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FDTD MODELING AND SIMULATION OF MICROWAVE HEATING OF IN-SHELL EGGS

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Abstract—Considering microwaves as a viable alternative for the pasteurization of In-shell eggs, preliminary trials performed had confirmed that microwave at 2450 MHz can be successfully used to raise the temperature of in-shell eggs to the required pasteurization temperatures in a few minutes. Based on these trials a finite difference time domain (FDTD) model was developed using C language and MATLAB to simulate the E field and power distribution in lossy dielectric media like that of the egg components (egg white and yolk) taking into consideration the complex shape, dielectric properties and heterogeneous composition of the in-shell egg. This can be used to assist in the design and development of an industrial microwave in-shell eggs pasteurization unit.

1. INTRODUCTION

Eggs have a rich nutritive value. Thus eggs are potential hosts and carriers for pathogenic microbes like *Salmonella enteritidis* and the most deadly strain (H5N1) of the avian flu virus. Heat pasteurization is a well-known process for enhancing food safety. However, egg proteins are extremely heat sensitive. Therefore, heat pasteurization with minimal changes to the egg proteins needs consideration. Conventional methods of heat pasteurization using hot air and hot water bath severely affect the functional quality of the eggs. Microwaves provide a viable alternative for the pasteurization of In-shell eggs [1].

The following are a few among the several factors to be taken into account, while considering microwaves to do the job.

- Microwave heating is fairly non-uniform

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- Heterogeneity of the egg
- Complexity in locating the points of overheating
- Remediation of cold spots through specific design alternatives

Finite element and Finite Difference Time Domain (FDTD) are two commonly used methods for solving Maxwell's equation to describe the energy distribution in a complex object or within a multimode cavity, and both methods are capable of simulating power density distribution in a 3-D space [2–6]. The finite element method is suitable for arbitrarily shaped non-homogeneous objects and requires the solution of a sparse matrix which can prove very complicated. Comparatively, FDTD is a very straight forward method that can readily model non-homogeneous and anisotropic materials as well as arbitrarily shaped geometries; it can also provide both time and frequency domain analyses which are important to microwave heating problems like field distribution, scattering parameters and dissipated power distribution for various materials and geometries [7–9].

A normal egg would explode during microwave (MW) heating due to water vapour pressure build-up inside the shell. Besides this, when it comes to pasteurization, heating uniformity is a critical factor, but microwave heating is fairly non uniform. In this study, a FDTD method was used for the numerical simulation of microwave heating of a heterogeneous multilayered complex geometry like that of in-shell eggs and experimental validation of the simulation process was undertaken.

2. MATERIALS & METHODS

MATLAB version R2007a was used for the FDTD simulation of the Microwave heating of eggs. First the electromagnetic model for a microwave oven with a waveguide and the egg placed inside it was created and FDTD method was used to solve the model numerically and the electric field inside the cavity was traced. This field was then used to compute the power loss within the egg. This power loss was used as the source term for the heat equations which were, in turn, solved in order to calculate the temperature variation within the egg.

2.1. Electromagnetic Model for Field Distribution

At the macroscopic level, electromagnetic phenomena were defined using Maxwell Equations. The electromagnetic field distribution inside the microwave oven was traced out by solving the following Maxwell's

Equations [10].

$$\begin{aligned} &\nabla \times E = -j\omega\mu H \\ &\nabla \times H = j\omega\varepsilon_0\varepsilon E \\ &\nabla \cdot E = 0 \\ &\nabla \cdot H = 0 \end{aligned} \tag{1}$$

where, E is electric field intensity in Vm^{-1} and H is the magnetic field intensity in Am^{-1}. Also the Time Harmonic Function can be written as:

$$\begin{aligned} E(x, y, z, t) &= E_0(x, y, z)e^{j\omega t} \\ H(x, y, z, t) &= H_0(x, y, z)e^{j\omega t} \end{aligned} \tag{2}$$

The relative permittivity ε can be expressed in complex form as

$$\varepsilon = \varepsilon' - j\varepsilon'' \tag{3}$$

where ε' is dielectric constant and ε'' is the dielectric loss factor.

For an isotropic lossy material, the loss characteristics of the material are given by the loss tangent, as defined by:

$$\tan\delta = \frac{\sigma}{2\pi f\varepsilon_0\varepsilon_r} \tag{4}$$

where, f = frequency (Hz), σ = conductivity (Sm^{-1}), ε_o = free-space permittivity (Fm^{-1}), and ε_r = relative permittivity (unitless).

For a material with complex permittivity as in (3), the loss tangent is defined by:

$$\tan\delta = \frac{-\varepsilon''}{\varepsilon'} \tag{5}$$

The following relationships exist between electric and magnetic field intensity and flux density,

$$\begin{aligned} \tilde{B} &= \mu_o\tilde{H} \\ \tilde{D} &= \epsilon_o\tilde{E} \end{aligned} \tag{6}$$

where, $\mu_0 = 4\pi \times 10^{-7}$ henry m^{-1}, is the free space permittivity, and $\varepsilon = 8.85 \times 10^{12}$ farad m^{-1}, is the free space permeability.

The following scalar equations (Equations (7)–(12)) can be produced from the time dependent Maxwell's equations [8].

$$\frac{\partial \tilde{D}_x}{\partial t} = \frac{1}{\sqrt{\varepsilon_o\mu_o}}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) \tag{7}$$

$$\frac{\partial \tilde{D}_y}{\partial t} = \frac{1}{\sqrt{\varepsilon_o \mu_o}} \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \right) \tag{8}$$

$$\frac{\partial \tilde{D}_z}{\partial t} = \frac{1}{\sqrt{\varepsilon_o \mu_o}} \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) \tag{9}$$

$$\frac{\partial H_x}{\partial t} = \frac{1}{\sqrt{\varepsilon_o \mu_o}} \left(\frac{\partial \tilde{E}_y}{\partial z} - \frac{\partial \tilde{E}_z}{\partial y} \right) \tag{10}$$

$$\frac{\partial H_y}{\partial t} = \frac{1}{\sqrt{\varepsilon_o \mu_o}} \left(\frac{\partial \tilde{E}_z}{\partial x} - \frac{\partial \tilde{E}_x}{\partial z} \right) \tag{11}$$

$$\frac{\partial H_z}{\partial t} = \frac{1}{\sqrt{\varepsilon_o \mu_o}} \left(\frac{\partial \tilde{E}_x}{\partial y} - \frac{\partial \tilde{E}_y}{\partial x} \right) \tag{12}$$

Using the electric field obtained by electromagnetic analysis, the power absorbed from the microwaves per unit volume of egg was calculated for specifying different input power densities of 1, 2 and 3 Wg^{-1} using the following equation:

$$P(x, y, z, t) = \frac{\omega \varepsilon_0 \varepsilon'' \left| E \right|^2}{2} \tag{13}$$

2.2. Heat Transfer Model for Heat Generation

As the source term, temperatures inside the egg were obtained, the heat transfer equation (Equation (14)) was solved with the calculated power loss added [11]

$$\rho C_p \frac{\partial T}{\partial t} = k \nabla^2 T + P(x, y, z, t) \tag{14}$$

2.3. Computer Simulation of Microwave Heating of an Egg

2.3.1. Material Models & Boundary Conditions

First the material properties were specified. The model consisted of the oven cavity, the egg yolk and the egg white. The egg shell is almost transparent to the microwaves. However the dielectric properties of the shell were also taken into account for the simulation. Hence the model had three layers.

2.3.2. Permittivity

- For the cavity and waveguide the permittivity is taken as being the same as that of a vacuum.

- For egg white [1]:

$$\varepsilon' = 72.38 - 0.17T - 1.75F \text{ and } \varepsilon'' = 17.22 - 0.19T + 1.58F \quad (15)$$

- for egg yolk [1]:

$$\varepsilon' = 50.085 - 0.13T - 1.72F \text{ and } \varepsilon'' = 13.55 - 0.11T + 0.65F \quad (16)$$

where, T is the temperature in °C, and F is the frequency in GHz. The initial (room) temperature was taken as 22°C and the frequency of operation was taken as 2.450 GHz. Figure 1 shows the geometry and dimensions of the cavity and waveguide adjusted for resonating in TE_{10} mode with the position of egg in it and Figure 2 along with Table 1 detail the dimensions of the typical egg used for simulation studies.

A grid size of 4.7 mm × 4.7 mm was used to divide the XY plane of the cavity into a 100 × 100 mesh grid.

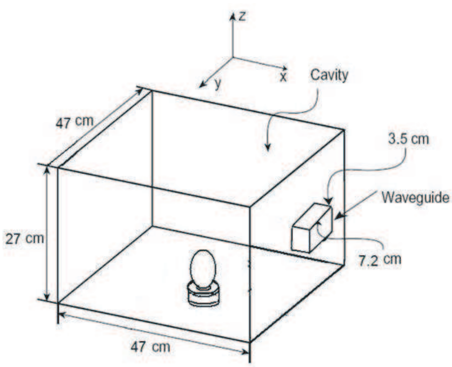

Figure 1. Egg in the Microwave Cavity.

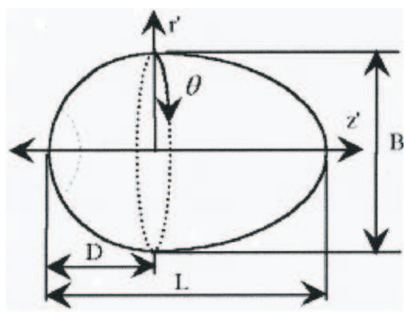

Figure 2. Egg geometry.

Table 1. Dimensions of the egg.

Description	**Symbol**	**Value**
Thickness of shell	h_s	1 mm
Length of shell	L	5.7 cm
Largest breadth of shell	B	4.2 cm
Distance from largest breadth to blunt end	D	2.5 cm
Radius of spherical shell		
Radius of yolk/embryo	a	2.3 cm
Area of contact	a_e	1.5 cm
	A_R	0.15 cm^2

2.3.3. Boundary Conditions and Excitations (Loads)

The walls of the cavity were specified as perfect electrical conductors (Figure 3). An exterior waveguide port (Figure 4) was used for the excitation of the port.

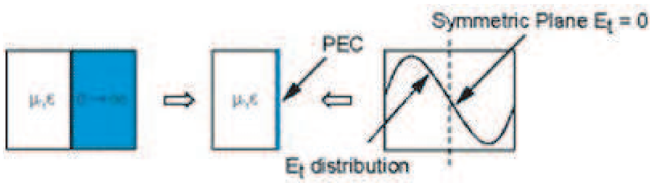

Figure 3. Perfect electrical conductor.

Figure 4. Exterior waveguide port excitation.

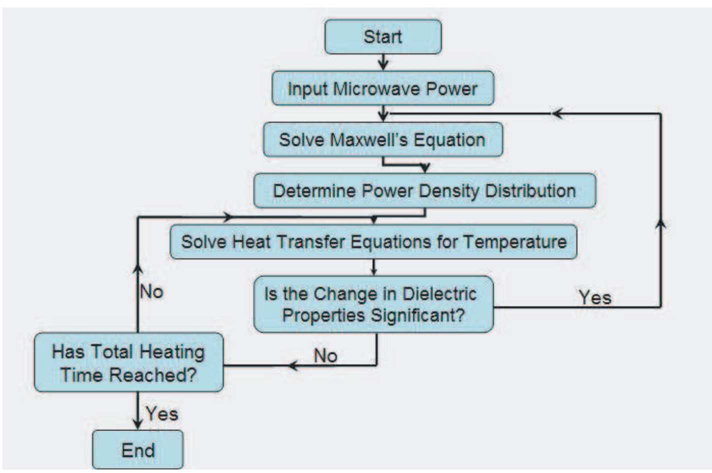

Figure 5. Flow diagram of the FDTD simulation process.

The excitation port option was specified in two steps. First the solid model area was selected to define the port location and assign a port number. The port number assigned must be between 1 and 50. For an exterior port, after assigning the port number, the port type was specified as a rectangular waveguide of TE_{10} (Transverse electric) mode.

The computational geometry of the egg was staggered with rectangular elements. A simplified self-explanatory logical flow diagram of the entire simulation process is given in Figure 5.

2.4. Innovations in the Simulation Process

The following factors were taken into account for the first time in the simulation of microwave heating of a heterogeneous lossy dielectric medium.

- A coupled approach was used for the temperature and power distribution
- The dependency of dielectric properties on temperature of the material and frequency of the microwaves was taken into account.
- A microwave heat generation factor was introduced
- The conduction, convection and radiation modes of heat transfer (though negligible) were included.

2.5. Evaluation of the Simulation

The simulations were qualitatively assessed for their validity by microwave heating of commercially available large sized grade A eggs and then quick freezing them using liquid nitrogen. A regular domestic microwave oven (Panasonic Model NNSN968B, Panasonic Inc, Canada) was used. Temperature was measured at two point (one in broad end and one in the narrow end of the egg). Quick freezing preserved the location of the coagulation and the egg retained its shape even after peeling off the shell. This also provided the required transparency to determine the position and size of the coagulation inside the egg white, which in turn indicated the region(s) of overheating.

3. RESULTS & DISCUSSION

3.1. Electric Field Intensity

Figures 6, 7 and 8 depict the distribution of electric field at the central transverse section of an egg stratum in the cavity for the power densities of 1, 2 and 3 Wg^{-1} respectively. These clearly indicate that the pattern of the electric field distribution was not affected by a change in power density, but that electric field intensity varied exponentially with a linear increase in the power density. The Figures 6, 7 and

8 also clearly indicate that the distribution of the electric field is strongly extensively affected by the presence of the egg inside the cavity, which can be seen from the drastically decreased electric field intensity immediately adjacent the egg in the cavity.

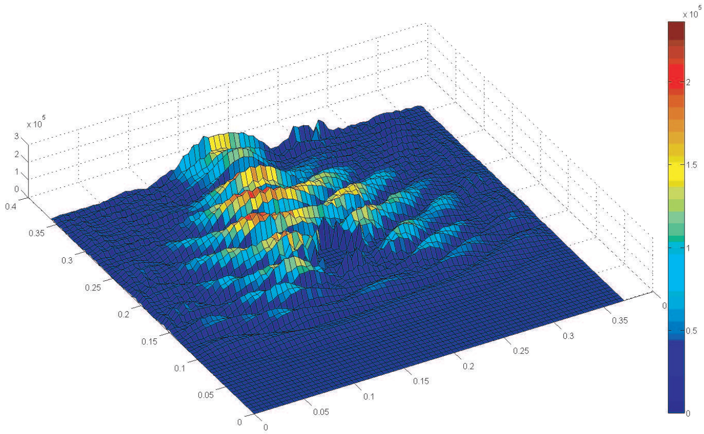

Figure 6. Distribution of electric field $(\mathrm{V \cdot m^{-1}})$ at the central transverse section of an egg stratum in the cavity for $1\,\mathrm{Wg^{-1}}$.

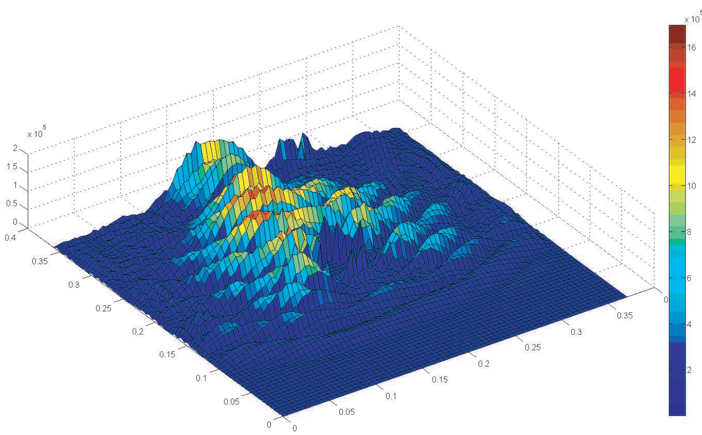

Figure 7. Distribution of electric field $(\mathrm{V \cdot m^{-1}})$ at the central transverse section of an egg stratum in the cavity for $2\,\mathrm{Wg^{-1}}$.

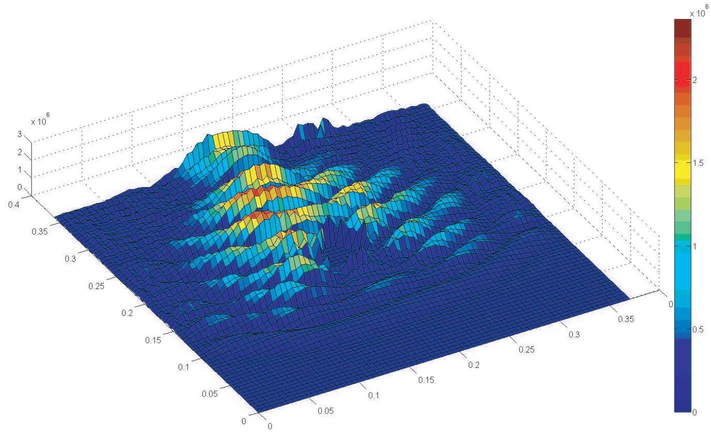

Figure 8. Distribution of electric field $(\mathrm{V \cdot m^{-1}})$ at the central transverse section of an egg stratum in the cavity for $3\,\mathrm{Wg^{-1}}$.

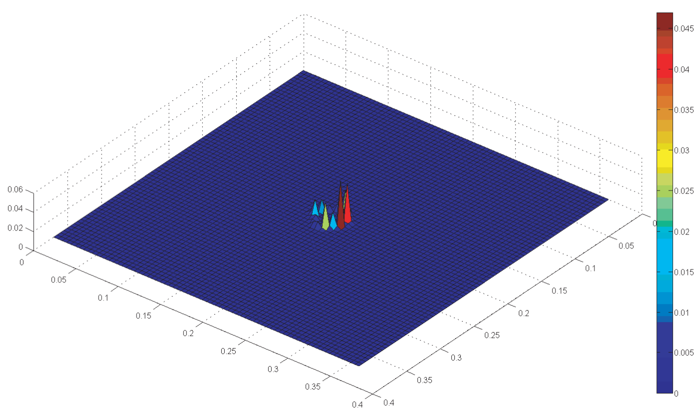

Figure 9. Power loss (W) inside the egg at the central transverse section for $1\,\mathrm{Wg^{-1}}$.

3.2. Power Loss Inside the Egg Leading to Heat Generation

Power loss is the primary factor contributing to the microwave heating of the lossy dielectric media. Figures 9, 10 and 11 denote the power loss in watts inside the egg at the central transverse section for power densities of 1, 2 and 3 $\mathrm{Wg^{-1}}$ respectively. (The images are rotated by 180 degrees to present a better view). The power loss increased linearly with the linear increase in power density, indicating that for a stationary (non-rotating) egg in a microwave cavity the power loss was at its maximum at the point of incidence and that the microwaves soon dissipated most of their energy.

Fu Metexas [2] and Dai [7] followed similar approach to estimate the electric field intensity and power loss in a multimode cavity, whereas their models did not take into consideration a heterogeneous substance like the shell eggs and their thermodynamically changing properties.

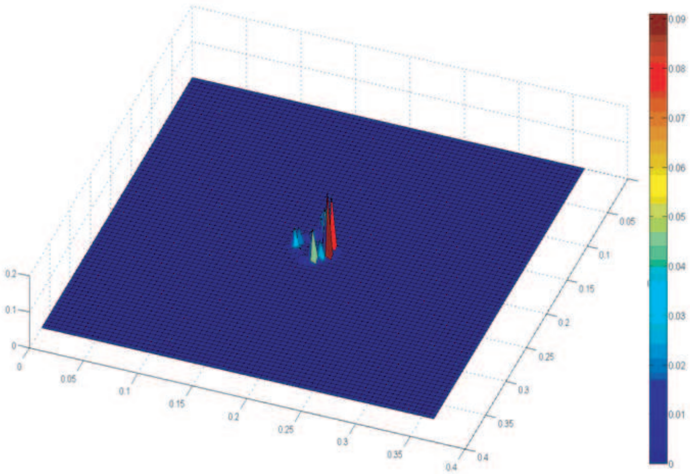

Figure 10. Power loss (W) inside the egg at the central transverse section for $2\,\mathrm{Wg}^{-1}$.

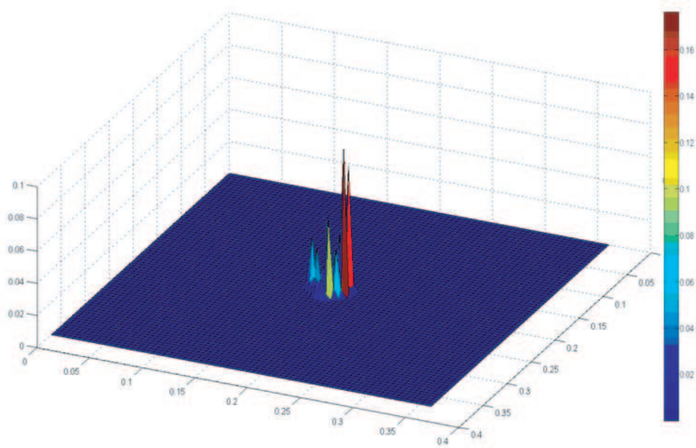

Figure 11. Power loss (W) inside the egg at the central transverse section for $3\,\mathrm{Wg}^{-1}$.

3.3. Temperature Distribution

The power distribution inside the egg was directly related to the extent of heating at a particular location inside the egg. Figures 12, 13 and 14 represent the simulated temperature values inside the egg at the central transverse section of the egg in °C for power densities of 1, 2 and 3 Wg^{-1} respectively, after 100 seconds heating time for eggs with an intial temperature of 5°C.

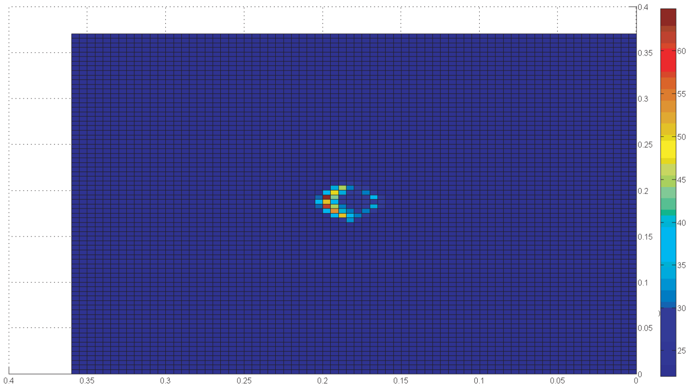

Figure 12. 2D -Temperature distribution (°C) inside the egg at the central transverse section for 1 Wg^{-1}.

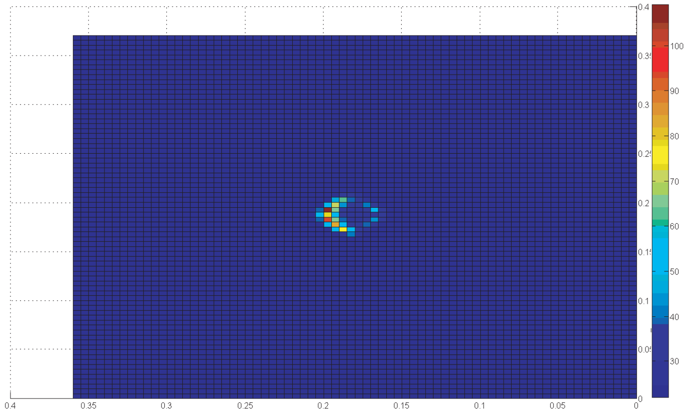

Figure 13. 2D -Temperature distribution (°C) inside the egg at the central transverse section for 2 Wg^{-1}.

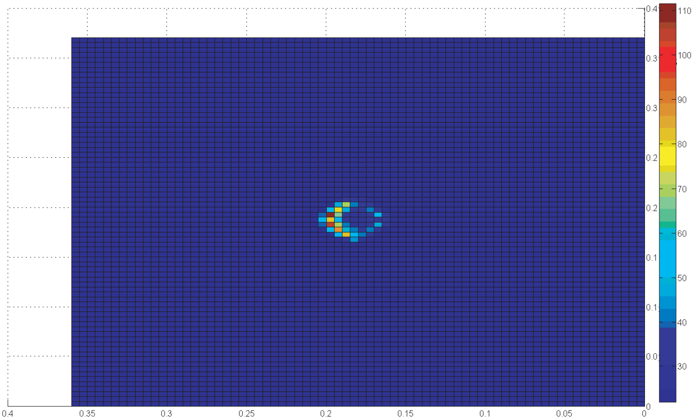

Figure 14. 2D-Temperature distribution (°C) inside the egg at the central transverse section for $3\,\mathrm{Wg}^{-1}$.

The heating pattern of the eggs clearly had the same non-uniformity as that of the power loss, shedding some light on the extent and pattern of overheating and non-uniform temperatures inside the egg. The temperature gradient within the egg increased drastically with an increase in power level, as can be seen by comparing values in plots generated for increasing power levels.

The simulations showed that at some locations the temperature shot up to 110°C within 100 seconds at a power density of $3\,\mathrm{Wg}^{-1}$, while other portions of the egg that remained below 30°C. Comparatively, at lower power densities the temperature gradient was much lesser due to the slower heating rates and greater quantities of heat transfer occurring through conduction. Harms et al. (1996) obtained results with good precision but the variability of the validation data was considerably high, leading to 15% error in estimation. As this model involves simultaneous solutions for all the three modes of heat transfer, the error was kept within 10% for all the measured values. Rammen et al. [12] found similar differential temperatures and temperature gradients for regular shaped food materials.

3.4. Experimental Validation:

Actual microwave-heated eggs showed coagulation of egg white exactly in the regions predicted by the simulation, indicating an excellent corroboration of the experimental results by those of simulated microwave heating (Figure 15).

Figure 15. Quick frozen Microwave heated egg showing coagulation of egg white (right hand side) compared to the control (left hand side).

At the higher power levels the eggs exploded quickly in the microwave indicating that the temperature had risen above the boiling point in certain parts of the egg. Such an explosion can be avoided by the slower heating of the egg which occurs under lower power densities.

4. CONCLUSIONS

Results of the actual microwave heating and numerical simulations corroborate each other very well, thereby confirming the accuracy of this approach in simulating novel wave guide and/or microwave cavity designs. The non-uniformity of heating was more pronounced at higher power levels, thereby suggesting lower power levels to be better in producing a quality pasteurized product.

Thus, microwave heating can be considered as an alternative method for the pasteurization of in-shell eggs, but it requires further research to make it industrially viable.

5. RECOMMENDATIONS FOR FURTHER RESEARCH

FDTD Simulation of a rotating egg inside a microwave cavity can help better understanding of the heating behaviour of rotating objects. Also Finite Element Modeling and Simulation of the microwave heating of in-shell eggs would bring about better and more accurate simulation results. Hyperspectral imaging of the coagulation of egg protein due to microwave heating would reveal any non-uniform heating with greater discrimination.

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