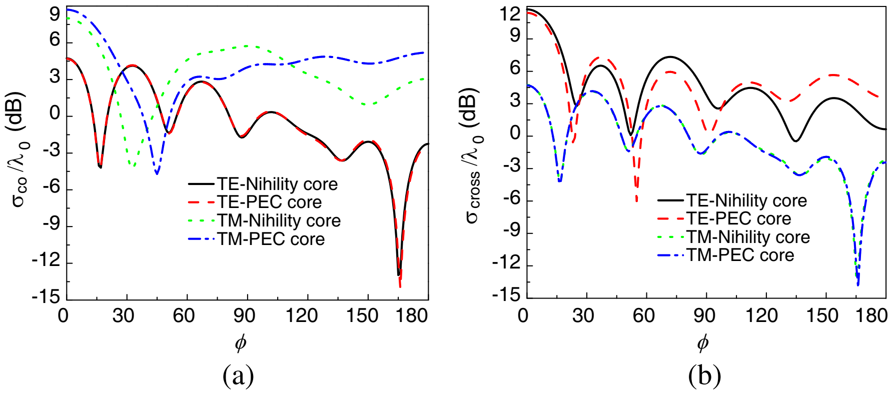

Figure 3. (a) Co-pol component of bistatic echo width of a chiral-coated circular cylinder ($\mu_c = 2$, $\varepsilon_c = 3$, $\xi_c = 0.002$, $a = 0.5\,\text{m}$, $b = 1.0\,\text{m}$, $\lambda_0 = 1\,\text{m}$). (b) Cross-pol component of bistatic echo width of a chiral-coated circular cylinder ($\mu_c = 2$, $\varepsilon_c = 3$, $\xi_c = 0.002$, $a = 0.5\,\text{m}$, $b = 1.0\,\text{m}$, $\lambda_0 = 1\,\text{m}$).

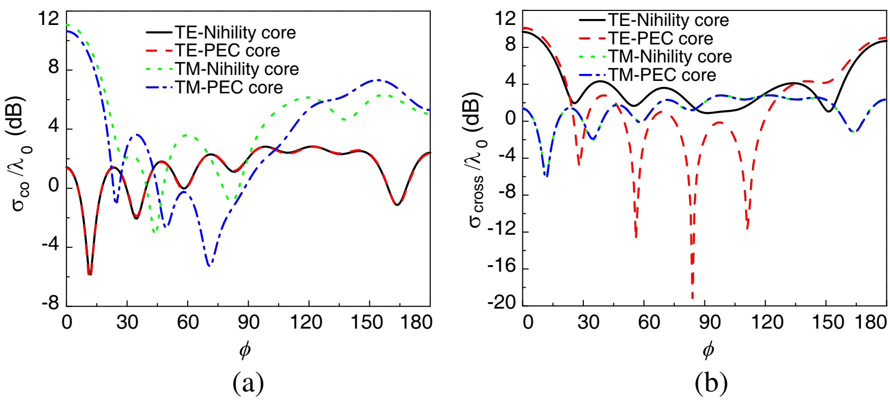

Figure 4. (a) Co-pol component of bistatic echo width of a chiral-coated circular cylinder ($\mu_c = -2$, $\varepsilon_c = -3$, $\xi_c = 0.002$, $a = 0.5\,\text{m}$, $b = 1.0\,\text{m}$, $\lambda_0 = 1\,\text{m}$). (b) Cross-pol component of bistatic echo width of a chiral-coated circular cylinder ($\mu_c = -2$, $\varepsilon_c = -3$, $\xi_c = 0.002$, $a = 0.5\,\text{m}$, $b = 1.0\,\text{m}$, $\lambda_0 = 1\,\text{m}$).

case of TM polarization the two geometries show similar behavior and response is different in case of TE polarization. Similarly Fig. 4 exhibits the co- and cross-polarized components of the bistatic echo width chiral-coated nihility and PEC cores when $\mu_c = -2$, $\epsilon_c = -3$ and $\xi_c = 0.002$. Again, the co-polarized components of the two

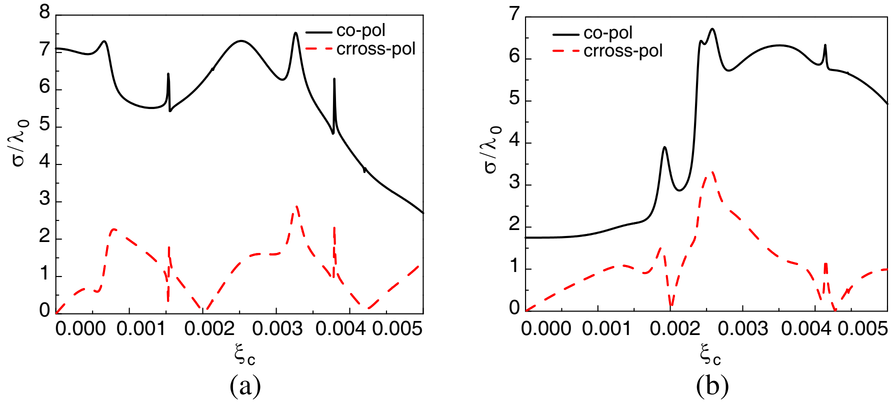

Figure 5. (a) Co and cross-polarized components of monostatic echo width versus variation in chirality ($a = 0.15\lambda_0$, $b = 0.3\lambda_0$, $\varepsilon_c = 3$, $\mu_c = 2$, $\lambda_0 = 1\,\text{m}$). (a) Co and cross-polarized components of monostatic echo width versus variation in chirality ($a = 0.15\lambda_0$, $b = 0.3\lambda_0$, $\varepsilon_c = -3$, $\mu_c = -2$, $\lambda_0 = 1\,\text{m}$).

configurations are similar in case of TE polarization and different for TM polarization. And cross-polarized component of bistatic echo width shows same response for TM polarization while different for TE polarization.

Figure 5 shows the co- and cross-polarized components of the monostatic echo width of the chiral-coated nihility cylinder versus the chirality parameter when $a = 0.15\,\text{m}$, $b = 0.3\,\text{m}$, and $\lambda_0 = 1\,\text{m}$. Fig. 5(a) presents the behavior of monostatic echo width when $\mu_c = 2$, $\epsilon_c = 3$ while Fig. 5(b) shows the behavior when $\mu_c = -2$, $\epsilon_c = -3$.

4. CONCLUSION

It has been observed that by choosing negative permittivity and permeability of the chiral cylinder in free space the co- and cross-polarized components of forward and backward echo widths may be altered as obvious from Fig. 2. Moreover, it is seen that the co-polarized component of bistatic echo width of the chiral-coated PEC and nihility cores are exactly the same in case of TE polarization while it is different in the case of TM polarization. And the cross-polarized component of bistatic echo width of the chiral-coated PEC and nihility cores are exactly the same in case of TM polarization while it is different in the case of TE polarization. Analysis shows that co-polarized component of the monostatic echo width increases from minimum to maximum with the increase in the chirality when negative permittivity and permeability is chosen, which is opposite to the case when it decreases from a maximum to minimum for positive permittivity and permeability of the coating chiral layer.

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