

A METRIC FUNCTION FOR FAST AND ACCURATE PERMITTIVITY DETERMINATION OF LOW-TO-HIGH-LOSS MATERIALS FROM REFLECTION MEASUREMENTS

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Abstract—We have derived a one-variable metric function for fast and accurate complex permittivity extraction of low-to-high-loss materials using reflection-only microwave non-resonant measurements at one frequency. The metric function can be modified to facilitate fast computation of the complex permittivity of materials for various applications (e.g., relative complex permittivity measurement of low-loss materials). It is useful as a measurement tool for broadband measurements of complex permittivity of samples with substantiate lengths. In addition, the method is applicable for measurement of complex permittivity of dispersive materials or complex permittivity of non-dispersive samples in limited frequency-band applications, since it is based on point-by-point (or frequency-by-frequency) extraction. It is validated by a numerical analysis and measurements of a liquid sample.

1. INTRODUCTION

Material characterization is an important issue in many material production, processing, and management applications in agriculture, food engineering, medical treatments, bioengineering, and the concrete industry [1]. In addition, microwave engineering requires precise knowledge of electromagnetic properties of materials at microwave frequencies since microwave communications are playing more and

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more important roles in military, industrial, and civilian life [1]. For these reasons, various microwave techniques have been introduced to characterize the electrical properties of materials. These methods can roughly be divided into resonant and non-resonant methods [1].

Resonant methods have much better accuracy and sensitivity than nonresonant methods [1, 2, 16, 17]. They are generally applied to characterization of low-loss materials and require a meticulous sample preparation before measurements. In addition, for an analysis over a broad frequency band, a new measurement set-up (a cavity) must be made. On the other hand, non-resonant methods have relatively higher accuracy over a broad frequency band and necessitate less sample preparation compared to resonant methods [1, 2]. Due to their relative simplicity, nonresonant waveguide (or coaxial) transmission/reflection methods are presently the most widely used broadband measurement techniques [1, 18].

Various non-resonant transmission-reflection methods have been proposed for electrical characterization of low-, medium, and high-loss materials [2–46]. Transmission measurements are convenient for gathering whole volume information [22–26], do not suffer much from surface roughness at high frequencies [22–26], and provide longitudinal averaging of variations in sample properties [21, 32]. On the other hand, reflection measurements are feasible for measurements where only one side of the sample is accessible [27] and provide higher accuracy over transmission measurements for electrical property extraction of high-loss samples. In a recent study, we proposed a generalized formulation for relative complex permittivity ($\varepsilon_r = \varepsilon'_r - j\varepsilon''_r$) extraction of low-to-high-loss samples using transmission-only scattering (S -) parameter measurements [32]. Although this study is important for fast and accurate computations of ε_r , there are some situations where reflection-only measurements are a necessity when only one side of the sample is accessible [23] and/or when the measured level of transmission measurements is lower than the threshold level of the measurement instrument for electrical property extraction of high-loss samples [29, 39]. To meet the demand for accurate ε_r measurement of materials in these circumstances, we have recently proposed two different methods [27, 28]. Although these methods are attractive in above-discussed circumstances, they do not present closed-form expressions for ε_r using reflection-only S -parameter measurements and thus are not much suitable for fast computations of ε_r . In this research paper, we present a metric function for fast and accurate ε_r extraction of low-to-high-loss samples using reflection-only measurements.

2. THEORETICAL BACKGROUND

It is assumed that a flat, isotropic and homogeneous dielectric sample with length L is positioned into a waveguide, as shown in Fig. 1, and higher-order modes appearing at sample front and end surfaces (and coupling of these modes) are negligible. We also assume that only the dominant mode (TE₁₀) is present inside the waveguide, which is effectively provided when measurements are taken away from the sample end surfaces.

Applying the boundary conditions at sample surfaces (the continuity of tangential electric and magnetic fields) for the vector potentials in sample and empty waveguide sections, we obtain reflection S -parameter $S_{11} = |S_{11}| e^{j\theta_{11}}$ as [41]

$$|S_{11}| = \sqrt{(\Lambda_1^2 + \Lambda_2) (1 + B^2 - 2B \cos(A)) / \psi}, \tag{1}$$

$$\begin{aligned} \theta_{11} = & -\arctan [\xi / (\chi - \kappa)] - \arctan [\xi / (\chi + \kappa)] \\ & + \arctan \left(\frac{B \sin(A)}{1 - B \cos(A)} \right) - \arctan \left(\frac{\Omega_1}{\Omega_2} \right), \end{aligned} \tag{2}$$

where $|S_{11}|$ and θ_{11} denote the magnitude and the phase of S_{11} , and

$$\chi - j\xi = \sqrt{\epsilon_r - (\lambda_0/\lambda_c)^2}, \quad B = \exp(-4\pi\xi L/\lambda_0), \tag{3}$$

$$A = 4\pi\chi L/\lambda_0, \quad \kappa = \sqrt{1 - (\lambda_0/\lambda_c)^2} \tag{4}$$

$$\psi = B^2\Lambda_3^2 + \Lambda_4^2 + 8\kappa\xi B \sin(A) \Lambda_1 - 2B \cos(A) (\Lambda_1^2 - \Lambda_2), \tag{5}$$

$$\Lambda_1 = \chi^2 + \xi^2 - \kappa^2, \quad \Lambda_2 = 4\kappa^2\xi^2, \tag{6}$$

$$\Lambda_3 = (\chi - \kappa)^2 + \xi^2, \quad \Lambda_4 = (\chi + \kappa)^2 + \xi^2, \tag{7}$$

$$\Omega_1 = B \left\{ 2\xi \cos(A) (\chi - \kappa) + \sin(A) [(\chi - \kappa)^2 - \xi^2] \right\} - 2\xi (\chi + \kappa), \tag{8}$$

$$\Omega_2 = B \left\{ 2\xi \sin(A) (\chi - \kappa) - \cos(A) [(\chi - \kappa)^2 - \xi^2] \right\} + (\chi + \kappa)^2 - \xi^2. \tag{9}$$

Here, $\lambda_0 = c/f$ and $\lambda_c = c/f_c$ correspond to the free-space and cut-off wavelengths; and f , f_c , and c are the operating and cut-off frequencies and the speed of light, respectively.

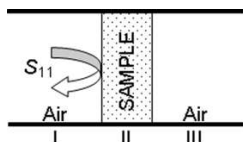


Figure 1. The configuration for reflection S -parameter measurements in a waveguide.

3. CLOSED-FORM EXPRESSIONS

In contrast to the expectation, a unique solution is generally not obtained for ε_r of a thicker sample at one fixed frequency using (1)–(9) because of the presence of transcendental terms in (1)–(9). However, a good initial guess for ε_r can provide a unique and accurate solution. Nonetheless, fast and precise computation of this unique solution is important. For this purpose, in this section, we derive a metric function based on the reflection-only S -parameter measurements. Using (1), we obtain an equation for B as

$$\begin{aligned} & \left[|S_{11}|^2 \Lambda_3^2 - (\Lambda_1^2 + \Lambda_2) \right] B^2 + 2 \left\{ 4 |S_{11}|^2 \kappa \xi \sin(A) \Lambda_1 + \left[(1 - |S_{11}|^2) \Lambda_1^2 \right. \right. \\ & \left. \left. + (1 + |S_{11}|^2) \Lambda_2 \right] \cos(A) \right\} B + |S_{11}|^2 \Lambda_4^2 - (\Lambda_1^2 + \Lambda_2) = 0 \end{aligned} \quad (10)$$

In the same manner, using (2), we find

$$\begin{aligned} & \Omega_3 \Omega_2 (1 - B \cos(A)) + B \sin(A) \Omega_1 \Omega_3 \\ & = \Omega_2 B \sin(A) - \Omega_1 (1 - B \cos(A)) \end{aligned} \quad (11)$$

where

$$\begin{aligned} \Omega_3 & = \tan \left(\theta_{11} + \arctan \left(\frac{\xi}{\chi - \kappa} \right) + \arctan \left(\frac{\xi}{\chi + \kappa} \right) \right) \\ & = \frac{-\tan(\theta_{11}) \xi^2 + 2\chi\xi + \tan(\theta_{11}) (\chi^2 - \kappa^2)}{-\xi^2 - 2\chi \tan(\theta_{11}) \xi + (\chi^2 - \kappa^2)}. \end{aligned} \quad (12)$$

After some manipulations and using (8) and (9), we express (11) as

$$(\alpha_2 \cos(A) + \alpha_4) B^2 + (\alpha_1 \cos(A) + \alpha_3 - \alpha_2) B - \alpha_1 = 0 \quad (13)$$

where

$$\alpha_1 = \left[(\chi + \kappa)^2 - \xi^2 \right] \Omega_3 - 2\xi(\chi + \kappa), \quad (14)$$

$$\begin{aligned} \alpha_2 & = 2\xi [\cos(A) + \Omega_3 \sin(A)] (\chi - \kappa) \\ & \quad + [\sin(A) - \Omega_3 \cos(A)] \left[(\chi - \kappa)^2 - \xi^2 \right], \end{aligned} \quad (15)$$

$$\alpha_3 = \sin(A) \left[\left((\chi + \kappa)^2 - \xi^2 \right) + 2\Omega_3 (\chi + \kappa) \xi \right], \quad (16)$$

$$\begin{aligned} \alpha_4 & = \sin(A) \left\{ 2\xi [\sin(A) - \Omega_3 \cos(A)] (\chi - \kappa) \right. \\ & \quad \left. - (\cos(A) + \sin(A) \Omega_3) \left[(\chi - \kappa)^2 - \xi^2 \right] \right\} \end{aligned} \quad (17)$$

The equations for B in (10) and (13) are quadratic functions, and the selection of a correct root for B from either requires an elaborate analysis and thus is not feasible. Instead, (10) and (13) can be

simultaneously utilized, the terms containing B^2 can be eliminated, and an explicit expression for B can be derived. Following these steps, we obtain B as

$$B = \frac{\alpha_6 (\alpha_2 \cos(A) + \alpha_4) - \alpha_1 \alpha_7}{\alpha_5 (\alpha_2 \cos(A) + \alpha_4) - \alpha_7 (\alpha_1 \cos(A) + \alpha_3 - \alpha_2)}, \quad (18)$$

where

$$\alpha_5 = 2 \left\{ 4 |S_{11}|^2 \kappa \xi \sin(A) \Lambda_1 + \left[(1 - |S_{11}|^2) \Lambda_1^2 + (1 + |S_{11}|^2) \Lambda_2 \right] \cos(A) \right\}, \quad (19)$$

$$\alpha_6 = (\Lambda_1^2 + \Lambda_2) - |S_{11}|^2 \Lambda_4^2, \quad \alpha_7 = -(\Lambda_1^2 + \Lambda_2) + |S_{11}|^2 \Lambda_3^2. \quad (20)$$

Then, substituting (18) into (10), we derive a metric function in polynomials of ξ as

$$F(\chi, \xi) = P_1 \xi^{18} + P_2 \xi^{17} + P_3 \xi^{16} + P_4 \xi^{15} + P_5 \xi^{14} + P_6 \xi^{13} + P_7 \xi^{12} + P_8 \xi^{11} + P_9 \xi^{10} + P_{10} \xi^9 + P_{11} \xi^8 + P_{12} \xi^7 + P_{13} \xi^6 + P_{14} \xi^5 + P_{15} \xi^4 + P_{16} \xi^3 + P_{17} \xi^2 + P_{18} \xi + P_{19} = 0, \quad (21)$$

where

$$P_1 = z_1 z_9 g_1 - h_1 z_1^2 - h_1 z_9^2, \quad (22)$$

$$P_2 = g_1 (z_1 z_{10} + z_2 z_9) + g_2 z_1 z_9 - 2h_1 (z_1 z_2 - z_9 z_{10}),$$

$$P_3 = g_1 (z_1 z_{11} + z_2 z_{10} + z_3 z_9) + g_2 (z_1 z_{10} + z_2 z_9) + g_3 z_1 z_9 - h_1 (z_2^2 + 2z_1 z_3) - (h_2 + h_3) z_1^2 - h_1 (z_{10}^2 + 2z_9 z_{11}) - (h_2 - h_3) z_9^2, \quad (23)$$

$$P_4 = g_1 (z_1 z_{12} + z_2 z_{11} + z_3 z_{10} + z_4 z_9) + g_2 (z_1 z_{11} + z_2 z_{10} + z_3 z_9) + g_3 (z_1 z_{10} + z_2 z_9) + g_4 z_1 z_9 - 2 [h_1 (z_1 z_4 + z_2 z_3) + (h_2 + h_3) z_1 z_2] - 2 [h_1 (z_9 z_{12} + z_{10} z_{11}) + (h_2 - h_3) z_9 z_{10}], \quad (24)$$

$$P_5 = g_1 (z_1 z_{13} + z_2 z_{12} + z_3 z_{11} + z_4 z_{10} + z_5 z_9) + g_2 (z_1 z_{12} + z_2 z_{11} + z_3 z_{10} + z_4 z_9) + g_5 z_1 z_9 + g_4 (z_1 z_{10} + z_2 z_9) + g_3 (z_1 z_{11} + z_2 z_{10} + z_3 z_9) - h_1 (z_3^2 + 2z_1 z_5 + 2z_2 z_4) - h_5 z_1^2 - (h_2 + h_3) (z_2^2 + 2z_1 z_3) - h_1 (z_{11}^2 + 2z_9 z_{13} + 2z_{10} z_{12}) - (h_2 - h_3) (z_{10}^2 + 2z_9 z_{11}) - h_4 z_9^2, \quad (25)$$

$$P_6 = g_1 (z_1 z_{14} + z_2 z_{13} + z_3 z_{12} + z_4 z_{11} + z_5 z_{10} + z_6 z_9) + g_3 (z_1 z_{12} + z_2 z_{11} + z_3 z_{10} + z_4 z_9) + g_2 (z_1 z_{13} + z_2 z_{12} + z_3 z_{11} + z_4 z_{10} + z_5 z_9) + g_5 (z_1 z_{10} + z_2 z_9) - 2h_1 (z_1 z_6 + z_2 z_5 + z_3 z_4) + g_4 (z_1 z_{11} + z_2 z_{10} + z_3 z_9) - 2 (h_2 + h_3) (z_1 z_4 + z_2 z_3) - 2h_5 z_1 z_2 - 2h_4 z_9 z_{10} - 2h_1 (z_9 z_{14} + z_{10} z_{13} + z_{11} z_{12}) - 2 (h_2 - h_3) (z_9 z_{12} + z_{10} z_{11}), \quad (26)$$

$$\begin{aligned}
P_7 = & g_1 (z_1 z_{15} + z_2 z_{14} + z_3 z_{13} + z_4 z_{12} + z_5 z_{11} + z_6 z_{10} + z_7 z_9) \\
& + g_4 (z_1 z_{12} + z_2 z_{11} + z_3 z_{10} + z_4 z_9) \\
& + g_2 (z_1 z_{14} + z_2 z_{13} + z_3 z_{12} + z_4 z_{11} + z_5 z_{10} + z_6 z_9) \\
& + g_5 (z_1 z_{11} + z_2 z_{10} + z_3 z_9) \\
& + g_3 (z_1 z_{13} + z_2 z_{12} + z_3 z_{11} + z_4 z_{10} + z_5 z_9) \\
& - h_1 (z_4^2 + 2z_1 z_7 + 2z_2 z_6 + 2z_3 z_5) \\
& - (h_2 + h_3) (z_3^2 + 2z_1 z_5 + 2z_2 z_4) - h_5 (z_2^2 + 2z_1 z_3) \\
& - h_1 (z_{12}^2 + 2z_9 z_{15} + 2z_{10} z_{14} + 2z_{11} z_{13}) \\
& - (h_2 - h_3) (z_{11}^2 + 2z_9 z_{13} + 2z_{10} z_{12}) - h_4 (z_{10}^2 + 2z_9 z_{11}), \quad (27)
\end{aligned}$$

$$\begin{aligned}
P_8 = & g_1 (z_1 z_{16} + z_2 z_{15} + z_3 z_{14} + z_4 z_{13} + z_5 z_{12} + z_6 z_{11} + z_7 z_{10} + z_8 z_9) \\
& - 2h_5 (z_1 z_4 + z_2 z_3) \\
& + g_2 (z_1 z_{15} + z_2 z_{14} + z_3 z_{13} + z_4 z_{12} + z_5 z_{11} + z_6 z_{10} + z_7 z_9) \\
& - 2h_1 (z_1 z_8 + z_2 z_7 + z_3 z_6 + z_4 z_5) \\
& + g_3 (z_1 z_{14} + z_2 z_{13} + z_3 z_{12} + z_4 z_{11} + z_5 z_{10} + z_6 z_9) \\
& - 2h_1 (z_9 z_{16} + z_{10} z_{15} + z_{11} z_{14} + z_{12} z_{13}) \\
& + g_4 (z_1 z_{13} + z_2 z_{12} + z_3 z_{11} + z_4 z_{10} + z_5 z_9) \\
& - 2(h_2 - h_3) (z_9 z_{14} + z_{10} z_{13} + z_{11} z_{12}) \\
& + g_5 (z_1 z_{12} + z_2 z_{11} + z_3 z_{10} + z_4 z_9) \\
& - 2(h_2 + h_3) (z_1 z_6 + z_2 z_5 + z_3 z_4) - 2h_4 (z_9 z_{12} + z_{10} z_{11}), \quad (28)
\end{aligned}$$

$$\begin{aligned}
P_9 = & g_1 (z_2 z_{16} + z_3 z_{15} + z_4 z_{14} + z_5 z_{13} + z_6 z_{12} + z_7 z_{11} + z_8 z_{10}) \\
& - h_4 (z_{11}^2 + 2z_9 z_{13} + 2z_{10} z_{12}) \\
& + g_2 (z_1 z_{16} + z_2 z_{15} + z_3 z_{14} + z_4 z_{13} + z_5 z_{12} + z_6 z_{11} + z_7 z_{10} + z_8 z_9) \\
& - h_5 (z_3^2 + 2z_1 z_5 + 2z_2 z_4) \\
& + g_3 (z_1 z_{15} + z_2 z_{14} + z_3 z_{13} + z_4 z_{12} + z_5 z_{11} + z_6 z_{10} + z_7 z_9) \\
& - h_1 (z_5^2 + 2z_2 z_8 + 2z_3 z_7 + 2z_4 z_6) \\
& + g_4 (z_1 z_{14} + z_2 z_{13} + z_3 z_{12} + z_4 z_{11} + z_5 z_{10} + z_6 z_9) \\
& - (h_2 + h_3) (z_4^2 + 2z_1 z_7 + 2z_2 z_6 + 2z_3 z_5) \\
& + g_5 (z_1 z_{13} + z_2 z_{12} + z_3 z_{11} + z_4 z_{10} + z_5 z_9) \\
& - h_1 (z_{13}^2 + 2z_{10} z_{16} + 2z_{11} z_{15} + 2z_{12} z_{14}) \\
& - (h_2 - h_3) (z_{12}^2 + 2z_9 z_{15} + 2z_{10} z_{14} + 2z_{11} z_{13}), \quad (29)
\end{aligned}$$

$$\begin{aligned}
 P_{10} = & g_3 (z_1 z_{16} + z_2 z_{15} + z_3 z_{14} + z_4 z_{13} + z_5 z_{12} + z_6 z_{11} + z_7 z_{10} + z_8 z_9) \\
 & - 2h_5 (z_1 z_6 + z_2 z_5 + z_3 z_4) \\
 & + g_1 (z_3 z_{16} + z_4 z_{15} + z_5 z_{14} + z_6 z_{13} + z_7 z_{12} + z_8 z_{11}) \\
 & - 2(h_2 + h_3) (z_1 z_8 + z_2 z_7 + z_3 z_6 + z_4 z_5) \\
 & + g_2 (z_2 z_{16} + z_3 z_{15} + z_4 z_{14} + z_5 z_{13} + z_6 z_{12} + z_7 z_{11} + z_8 z_{10}) \\
 & - 2h_1 (z_3 z_8 + z_4 z_7 + z_5 z_6) \\
 & + g_4 (z_1 z_{15} + z_2 z_{14} + z_3 z_{13} + z_4 z_{12} + z_5 z_{11} + z_6 z_{10} + z_7 z_9) \\
 & - 2h_4 (z_9 z_{14} + z_{10} z_{13} + z_{11} z_{12}) \\
 & + g_5 (z_1 z_{14} + z_2 z_{13} + z_3 z_{12} + z_4 z_{11} + z_5 z_{10} + z_6 z_9) \\
 & - 2h_1 (z_{11} z_{16} + z_{12} z_{15} + z_{13} z_{14}) \\
 & - 2(h_2 - h_3) (z_9 z_{16} + z_{10} z_{15} + z_{11} z_{14} + z_{12} z_{13}), \tag{30}
 \end{aligned}$$

$$\begin{aligned}
 P_{11} = & g_4 (z_1 z_{16} + z_2 z_{15} + z_3 z_{14} + z_4 z_{13} + z_5 z_{12} + z_6 z_{11} + z_7 z_{10} + z_8 z_9) \\
 & - h_1 (z_6^2 + 2z_4 z_8 + 2z_5 z_7) \\
 & + g_1 (z_4 z_{16} + z_5 z_{15} + z_6 z_{14} + z_7 z_{13} + z_8 z_{12}) \\
 & - h_4 (z_{12}^2 + 2z_9 z_{15} + 2z_{10} z_{14} + 2z_{11} z_{13}) \\
 & + g_2 (z_3 z_{16} + z_4 z_{15} + z_5 z_{14} + z_6 z_{13} + z_7 z_{12} + z_8 z_{11}) \\
 & - h_5 (z_4^2 + 2z_1 z_7 + 2z_2 z_6 + 2z_3 z_5) \\
 & + g_3 (z_2 z_{16} + z_3 z_{15} + z_4 z_{14} + z_5 z_{13} + z_6 z_{12} + z_7 z_{11} + z_8 z_{10}) \\
 & - h_1 (z_{14}^2 + 2z_{12} z_{16} + 2z_{13} z_{15}) \\
 & + g_5 (z_1 z_{15} + z_2 z_{14} + z_3 z_{13} + z_4 z_{12} + z_5 z_{11} + z_6 z_{10} + z_7 z_9) \\
 & - (h_2 + h_3) (z_5^2 + 2z_2 z_8 + 2z_3 z_7 + 2z_4 z_6) \\
 & - (h_2 - h_3) (z_{13}^2 + 2z_{10} z_{16} + 2z_{11} z_{15} + 2z_{12} z_{14}), \tag{31}
 \end{aligned}$$

$$\begin{aligned}
 P_{12} = & g_1 (z_5 z_{16} + z_6 z_{15} + z_7 z_{14} + z_8 z_{13}) \\
 & + g_2 (z_4 z_{16} + z_5 z_{15} + z_6 z_{14} + z_7 z_{13} + z_8 z_{12}) \\
 & + g_3 (z_3 z_{16} + z_4 z_{15} + z_5 z_{14} + z_6 z_{13} + z_7 z_{12} + z_8 z_{11}) \\
 & - 2h_5 (z_1 z_8 + z_2 z_7 + z_3 z_6 + z_4 z_5) \\
 & + g_4 (z_2 z_{16} + z_3 z_{15} + z_4 z_{14} + z_5 z_{13} + z_6 z_{12} + z_7 z_{11} + z_8 z_{10}) \\
 & - 2h_1 (z_{13} z_{16} + z_{14} z_{15}) \\
 & + g_5 (z_1 z_{16} + z_2 z_{15} + z_3 z_{14} + z_4 z_{13} + z_5 z_{12} + z_6 z_{11} + z_7 z_{10} + z_8 z_9) \\
 & - 2h_1 (z_5 z_8 + z_6 z_7) - 2(h_2 + h_3) (z_3 z_8 + z_4 z_7 + z_5 z_6) \\
 & - 2h_4 (z_9 z_{16} + z_{10} z_{15} + z_{11} z_{14} + z_{12} z_{13}) \\
 & - 2(h_2 - h_3) (z_{11} z_{16} + z_{12} z_{15} + z_{13} z_{14}), \tag{32}
 \end{aligned}$$

$$\begin{aligned}
P_{13} = & g_1 (z_6 z_{16} + z_7 z_{15} + z_8 z_{14}) + g_2 (z_5 z_{16} + z_6 z_{15} + z_7 z_{14} + z_8 z_{13}) \\
& - h_1 (z_7^2 + 2z_6 z_8) + g_3 (z_4 z_{16} + z_5 z_{15} + z_6 z_{14} + z_7 z_{13} + z_8 z_{12}) \\
& - (h_2 - h_3) (z_{14}^2 + 2z_{12} z_{16} + 2z_{13} z_{15}) \\
& + g_4 (z_3 z_{16} + z_4 z_{15} + z_5 z_{14} + z_6 z_{13} + z_7 z_{12} + z_8 z_{11}) \\
& - (h_2 + h_3) (z_6^2 + 2z_4 z_8 + 2z_5 z_7) \\
& + g_5 (z_2 z_{16} + z_3 z_{15} + z_4 z_{14} + z_5 z_{13} + z_6 z_{12} + z_7 z_{11} + z_8 z_{10}) \\
& - h_1 (z_{15}^2 + 2z_{14} z_{16}) - h_5 (z_5^2 + 2z_2 z_8 + 2z_3 z_7 + 2z_4 z_6) \\
& - h_4 (z_{13}^2 + 2z_{10} z_{16} + 2z_{11} z_{15} + 2z_{12} z_{14}), \tag{33}
\end{aligned}$$

$$\begin{aligned}
P_{14} = & g_1 (z_7 z_{16} + z_8 z_{15}) + g_2 (z_6 z_{16} + z_7 z_{15} + z_8 z_{14}) \\
& + g_3 (z_5 z_{16} + z_6 z_{15} + z_7 z_{14} + z_8 z_{13}) \\
& + g_4 (z_4 z_{16} + z_5 z_{15} + z_6 z_{14} + z_7 z_{13} + z_8 z_{12}) - 2h_1 z_{15} z_{16} \\
& - 2(h_2 - h_3) (z_{13} z_{16} + z_{14} z_{15}) \\
& + g_5 (z_3 z_{16} + z_4 z_{15} + z_5 z_{14} + z_6 z_{13} + z_7 z_{12} + z_8 z_{11}) \\
& - 2h_1 z_7 z_8 - 2(h_2 + h_3) (z_5 z_8 + z_6 z_7) \\
& - 2h_4 (z_{11} z_{16} + z_{12} z_{15} + z_{13} z_{14}) - 2h_5 (z_3 z_8 + z_4 z_7 + z_5 z_6), \tag{34}
\end{aligned}$$

$$\begin{aligned}
P_{15} = & g_2 (z_7 z_{16} + z_8 z_{15}) + g_3 (z_6 z_{16} + z_7 z_{15} + z_8 z_{14}) \\
& + g_4 (z_5 z_{16} + z_6 z_{15} + z_7 z_{14} + z_8 z_{13}) - h_1 z_8^2 \\
& + g_1 z_8 z_{16} + g_5 (z_4 z_{16} + z_5 z_{15} + z_6 z_{14} + z_7 z_{13} + z_8 z_{12}) \\
& - (h_2 + h_3) (z_7^2 + 2z_6 z_8) - h_1 z_{16}^2 - h_5 (z_6^2 + 2z_4 z_8 + 2z_5 z_7) \\
& - (h_2 - h_3) (z_{15}^2 + 2z_{14} z_{16}) - h_4 (z_{14}^2 + 2z_{12} z_{16} + 2z_{13} z_{15}), \tag{35}
\end{aligned}$$

$$\begin{aligned}
P_{16} = & +g_3 (z_7 z_{16} + z_8 z_{15}) + g_4 (z_6 z_{16} + z_7 z_{15} + z_8 z_{14}) \\
& - 2h_4 (z_{13} z_{16} + z_{14} z_{15}) - 2(h_2 + h_3) z_7 z_8 \\
& + g_5 (z_5 z_{16} + z_6 z_{15} + z_7 z_{14} + z_8 z_{13}) - 2h_5 (z_5 z_8 + z_6 z_7) \\
& + g_2 z_8 z_{16} - 2(h_2 - h_3) z_{15} z_{16}, \tag{36}
\end{aligned}$$

$$\begin{aligned}
P_{17} = & g_3 z_8 z_{16} + g_4 (z_7 z_{16} + z_8 z_{15}) + g_5 (z_6 z_{16} + z_7 z_{15} + z_8 z_{14}) \\
& - (h_2 + h_3) z_8^2 - h_5 (z_7^2 + 2z_6 z_8) - (h_2 - h_3) z_{16}^2 \\
& - h_4 (z_{15}^2 + 2z_{14} z_{16}), \tag{37}
\end{aligned}$$

$$P_{18} = g_4 z_8 z_{16} + g_5 (z_7 z_{16} + z_8 z_{15}) - 2h_5 z_7 z_8 - 2h_4 z_{15} z_{16}, \tag{38}$$

$$P_{19} = g_5 z_8 z_{16} - h_5 z_8^2 - h_4 z_{16}^2.$$

In (22)–(38), the intermediate variables are given as

$$\begin{aligned}
z_1 = 4\kappa h_1, \quad z_2 = 4\chi\kappa \tan(\theta_{11}) \left(1 + |S_{11}|^2\right), \\
z_{10} = 4\kappa \left(\chi y_3 - |S_{11}|^2 (\chi - 4\kappa) y_1\right), \tag{39}
\end{aligned}$$

$$z_5 = 4\kappa (3\chi^4 + 2\chi^2\kappa^2 + 3\kappa^4) h_1, \tag{40}$$

$$z_7 = 4\kappa (\chi^2 - \kappa^2)^2 (\chi^2 + \kappa^2) h_1,$$

$$z_8 = 4\chi\kappa \tan(\theta_{11}) \left(1 + |S_{11}|^2\right) (\chi^2 - \kappa^2)^3, \tag{41}$$

$$z_4 = 4\chi\kappa \tan(\theta_{11}) (3\chi^2 + \kappa^2) \left(1 + |S_{11}|^2\right),$$

$$z_9 = 4\kappa \left(y_4 - |S_{11}|^2 y_2\right), \tag{42}$$

$$z_{11} = 4\kappa \left[3(\chi^2 + \kappa^2) y_4 - |S_{11}|^2 (3\chi^2 - 5\kappa^2) y_2\right],$$

$$z_{12} = 4\chi\kappa \left[y_3 (3\chi^2 + \kappa^2) - 3y_1 |S_{11}|^2 (\chi^2 - 4\chi\kappa + 3\kappa^2)\right], \tag{43}$$

$$z_{13} = 4\kappa \left[(3\chi^4 + 2\chi^2\kappa^2 + 3\kappa^4) y_4 - |S_{11}|^2 y_2 (3\chi^4 - 14\chi^2\kappa^2 + 16\chi\kappa^3 - 5\kappa^4)\right], \tag{44}$$

$$z_{14} = 4\kappa (\chi - \kappa) \left[\chi(\chi + \kappa) (3\chi^2 + \kappa^2) y_3 - |S_{11}|^2 (3\chi^4 - 9\chi^3\kappa + 5\chi^2\kappa^2 + 5\chi\kappa^3 - 4\kappa^4) y_1\right], \tag{45}$$

$$z_{15} = 4\kappa (\chi - \kappa)^2 \left[(\chi + \kappa)^2 (\chi^2 + \kappa^2) y_4 - |S_{11}|^2 (\chi - \kappa)^2 (\chi^2 + 4\chi\kappa + \kappa^2) y_2\right], \tag{46}$$

$$z_{16} = 4\chi\kappa (\chi^2 - \kappa^2) (\chi - \kappa)^2 \left[(\chi + \kappa)^2 y_3 - |S_{11}|^2 (\chi - \kappa)^2 y_1\right], \tag{47}$$

$$y_1 = \sin(A) - \cos(A) \tan(\theta_{11}), \quad y_2 = \cos(A) + \sin(A) \tan(\theta_{11}), \tag{48}$$

$$y_3 = \sin(A) + \cos(A) \tan(\theta_{11}), \quad y_4 = \cos(A) - \sin(A) \tan(\theta_{11}), \tag{49}$$

$$g_1 = 2 \cos(A) h_1, \quad g_2 = 8 |S_{11}|^2 \kappa \sin(A),$$

$$g_3 = 2g_1\chi^2 + 4 \cos(A) \left(1 + 3 |S_{11}|^2\right) \kappa^2, \tag{50}$$

$$g_4 = g_2 (\chi^2 - \kappa^2), \quad g_5 = g_1 (\chi^2 - \kappa^2)^2,$$

$$h_1 = 1 - |S_{11}|^2, \quad h_2 = 2h_1 (\chi^2 + \kappa^2), \quad h_3 = 4 |S_{11}|^2 \kappa\chi,$$

$$h_4 = (\chi^2 - \kappa^2)^2 - |S_{11}|^2 (\chi + \kappa)^4, \quad h_5 = (\chi^2 - \kappa^2)^2 - |S_{11}|^2 (\chi - \kappa)^4, \tag{51}$$

At first moment, it seems that some redundancy appear in the derivation of the metric function, $F(\chi, \xi)$, in (21), since we invoke (10) two times for the derivation of $F(\chi, \xi)$. However, since we have two equations for B in (10) and (13), this invoking is a requirement to eliminate an elaborate analysis for assigning a correct root for B from either (10) or (13).

We note that, in the derivation of the metric function in (21) and of its coefficients in (22)–(51), symbolic functions of MATLAB can be utilized with the help of MAPLE. However, the symbolic functions of MATLAB fail to produce any result if the analyzed expressions are intricate. In this paper, because the expressions in our analysis are complex, we solely utilized paper and pencil for the derivation of the metric function and its coefficients.

The roots of ξ in $F(\chi, \xi)$ in (21) can easily be computed using the 'roots' function of MATLAB. After obtaining these roots using the following constraints $\xi > 0$ and ξ : real, χ can be determined from (1). Eventually, ε_r can be found using (3).

4. VALIDATION OF THE METRIC FUNCTION

For validation of the closed-form expression in (21), we perform a numerical analysis. We first assume a test ε_r value of a sample with a known length, then obtain $|S_{11}|$ and θ_{11} for a given f and f_c using (1)–(9), and finally use $F(\chi, \xi)$ in (21) with different powers of ξ to inverse the test value. For example, Table 1 illustrates various test and inversed values of ε_r using $F(\chi, \xi)$ with various powers for $L = 20$ mm, $f = 10$ GHz, and $f_c = 6.555$ GHz.

It is obvious from Table 1 that, as the loss tangent of the sample decreases (first row of the entry), we can utilize lower powers of ξ in $F(\chi, \xi)$ for determining accurate ε_r . On contrary, we must utilize higher powers of ξ in $F(\chi, \xi)$ for correct inversion of ε_r for lossy samples (last row of the entry). A closer investigation of the extracted ε_r values in Table 1 demonstrates that sometimes using lower powers of ξ in $F(\chi, \xi)$ can result in better accuracy than using higher powers of ξ in $F(\chi, \xi)$ (last four rows of entry). This can be explained by considering the Taylor series expansion with fewer terms than the necessary ones.

Table 1. Computed ε_r using the derived metric function, $F(\chi, \xi)$, in (21) for its various degrees of power.

Test value, ε_r	Degree of powers of $F(\chi, \xi)$ in (21)				
	18	15	10	5	2
$10 - j0.05$	$10.000 - j0.050$	$10.000 - j0.050$	$10.000 - j0.050$	$10.000 - j0.050$	$10.000 - j0.050$
$10 - j0.5$	$10.000 - j0.500$	$10.000 - j0.500$	$10.000 - j0.500$	$10.000 - j0.500$	$10.000 - j0.500$
$10 - j5$	$10.000 - j5.000$	$10.000 - j5.000$	$10.000 - j5.000$	$9.944 - j5.227$	$9.554 - j6.571$
$10 - j10$	$10.000 - j10.000$	$10.000 - j10.000$	$9.989 - j10.008$	$9.887 - j10.244$	$9.997 - j9.999$
$10 - j15$	$10.000 - j15.000$	$10.000 - j15.000$	$9.841 - j15.290$	$10.555 - j13.933$	$9.997 - j14.998$
$10 - j20$	$10.000 - j20.000$	$9.999 - j20.004$	$9.694 - j20.482$	$0.622 - j31.551$	$9.996 - j19.998$

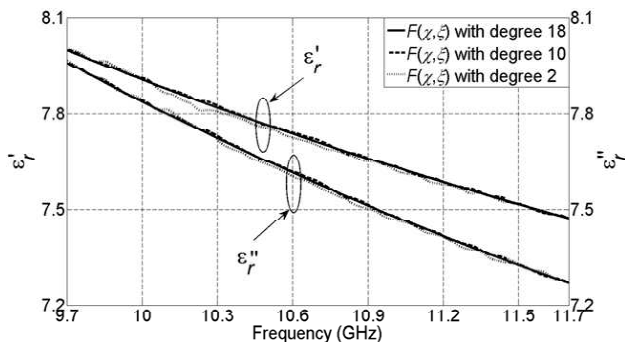


Figure 2. Extracted ε_r of a binary mixture of ethyl alcohol (75%) and water (25%) using various powers of ξ in $F(\chi, \xi)$ in (21).

As a result, comparing the values of ξ obtained from successive lower orders of ξ in $F(\chi, \xi)$ at a given frequency in the band, one can quickly determine the accurate value of ε_r using reflection measurements in a whole frequency band.

5. MEASUREMENTS

For validation of the proposed method, we utilized the measurement data of a binary mixture of ethyl alcohol (75%) and water (25%) solution [31]. For example, Fig. 2 illustrates the extracted ε_r by the proposed method using $F(\chi, \xi)$ in (21) with various degrees of power of ξ . In the application of the proposed method, we first utilize a guess value for the ε_r from the data in the literature, then refine this guess, and finally determine the ε_r using (21)–(51).

It is seen from Fig. 2 that, the extracted ε_r values especially for the used higher order of ξ in (21) are in good agreement with the theoretical data obtained from Debye model. This is because, for lossy materials, the effect of ξ is dominant, and higher-order terms of ξ in (21) cannot be directly ignored for the inversion of ε_r .

6. CONCLUSION

A one-variable metric function has been derived for fast complex permittivity determination of low-to-high-loss materials. It can be simplified or modified based on the nature of the problem to facilitate fast permittivity extraction. It is useful as a measurement tool for broadband measurements of complex permittivity of samples with substantiate lengths. It has been shown that comparing the values

of ξ obtained from successive lower orders of ξ in $F(\chi, \xi)$ at a given frequency in the band, one can quickly determine the accurate value of ε_r using reflection measurements in a whole frequency band. The expressions were validated by measurements of ε_r of ethyl alcohol and water solution.

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