TAMM STATES OF A NONLINEAR SLAB SANDWICHED BETWEEN A UNIFORM MEDIUM AND A ONE-DIMENSIONAL PHOTONIC CRYSTAL

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Abstract—In this paper, the surface states (so called Tamm states) of a nonlinear self-focusing slab sandwiched between a uniform medium and a one-dimensional photonic crystal is investigated based on the first integral of the nonlinear Helmholtz wave equation. The considered slab can be a left-handed or a conventional material. It is shown that the structure can support the Tamm states with two different transverse electric structures. In one structure, the surface waves have one hump, and in the other one they have two humps at the surface of photonic crystal. We show that in the case of the self-focusing left-handed metamaterial slab, there is a possibility to change the direction of the total energy flow of the surface waves by adjusting the intensity of exciting electromagnetic field.

1. INTRODUCTION

There is a worldwide interest in photonic crystals (PCs) [1–4] because they can modulate the propagation of photons and control the properties of electromagnetic light in the same way that semiconductor materials do in controlling the propagation of electrons. The advent of PC materials spurred a lot of interest toward the existence of surface modes at the interfaces of such materials [5–8]. The motivation for studying the surface electromagnetic waves (SWs) in photonic band gap (PBG) materials is their potential use in sensors, modulators, atom mirrors, and in the enhancement of the surface nonlinear optical effects. For applications that make use of SWs, PBG are appealing because it is possible to engineer a sample that exhibits a band gap (and hence SWs) within any frequency range irrespective of the intrinsic material

properties. Furthermore, because PBG materials are constructed from pure dielectrics, their loss is very low. The low loss is a desirable property in SW applications because it leads to narrow coupling resonances and high surface-electromagnetic (EM) fields. Recently the linear SWs at the interface of a metamaterial and a semi-infinite 1DPC have been studied in the Reference [9] based on the transfer matrix method. Here, we study the properties of nonlinear type of SWs that can be excited at the interface between a uniform LH material and 1DPC capped by a nonlinear slab with finite thickness and we investigate a possibility to control the dispersion properties of SWs by adjusting the intensity of electromagnetic field. Our method is based on the first integral of the nonlinear Helmholtz wave equation [10–12]. In this paper, we consider two different types of nonlinear LH and right-handed (RH) slabs, and demonstrate a number of peculiar features of the dispersion properties of the nonlinear TE-polarized SWs. The results of this paper revealed that, there are two types of the SWs in the modeled structure, one with maximum amplitude at the surface of PC, and the other one with two-humps. Moreover it is shown that there is a possibility to switch from the forwarded SWs to backwarded ones only by adjusting the intensity of the exciting electromagnetic field in the case of self-focusing LH nonlinear slab. We have also compared our results with the Runge-Kutta method that validates the above results. This paper is organized as follow: In Section 2, we introduce the model of the system under consideration. In Section 3, the intensity dependent properties of the nonlinear SWs are studied. Finally, Section 4 concludes the paper.

2. THEORETICAL MODEL

In what follows, we study the TE-polarized SWs in a nonlinear LH or RH slab sandwiched between a uniform medium and a one-dimensional photonic crystal. We choose a coordinate system in which the layers have normal vector along OZ. In the 1DPC, each layer is characterized by dielectric permittivity ε_i, magnetic permeability μ_i and the thickness d_i $(i = 1, 2)$. In the chosen coordinate system the slab with thickness d_c extends from $z = -d_c$ to 0 and the uniform LH material that is characterized by $\varepsilon_0 = -1$ and $\mu_0 = -1$ is located to the left of $z = -d_c$. The order of magnitude of the lattice constant $d = (d_1 + d_2)$ depends on the desired spectral range. For the microwave region with frequency around 5–12 GHz (the suitable range for the left-handed metamaterials (LHM) [13]), d is on the order of centimeter, while it can be on the order of micrometers to nanometers for the optical range (e.g., see the experimental work of Ref. [14]). On the

other hand, because of the recent developments in designing negative refractive index materials at the optical range [15, 16] the use of the LHM as a homogeneous medium is realizable. The nonlinear slab is characterized by μ_c and a nonlinear dielectric permittivity:

$$\varepsilon_{NL} = \varepsilon_c + \alpha |E_c(z)|^2 \tag{1}$$

Here ε_c is the linear part of the relative dielectric permittivity, $E_c(z)$ is the electric field in the slab and parameter α describes Kerr-type nonlinearity. For a conventional (RH) material dielectric, positive α refers to a self-focusing nonlinear material while a self-focusing LH medium corresponds to a negative α [17]. Below, both types of LH and RH nonlinear slabs are considered and for simplicity it is assumed that the containing media are lossless, homogeneous and isotropic. For definiteness, we suppose that the slab possesses self-focusing property; i.e., $\alpha < 0$ for LH slab and $\alpha > 0$ for RH slab. We consider propagation of monochromatic TE-polarized waves as

$$\mathbf{E} = E_y(z)e^{i(k_x x - \omega t)}\hat{\mathbf{e}}_y \tag{2}$$

$$\mathbf{H} = (H_x(z)\hat{\mathbf{e}}_x + H_z(z)\hat{\mathbf{e}}_z)e^{i(k_x x - \omega t)} \tag{3}$$

where ω is the angular frequency, $k = \omega/c$ is the vacuum wave number, $k_x = \frac{\omega}{c}\beta$ and $\beta = n'\sin\theta_0$ where θ_0 is the angle of the incidence measured from the normal and n' is the refractive index of a dense medium used to excite the SWs [18]. The stationary solution of the TE wave equation for the nonlinear slab is of the form $E_y(z) = E_c(z)\exp(i(k\beta x - \omega t))$ which the amplitude field in the nonlinear slab is modulated along z due to the nonlinearity. Using this form of the stationary solution with $\mu(z) = \mu_c$ and $\varepsilon(z) = \varepsilon_{NL}$ we reach

$$\left[\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial x^2} + \left(\frac{\omega^2}{c^2} \right) \mu_c(\varepsilon_c + \alpha|E_c(z)|^2) \right] E_c(z) = 0 \tag{4}$$

It is well known that surface modes correspond to the localized solutions with the field E decaying from the interface in both directions. In the left-side homogeneous medium the fields are decaying provided $\beta > \sqrt{\varepsilon_0 \mu_0}$. So, the solution of the scalar Helmholtz-type wave equation in a homogeneous medium($z \leq -d_c$) that satisfies the boundary conditions at infinity is

$$E_d(z) = E_0 e^{q_0(z+d_c)} \tag{5}$$

where E_0 is the electric field amplitude at $z = -d_c$, $q_0 = k\sqrt{\beta^2 - n_0^2}$ and $n_0 = \sqrt{\varepsilon_0 \mu_0}$. In the periodic structure, the waves are the Bloch modes

$$E(z) = \psi(z)\exp(iK_b z) \tag{6}$$

where K_b is the Bloch wave number and $\psi(z)$ is the Bloch function which is periodic with the period of the photonic structure [19]. In the periodic structure, the waves will be decaying provided that K_b is complex and this condition defines the spectral band gaps of an infinite multilayered structure. In the next stage, we investigate the solution of the nonlinear wave equation in two types of the nonlinear slabs, LH material and RH ones based on the first integral of the basic equation technique. So, Eq. (4) can be integrated once to give

$$(E_c')^2 - k_c^2 E_c^2 + k^2 \mu_c \alpha/2 E_c^4 = C_c \tag{7}$$

Here, the prime represents the differentiation with respect to z, $k_c^2 = k^2(\beta^2 - \varepsilon_c \mu_c)$ and C_c is the constant of integration (with respect to z). The quantity C_c which has arisen from the first integral is determined by applying the TE boundary conditions at the interfaces of the slab.

$$E_d|_{z=-d_c} = E_c|_{z=-d_c} = E_0; \qquad \frac{1}{\mu_0}\frac{\partial E_d}{\partial z}|_{z=-d_c} = \frac{1}{\mu_c}\frac{\partial E_c}{\partial z}|_{z=-d_c} \tag{8}$$

$$E_c|_{z=0} = E_1|_{z=0} = E_b; \qquad \frac{1}{\mu_c}\frac{\partial E_c}{\partial z}|_{z=0} = \frac{1}{\mu_1}\frac{\partial E_1}{\partial z}|_{z=0} \tag{9}$$

where E_0, E_b are the values of the electric field at the lower and upper boundaries of the slab and E_1 is the electric field in the first layer of PC [19]: $E_1(z) = Be^{ik_{1z}z} + (\lambda - A)e^{-ik_{1z}z}$. Here, A and B are the elements of the transfer matrix of the PC [19]: $A = e^{ik_1d_1}\left(\cos k_2 d_2 + \frac{i}{2}\left(\frac{k_1}{k_2} + \frac{k_2}{k_1}\right)\sin k_2 d_2\right)$, $B = e^{ik_1d_1}\frac{i}{2}\left(\frac{k_2}{k_1} - \frac{k_1}{k_2}\right)\sin k_2 d_2$, where $k_i = k_{iz} = k\sqrt{n_i^2 - \beta^2}$ $(i = 1, 2)$ and λ is the eigenvalue of the transfer matrix in the photonic band gap: $\lambda = \text{Re}(A) \pm \sqrt{\text{Re}(A)^2 - 1}$.

Inserting Eqs. (8), (9) into Eq. (7) give

$$\gamma_0\left(\mu_c q_0^2 - \frac{k_c^2 \mu_0^2}{\mu_c} + \frac{k^2 \mu_0^2 \gamma_0}{2}\right) = \gamma_b\left(\mu_c k_{1z}^2 \widetilde{R}^2 - \frac{k_c^2 \mu_1^2}{\mu_c} + \frac{k^2 \mu_1^2 \gamma_b}{2}\right) \tag{10}$$

where $\gamma_0 = \alpha|E_0|^2$, $\gamma_b = \alpha|E_b|^2$, $\widetilde{R} = ik_{1z}\frac{B-(\lambda-A)}{B+(\lambda-A)}$, $k_{1z} = k\sqrt{n_1^2 - \beta^2}$. Eq. (10) determines the dispersion relation $k = k(\beta)$ for the nonlinear surface waves. The solution to Eq. (7) can be found prior to the integration via expressing Eq. (7) in the form

$$\frac{\partial E_c}{\partial z} = k\sqrt{|\mu_c||\alpha|/2}\left[E_c^2 + \frac{-k_c^2 + \sqrt{k_c^4 + 2C_c|\mu_c|k^2|\alpha|}}{|\mu_c|k^2|\alpha|}\right]^{\frac{1}{2}}$$

$$\times\left[-E_c^2 + \frac{k_c^2 + \sqrt{k_c^4 + 2c_c|\mu_c|k^2|\alpha|}}{|\mu_c|k^2|\alpha|}\right]^{\frac{1}{2}} \tag{11}$$

The factors on the right-hand side of Eq. (11) are real, so the solutions can be written in terms of the Jacobi functions $sd(p|m)$, where p is the relevant real argument and m is the relevant Jacobi parameter [10, 20]. The Jacobi elliptic functions satisfy nonlinear first order differential equations like Eq. (11). One can thus solve all such equations exactly, in closed form, in terms of elliptic functions. Elliptic functions open up a window of solvable nonlinear (polynomial) differential equations, all of which are related to the physical problems and physical phenomena. There are, in fact, 12 Jacobi functions and their selection depends upon the signs of C_c and k_c^2. Therefore we must determine the signs of C_c and k_c^2 in the band gap of the photonic crystal. Our calculations showed that the sign of C_c in the first photonic band gap with both types of slabs is positive and the sign of k_c^2 is positive (negative) in the case of LH (RH) slab. Accordingly, the solutions of the nonlinear wave equation that satisfy the TE boundary conditions, after second integration have the following forms for the case of LH and RH slabs, respectively [20]:

$$E_{c1}(z) = sd\left(k\sqrt{|\mu_c||\alpha|}(a_1^2 + b_1^2)^{\frac{1}{2}}(z + d_c) + z_{01}|m_1\right)(a_1^2 + b_1^2)^{1/2} \quad (12)$$

$$E_{c2}(z) = sd\left(z_{02} - k\sqrt{\mu_c\alpha}(a_2^2 + b_2^2)^{\frac{1}{2}}(z + d_c)|m_2\right)(a_2^2 + b_2^2)^{1/2} \quad (13)$$

where $a_1^2 = \frac{-k_c^2 + \sqrt{k_c^4 + 2C_c|\mu_c|k^2|\alpha|}}{|\mu_c|k^2|\alpha|}$, $b_1^2 = \frac{k_c^2 + \sqrt{k_c^4 + 2C_c|\mu_c|k^2|\alpha|}}{|\mu_c|k^2|\alpha|}$, $z_{01} = sd^{-1}(\frac{E_0\sqrt{a_1^2 + b_1^2}}{a_1 b_1}|m_1)$, $m_1 = \frac{b_1^2}{(a_1^2 + b_1^2)}$, $a_2^2 = \frac{|k_c^2| + \sqrt{|k_c^2|^2 + 2C_c\mu_c k^2\alpha}}{\mu_c k^2\alpha}$, $b_2^2 = \frac{-|k_c^2| + \sqrt{|k_c^2|^2 + 2C_c\mu_c k^2\alpha}}{\mu_c k^2\alpha}$. $z_{02} = sd^{-1}(\frac{E_0\sqrt{a_2^2 + b_2^2}}{a_2 b_2}|m_2)$ and $m_2 = \frac{b_2^2}{(a_2^2 + b_2^2)}$. Here, sd is one of the Jacobi elliptic functions, z_{0i} is the second constant of integration and m_i is the nonlinear period of the Jacobi elliptic function $(i = 1, 2)$.

3. RESULTS AND DISCUSSION

In this section, we use the dispersion relation of Eq. (10) and Eqs. (12) and (13) to study the effect of the dimensionless parameter γ_0 on the dispersion properties of the SWs. The dispersion properties of the nonlinear SWs are plotted in Fig. 1 in the first spectral gap on the plane (k, β) for different values of the dimensionless parameter γ_0. From Fig. 1, it is clear that the dispersion curves of the SWs move to the upper (lower) edge of the band gap by increasing the parameter γ_0 in the case of LH (RH) slab. So, the dispersion properties of SWs depend on the parameter γ_0. Our investigations reveal that the mode structure of the SWs depends on the parameters γ_0 in the case of nonlinear LH
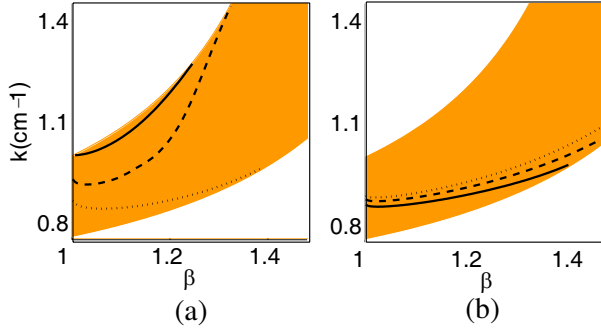
Figure 1. Dispersion curves of the SWs for (a) the LH slab with $\mu_c = -1$, $\varepsilon_c = -1$ and (b) the RH slab with $\mu_c = 1$, $\varepsilon_c = 4$ in the first spectral gap of the PC for the cases of $|\gamma_0| = 0.5$ (solid lines), $|\gamma_0| = 0.2$ (dashed lines) and $|\gamma_0| \rightarrow 0$ (dotted lines). Here, the shaded area shows the band gap of PC and the used parameters are: $d_c = 2d_1$, $d_1 = 1$ cm, $d_2 = 1.65$ cm, $n_1 = 2$, $\mu_1 = 1$, $n_2 = 1.5$ and $\mu_2 = 1$.
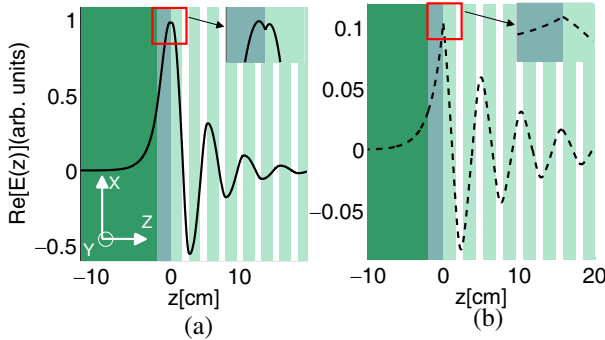

Figure 2. The transverse profile of SWs vs coordinate z for the nonlinear LH slab sandwiched between the uniform medium and the one-dimensional photonic crystal with (a) $|\gamma_0| = 0.2$ (two-humped structure) and (b) $\gamma_0 \rightarrow 0$ (one-humped structure). Here, we used $d_c = 2d_1$ and $\beta = 1.211$. The insets show blow-up regions of profiles at the surface of PC.

slab. To show the modes with different structures the transverse profile of some typical SWs corresponding to Fig. 1 are plotted in Fig. 2 as a function of coordinate z. As one can see from Fig. 2(a), the modes corresponding to $|\gamma_0| = 0.2$ have a two-humps structure around the interface while the modes corresponding to $|\gamma_0| = 0.0$ have a one-humped structure [see Fig. 2(b)]. Moreover, the structure supports

only the SW modes without any nodes within the slab (order of zero modes). Investigation of Fig. 1(b) shows that the RH slab can support the SWs with one or more nodes, the so-called order 1 or higher orders of SWs depending on the thicknesses of the nonlinear slab (see Fig. 3). We have also compared our results based on the first integral method with Runge-Kutta method. Our studies show good agreement between the analytical and the numerical results. The corresponding results are presented in Fig. 4. As was recently shown, the direction of the total
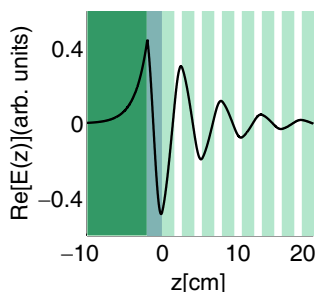

Figure 3. The transverse profile of order 1 SWs vs coordinate z for the nonlinear RH slab sandwiched between the uniform medium and the one-dimensional photonic crystal with $d_c = 2d_1$, $\gamma_0 = 0.2$ and $\beta = 1.211$. The other parameters are the same as Fig. 1.
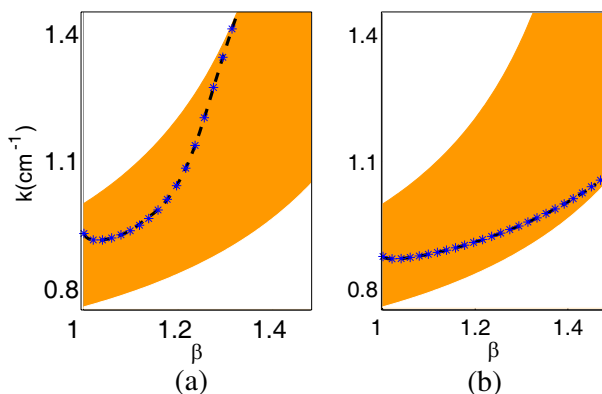

Figure 4. Dispersion curves of the SWs for (a) the LH slab with $\mu_c = -1$, $\varepsilon_c = -1$ and (b) the RH slab with $\mu_c = 1$, $\varepsilon_c = 4$ in the first spectral gap of the PC in the case of $|\gamma_0| = 0.2$. Here, the dashed and marked lines show dispersion from Jacobi elliptic functions and Runge-Kutta method, respectively. The other parameters are the same in Fig. 1.
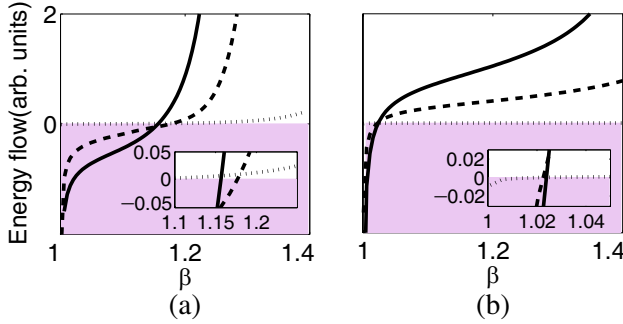
Figure 5. Total energy flow of the SWs for (a) the LH slab with $\mu_c = -1$, $\varepsilon_c = -1$ and (b) the RH slab with $\mu_c = 1$, $\varepsilon_c = 4$ vs β in the cases of $|\gamma_0| = 0.5$ (solid lines), $|\gamma_0| = 0.2$ (dashed lines), $|\gamma_0| \to 0$ (dotted lines). The other parameters are the same as Fig. 1.

energy flow of the SWs can be forward or backward in the presence of LHM [21]. Due to the localized nature of the SWs, the total energy flow of the SWs normal to the interface is zero. So, it is interesting to study the total energy flow of the nonlinear SWs along the interface. To demonstrate the backward or forward nature of the nonlinear surface modes we plotted the total energy flow of the modes as a function of β for different values of the parameter $|\gamma_0|$ (or different exciting intensity) in Fig. 5. As it is clear from Fig. 5(a), the energy flow of the SWs are in the backward or the forward directions at the region $1.065 < \beta < 1.177$ depending on the intensity. So, the forward SWs can be switched to the backward ones or vice versa by adjusting the intensity of the exciting field in the case of LH slab.

4. CONCLUSION

We have analytically studied the nonlinear TE-polarized SWs of a nonlinear LH or RH slab sandwiched between a homogeneous LH medium and a semi-infinite 1DPC. Our method is based on the first integral of the nonlinear Helmholtz wave equation. We have shown that these structures can support SWs with different transverse profiles. The intensity dependent properties of the nonlinear SWs for both cases of the LH or the RH slabs have been studied. It is shown that in the case of the self-focusing LH slab there is a possibility of switching between the forward and the backward SWs by varying the intensity of the electromagnetic field.

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