Progress In Electromagnetics Research Letters, Vol. 17, 85–93, 2010

INTERACTION OF NONLINEAR PULSES DEVELOPED IN COUPLED TRANSMISSION LINES REGULARLY SPACED SCHOTTKY VARACTORS

K. Narahara

Graduate School of Science and Engineering
Yamagata University
4-3-16 Jonan, Yonezawa, Yamagata 992-8510, Japan

Abstract—We numerically investigate the interaction of nonlinear pulses in coupled transmission lines with regularly spaced Schottky varactors. The c mode and π mode are two different propagation modes that can be developed on a coupled line. Recently, we have found that both modes can support soliton-like pulses due to the presence of the Schottky varactors and proposed a method of doubling repetition rate of the incident pulse stream. Through numerical evaluations, we find that small c-mode pulses are generated by colliding two π-mode pulses traveling in the opposite directions. Utilizing this unique property, the repetition rate of incident pulse stream can be increased by the factor greater than 2.

1. INTRODUCTION

A transmission line periodically loaded with Schottky varactors is called a nonlinear transmission line (NLTL) [1]. The operation bandwidth of carefully designed Schottky varactors goes beyond 100 GHz; therefore, they are employed in ultrafast electronic circuits including the subpicosecond electrical shock generator [2]. Moreover, several NLTL-based methods to manage short electrical pulses [3, 4] have been reported.

Recently, we consider the weakly dispersive coupled NLTLs in order to develop baseband pulses governed by the Korteweg de-Vries equation and propose a method of doubling the repetition rate of pulse stream input to the line. It is well-known that the c and π modes are two different propagation modes on a linear coupled line [5]. By

Received 26 July 2010, Accepted 23 August 2010, Scheduled 25 August 2010
Corresponding author: K. Narahara (narahara@yz.yamagata-u.ac.jp).

introducing Schottky varactors, a pulse traveling along a coupled NLTL preserves its original shape because of the balance between dispersion and nonlinearity, regardless of the propagation mode. When a pulse stream with a period of T is input to a coupled NLTL, each pulse is split to the c- and π-mode pulses. Generally, the c-mode pulse is faster than the π-mode pulse. Thus, the temporal separation between the c- and π-mode pulses becomes equal to $T/2$ by setting the length of a coupled NLTL properly. As a result, the repetition rate of a pulse stream can be doubled [6].

In order to examine the properties of interacting nonlinear pulses in a coupled NLTL, we numerically solve the transmission equations and find that the collision of two π-mode nonlinear pulses leads to the development of a pair of c-mode pulses (one travels forward, and the other travels backward). By proper terminations of a coupled NLTL, an incident π-mode pulse stream is reflected at the far end. The reflected π-mode pulse stream collides with an incident one repeatedly. As a result, c-mode pulses are newly generated at the collision points. Designing the line to guarantee the in-phase superposition of newly generated pulses, the c-mode pulses gain amplitude. By detecting these c-mode pulses together with original π-mode ones, we succeed in increasing the repetition rate of incident pulse stream by the factor greater than 2.

We first discuss the fundamental aspects of interacting pulses in a coupled NLTL including the circuit configuration, time-domain calculations that describe the collisions of two c-mode pulses and those of two π-mode pulses. We then propose a method of increasing the repetition rate of incident pulse stream by the factor greater than 2 with several numerical demonstrations.

2. INTERACTION OF NONLINEAR PULSES IN COUPLED NLTLS

Figure 1 shows the diagram of a unit cell of a coupled NLTL. Two NLTLs denoted by line 1 and line 2 are coupled via C_m and L_m. For line i ($i = 1, 2$), L_i, R_i and C_i represent the series inductor, series resistor, and shunt Schottky varactor of the unit cell, respectively. Lines 1 and 2 are biased at V_0 and W_0, respectively. The capacitance-voltage relationship of a Schottky varactor is generally given by

$$C(x) = \frac{C_0}{\left(1 - \frac{x}{V_J}\right)^m}, \tag{1}$$

where x is the voltage between the terminals. Moreover, C_0, V_J, and m are the zero-bias junction capacitance, junction potential, and

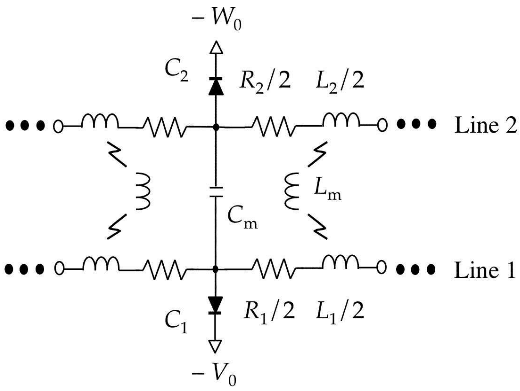

Figure 1. The unit cell of a coupled NLTL.

grading coefficient, respectively. The c and π modes are two different propagation modes that can be developed on a linear coupled line [5]. Each mode has its own velocity and voltage fraction between the lines (= line 2 voltage/line 1 voltage). Generally, the short-wavelength waves travel slower than the long-wavelength waves due to dispersion; this results in distortions of the baseband pulses having short temporal durations. By introducing Schottky varactors, this distortion can be compensated for by nonlinearity, regardless of the propagation mode. It has been found that the voltage fractions of the c- and π-mode nonlinear pulses are coincident with those of the linear c and π modes, respectively [6].

In order to investigate the properties of interacting nonlinear pulses, we numerically solve the transmission equations using a standard finite-difference time-domain method for a coupled NLTL with Schottky varactors having $C_0 = 0.1\,\mathrm{pF}$, $V_J = 0.7\,\mathrm{V}$ and $m = 0.5$. We set L_1, L_2, C_m, L_m, V_0, and W_0 to 0.1 nH, 0.12 nH, 0.1 pF, 0.0 nH, −0.3 V, and −3.0 V, respectively. For these line parameters, the c- and π-mode voltage fractions are calculated to be 1.15, and −1.04, respectively. Moreover, the c-mode pulse travels almost twice as fast as the π-mode pulse. The total cell size is set to 1000. To examine the properties of the colliding c- (π-) mode nonlinear pulses, the pulses having the identical c- (π-) mode one-soliton waveforms with an amplitude of 0.6 V (1.2 V) are input to both ends of a coupled NLTL. Figs. 2(a) and 2(b) show the collisions of the c- and π-mode pulses, respectively. The black and grey waveforms show the pulses on lines 1 and 2. The spatial waveforms monitored at six different temporal points with an increment of 0.22 ns are shown. It is observed

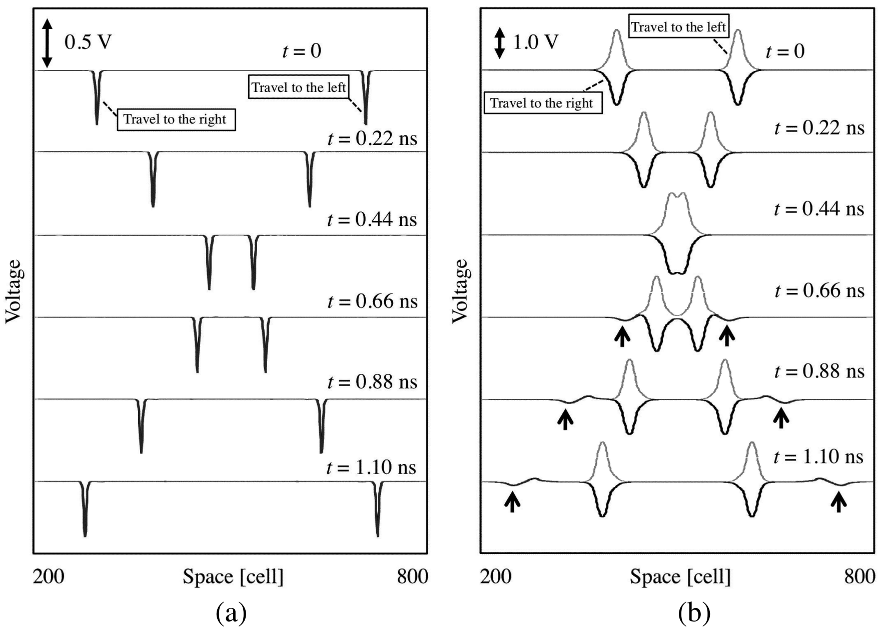

Figure 2. Interaction of two nonlinear pulses on a coupled NLTL. (a) The colliding c-mode nonlinear pulses and (b) the colliding π-mode nonlinear pulses. The arrows in Fig. 2(b) indicate the c-mode pulses generated by colliding two π-mode pulses traveling in the opposite directions.

that a pair of c- (π-) mode pulses develop when two π- (c-) mode pulses traveling in the opposite directions collide. Although a pair of π-mode pulses really develop after collision of c-mode pulses, their amplitudes are so small that we cannot identify them in Fig. 2(a). On the other hand, as indicated by the arrows in Fig. 2(b), small c-mode pulses develop after collision of two π-mode nonlinear pulses; these new c-mode pulses travel faster than the original π-mode pulses. In a linear coupled line, the c- and π-mode waves are independent, i.e., the c-mode wave never develops by the interaction of the π-mode waves and vice versa. The development of small c-mode pulses resulting from the collision of π-mode pulses is a unique property of a nonlinear coupled NLTL. Although L_m was set to zero, the generation of c-mode pulses can be observed even with a finite mutual inductance.

3. INCREASING REPETITION RATE OF INCIDENT PULSE STREAM

To operate a coupled NLTL as the platform to increase the repetition rate of incident pulse stream, one end of each line is connected with the transmission lines having the π-mode characteristic impedances [$Z_{\pi 1,\pi 2}$ in Fig. 3(a)], while the other end of each line is connected with the ones having the c-mode characteristic impedances [$Z_{c1,c2}$ in Fig. 3(a)]. Each transmission line is terminated with the corresponding matched resistance. By this arrangement, the multiple reflections of the waves

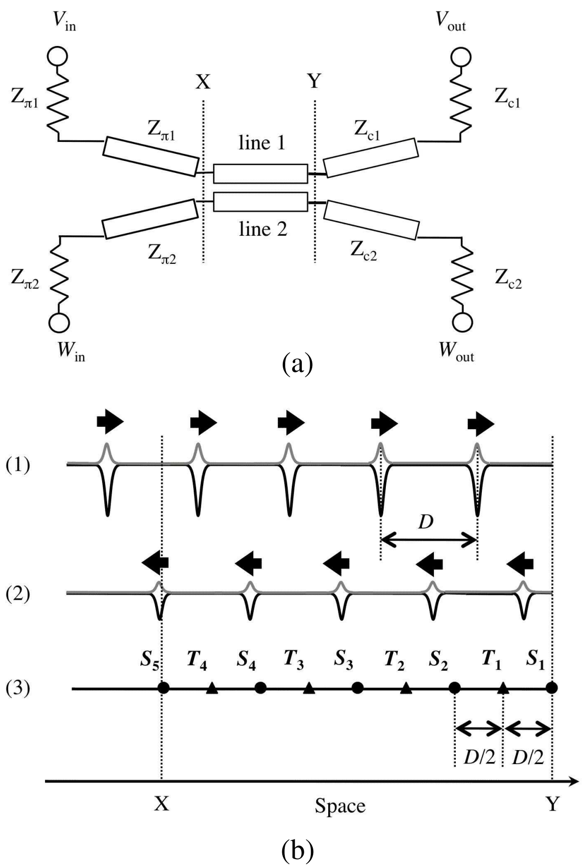

Figure 3. The method for increasing the repetition rate of incident pulse stream using coupled NLTLs. (a) The circuit configuration and (b) the operating principle.

carried by both the c and the π modes are suppressed; therefore, the outputs are free from the distortions caused by the reflections [6]. We consider the case where a π-mode pulse stream with a period of T is input to V_{in} and W_{in} as shown in Fig. 3(a). The pulse stream transmits through the interface X without any reflections [(1) in Fig. 3(b)], because the characteristic impedances of lines 1 and 2 are set to $Z_{\pi 1}$ and $Z_{\pi 2}$, respectively. On the other hand, it is greatly reflected at the interface Y because of the impedance mismatch. Generally, $Z_{c1,c2}$ is much greater than $Z_{\pi 1,\pi 2}$. The reflected π-mode pulses have the same parity as the input ones [(2) in Fig. 3(b)]. The π-mode pulses forming these two pulse streams collide in turn at several fixed points on a coupled NLTL, resulting in the development of small c-mode pulses starting to travel to both directions. The collision points are shown in (3) in Fig. 3(b). The spatial period of the incident pulse stream is given by T multiplied by the π-mode pulse velocity, denoted as D in Fig. 3(b). The π-mode pulses consisting of the incident and reflected streams collide at $S_1, S_2, \ldots$ at the same time, and then they collide at $T_1, T_2, \ldots$ after passing by $T/2$. When the velocity of the c-mode pulse is an integral multiple of the π-mode pulse, the original π-mode pulses and the newly developing c-mode pulses simultaneously arrive at the collision points. Thus, the c-mode pulse gains amplitude through the superposition of newly developing pulses. Moreover, when the c-mode pulse travels N times faster than the π-mode one, the repetition period of a pulse stream at V_{out} and W_{out} is reduced to be T/N by the c-mode pulses developed at the points S_i ($i = 1, 2, \ldots$). Moreover, the c-mode pulses developed at the points T_i ($i = 1, 2, \ldots$) doubles the repetition rate. As a result, the repetition rate increases $2N$ times. Through the in-phase superposition of c-mode pulses developed at distinct collision points, the c-mode pulses gain the amplitude at the output when the number of collision point increases. Note that the number of collision points increases when the period T decreases; therefore our method becomes more effective for higher incident repetition rate, as far as each pulse does not overlap with neighboring ones.

When the amplitude of the developed c-mode pulse becomes large enough, the varactors nonlinearity influences the pulse, such that the velocity increases when the amplitude increases. Because of this velocity variation, the in-phase superposition of developed c-mode pulses cannot be established for the present method of increasing pulses' repetition-rate; therefore, the instability caused by the turn-around pulse gain is naturally avoided. On the other hand, we must design a coupled NLTL to exhibit the maximal amplitude of developed c-mode pulse by optimizing the line length and number of collision points.

For numerical demonstration, we set V_J, m, C_0, C_m, L_1, L_2, and L_m to 0.7 V, 0.5, 0.1 pF, 0.2 pF, 50.0 pH, 10.0 pH, and 2.2 pH, respectively. We set V_0 and W_0 to -0.3 V and -5.0 V, respectively. Fig. 4 shows the results of a numerical demonstration of the method. The coupled NLTL is composed of 2752 cells; each of the four i/o ports is directly connected with a resistor having a characteristic impedance specified as in Fig. 3(a). Figs. 4(a), (b), and (c) show the waveforms monitored at the output of line 2 for the repetition period T of 2.616,

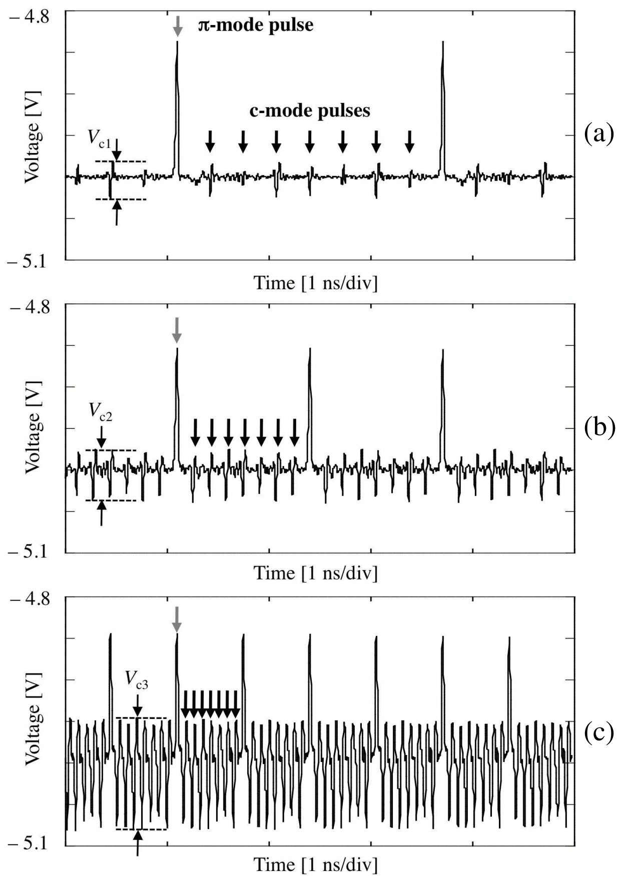

Figure 4. Numerical demonstration of increasing the repetition rate of incident pulse stream. The output waveforms at the repetition-period of (a) 2.616, (b) 1.308 and (c) 0.654 ns. Grey and black arrows indicate the incident π-mode pulses and newly developing c-mode pulses, respectively.

1.308, and 0.654 ns, respectively. The pulses specified by the grey arrows show the π-mode pulses transmitted along the line, and the ones specified by the black arrows show the superposed c-mode ones. For the present line parameters, the repetition rate increases eight times, because the velocity of the c-mode pulse is four times faster than that of the π-mode pulse.

For the present cell size, the number of collision points is seven ($S_{1,\dots,4}$ and $T_{1,\dots,3}$) for $T = 2.616$ ns, seventeen ($S_{1,\dots,9}$ and $T_{1,\dots,8}$) for $T = 1.308$ ns and thirty three ($S_{1,\dots,17}$ and $T_{1,\dots,16}$) for $T = 0.654$ ns; therefore, the c-mode amplitude should become about twice larger for $T = 1.308$ ns and about four times larger for $T = 0.654$ ns than for $T = 2.616$ ns. Actually, we can clearly see the relation: $V_{c3} > V_{c2} > V_{c1}$ in Figs. 4(a), (b) and (c).

4. DISCUSSION

The mismatch in the amplitudes of the c- and π-mode pulses has to be eliminated for versatile use of the output pulse streams. We expect that traveling-wave field effect transistors (TW-FETs) succeed in equalizing the amplitude of c- and π-mode pulses. By amplifying pulses carried by a unique propagation mode, a TW-FET can achieve broadband amplification.

In the presence of the parasitic resistors of the inductors, both modes are attenuated during propagation and the amplitudes of reflected pulses have to be decreased. On the other hand, the larger the cell size becomes, the more the incident pulse collides with reflected pulses. As a result, there is an optimal cell size that guarantees the required amplitude of reflected pulses and the required number of collision points on the line. Furthermore, we observed that the output amplitude of newly developing c-mode pulse increases proportionally as the repetition rate of the input pulse stream increases. We thus have to design the repetition rate of the input pulse stream high enough to compensate for attenuation.

Although a proper method for application and extraction of a pulse stream has to be devised, a circular coupled NLTL, where one π-mode pulse stream travels clockwise and another stream travels anti-clockwise, may give the best platform for increasing the repetition rate, because each π-mode pulse forming the clockwise stream repeatedly collides with the ones forming the anti-clockwise stream. The employment of the circular ring configuration for pulse generation using NLTLs has also been investigated in [7].

5. CONCLUSION

In a coupled NLTL, small c-mode pulses are generated by colliding two π-mode pulses traveling in the opposite directions. As an engineering application of this property, we propose a method to increase the repetition rate of incident pulse stream by the factor greater than 2. A numerical time-domain calculation demonstrates that the output repetition rate is eight times as much as the input one and the amplitude of c-mode pulse becomes larger for higher incident repetition rate. Generation of c-mode pulses can also be observed at the boundary of a coupled NLTL where a π-mode pulse is reflected. By these unique properties, coupled NLTLs have great potential for managing short electrical pulses. We believe that they may occupy a very important position in the field of high-speed electronics.

REFERENCES

1. Hirota, R. and K. Suzuki, "Studies on lattice solitons by using electrical networks," *J. Phys. Soc. Jpn.*, Vol. 28, 1366–1367, 1970.
2. Rodwell, M. J. W., S. T. Allen, R. Y. Yu, M. G. Case, U. Bhattacharya, M. Reddy, E. Carman, M. Kamegawa, Y. Konishi, J. Pusl, and R. Pullela, "Active and nonlinear wave propagation devices in ultrafast electronics and optoelectronics," *Proc. IEEE*, Vol. 82, 1037–1059, 1994.
3. Kintis, M., X. Lan, and F. Fong, "An MMIC pulse generator using dual nonlinear transmission lines," *IEEE Microwave Wireless Compo. Lett.*, Vol. 17, No. 6, 454–456, 2007.
4. Yildirim, O. O., D. S. Ricketts, and D. Ham, "Reflection soliton oscillator," *IEEE Trans. Microwave Theory Tech.*, Vol. 57, No. 10, 2344–2353, 2009.
5. Gupta, K. C., R. Garg, and I. J. Bahl, *Microstrip Lines and Slotlines*, Artech, 1979.
6. Narahara, K., "Coupled nonlinear transmission lines for doubling repetition rate of incident pulse streams," *Progress In Electromagnetics Research Letters*, Vol. 16, 69–78, 2010.
7. Ricketts, D. S., X. Li, N. Sun, K. Woo, and D. Ham, "On the self-generation of electrical soliton pulses," *IEEE J. Solid-state Circuits,* Vol. 42, No. 8, 1657–1668, 2007.