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COUPLED NONLINEAR TRANSMISSION LINES FOR DOUBLING REPETITION RATE OF INCIDENT PULSE STREAMS

K. Narahara

Graduate School of Science and Engineering
Yamagata University
4-3-16 Jonan, Yonezawa, Yamagata 992-8510, Japan

Abstract—We investigated the properties of pulse propagation on coupled nonlinear transmission lines to develop a method for doubling repetition rate of incident pulse streams. Coupled nonlinear transmission lines are two transmission lines with regularly spaced Schottky varactors coupled with each other. It is found that both of the modes developed in a coupled line can support soliton-like pulses because of Schottky varactors. We discuss the fundamental properties of each soliton-like pulse, including the width and velocity, and propose a method of doubling repetition rate of incident pulse streams by managing these soliton-like pulses.

1. INTRODUCTION

A nonlinear transmission line (NLTL) is defined as a lumped transmission line containing a series inductor and a shunt Schottky varactor in each section. NLTLs are used for the development of solitons [1]. The operation bandwidth of carefully designed Schottky varactors goes beyond 100 GHz; therefore, they are employed in ultrafast electronic circuits including the subpicosecond electrical shock generator [2]. Recently, Kintis et al. [3] proposed a NLTL pulse generator using two NLTLs. They arranged the diodes in these two NLTLs having opposite polarities in order to sharpen the two signals' rising and falling edges separately, and obtained a short pulse with 30 ps duration. Moreover, Yildirim et al. [4] proposed a method of generating a periodic short pulse train using an NLTL connected with an amplifier. They generated a stable periodic pulse train with the

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Corresponding author: K. Narahara (narahara@yz.yamagata-u.ac.jp).

repetition rate of 100 MHz. To extend the potential of NLTLs in ultrafast electronics, we consider two coupled NLTLs and find that both modes developed in a coupled NLTL preserved the original shape by compensation of dispersive distortions using nonlinearity. A coupled NLTL with strong dispersion has been investigated based on the nonlinear Schrödinger equation [5, 6]. In this article, we consider the weakly dispersive coupled NLTLs in order to develop baseband pulses governed by the Korteweg de-Vries (KdV) equation and propose a method of doubling repetition rate of pulse stream input to the line.

First, we discuss the fundamental properties of the nonlinear pulses in a coupled NLTL. We quantify how the velocity and width of the nonlinear pulse on a coupled NLTL depend on the amplitude for the first time. We then describe how to double repetition rate of pulse streams, including the analytically-obtained design criteria and also includes several results of numerical evaluations that validate this method.

2. COUPLED NLTLS

Figure 1 shows the diagram of a coupled NLTL. Two NLTLs, denoted by line 1 and line 2, are coupled via C_m. For the line i $(i = 1, 2)$, L_i and C_i represent the series inductor and shunt Schottky varactor of the unit cell, respectively. $V_n(W_n)$ and $I_n(J_n)$ show the line voltage and current, respectively, at the nth cell of line 1 (line 2). The capacitance–voltage relationship of a Schottky varactor is generally given by

$$C(x) = \frac{C_0}{\left(1 - \frac{x}{V_J}\right)^m}, \tag{1}$$

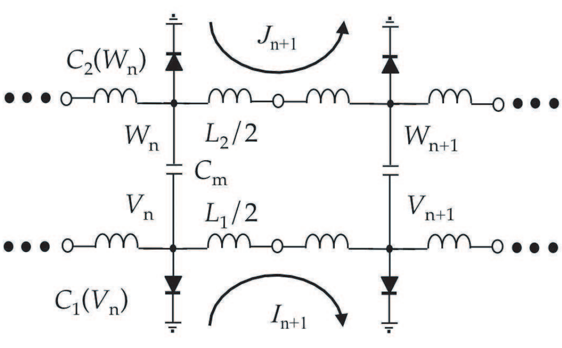

Figure 1. Two unit cells of coupled NLTLs.

where x is the voltage between the terminals. C_0, V_J, and m are the optimizing parameters. Note that $x < 0$ for reverse bias.

The transmission equations of a coupled NLTL are given by

$$L_1 \frac{dI_n}{dt} = V_{n-1} - V_n, \tag{2}$$

$$L_2 \frac{dJ_n}{dt} = W_{n-1} - W_n, \tag{3}$$

$$C_1(V_n) \frac{dV_n}{dt} + C_m \frac{d}{dt} (V_n - W_n) = I_n - I_{n+1}, \tag{4}$$

$$C_2(W_n) \frac{dW_n}{dt} + C_m \frac{d}{dt} (W_n - V_n) = J_n - J_{n+1}. \tag{5}$$

When the pulse spreads over many cells, the discrete spatial coordinate n can be replaced by a continuous one x, series-expanding $V_{n\pm1}$, and $W_{n\pm1}$ up to the fourth order of the cell length δ, we then obtain the evolution equation of the line voltage:

$$\begin{aligned} & l_2 \frac{dc_2(W)}{dW} \left(\frac{\partial W}{\partial t} \right)^2 + l_2 \left[c_2(W) + c_m \right] \frac{\partial^2 W}{\partial t^2} - l_2 c_m \frac{\partial^2 V}{\partial t^2} \\ = & \frac{\partial^2 W}{\partial x^2} + \frac{\delta^2}{12} \frac{\partial^4 W}{\partial x^4}, \end{aligned} \tag{6}$$

$$\begin{aligned} & l_1 \frac{dc_1(V)}{dV} \left(\frac{\partial V}{\partial t} \right)^2 + l_1 \left[c_1(V) + c_m \right] \frac{\partial^2 V}{\partial t^2} - l_1 c_m \frac{\partial^2 W}{\partial t^2} \\ = & \frac{\partial^2 V}{\partial x^2} + \frac{\delta^2}{12} \frac{\partial^4 V}{\partial x^4}, \end{aligned} \tag{7}$$

where $V = V(x, t)$ and $W = W(x, t)$ are the continuous counterparts of V_n and W_n. Moreover, $l_{1,2}$ and $c_{1,2,m}$ are the line inductance and capacitance per unit length defined as $l = L/\delta$ and $c = C/\delta$, respectively.

The c mode and π mode are two different propagation modes on a linear coupled line [7]. It is the same for a coupled NLTL when the voltage amplitude is sufficiently small. Each mode has its own velocity and voltage fraction between the lines (= line 2 voltage/line 1 voltage). The quantities u_c, u_π, R_c and R_π designate, respectively, the velocity of c mode, the velocity of π mode, the voltage fraction of c mode and the voltage fraction of π mode at long wavelengths. These quantities are explicitly written as:

$$u_{c,\pi} = \sqrt{\frac{x_1 + x_2 \pm \sqrt{(x_1 + x_2)^2 - 2x_3}}{x_3}}, \tag{8}$$

$$R_{c,\pi} = \frac{x_1 - x_2 \pm \sqrt{(x_1 + x_2)^2 - 2x_3}}{2c_m l_1}, \tag{9}$$

where the upper (lower) signs are for c (π) mode. For concise notations, we define $x_{1,2,3}$ as

$$x_1 = (c_1(V_0) + c_m)l_1, \tag{10}$$

$$x_2 = (c_2(W_0) + c_m)l_2, \tag{11}$$

$$x_3 = 2\left[c_1(V_0)c_2(W_0) + (c_1(V_0) + c_2(W_0))c_m\right] l_1 l_2, \tag{12}$$

for the case when lines 1 and 2 are biased at V_0 and W_0, respectively. In a linear line, the short-wavelength waves travel slower than the long-wavelength waves due to dispersion; this results in the distortions of the baseband pulses having short temporal durations. In a coupled NLTL, this distortion can be compensated by the nonlinearity introduced using Schottky varactors, regardless of the propagation mode. To quantify the compensation of dispersion by nonlinearity, we apply the reductive perturbation [8] to the transmission equation of a coupled NLTL.

We first series-expand the voltage variables as

$$V(x,t) = V_0 + \sum_{i=1}^{\infty} \epsilon^i V^{(i)}(x,t), \tag{13}$$

$$W(x,t) = W_0 + \sum_{i=1}^{\infty} \epsilon^i W^{(i)}(x,t), \tag{14}$$

for $\epsilon \ll 1$. Moreover, the transformations: $\xi = \epsilon^{1/2}(x - ut)$ and $\tau = \epsilon^{3/2} t$ are applied, where u is a parameter determined in the following. By evaluating Eqs. (6) and (7) for each order of ϵ, we can extract the equation that describes the developing soliton-like pulses. It has been shown that $O(\epsilon)$ terms give trivial identities, and $O(\epsilon^2)$ terms determine the allowed values of u: u has to be equal to either u_c or u_π. Moreover, $W^{(1)}$ becomes equal to $R_{c,\pi}V^{(1)}$ for $u = u_{c,\pi}$. Finally, we obtain the KdV equations for $W^{(1)}$ from $O(\epsilon^3)$ terms for both modes. They are given by

$$\frac{\partial W^{(1)}}{\partial \tau} + p_{c,\pi} W^{(1)} \frac{\partial W^{(1)}}{\partial \xi} + q_{c,\pi} \frac{\partial^3 W^{(1)}}{\partial \xi^3} = 0, \tag{15}$$

$$\begin{aligned} p_{c,\pi} = & \mp \frac{\sqrt{2}c_m}{\sqrt{(x_1+x_2)^2 - 2x_3}} \left[x_1 + x_2 \mp \sqrt{(x_1+x_2)^2 - 2x_3}\right]^{-3/2} \\ & \times \left[\frac{c_1(V_0) l_1^2 m_1}{V_0 - V_{J1}} \frac{x_1 - x_2 \mp \sqrt{(x_1+x_2)^2 - 2x_3}}{x_1 - x_2 \pm \sqrt{(x_1+x_2)^2 - 2x_3}} \right. \\ & \left. + \frac{c_2(W_0) l_2^2 m_2}{W_0 - V_{J2}} \frac{x_1 - x_2 \pm \sqrt{(x_1+x_2)^2 - 2x_3}}{x_1 - x_2 \mp \sqrt{(x_1+x_2)^2 - 2x_3}} \right], \end{aligned} \tag{16}$$

$$q_{c,\pi} = \frac{\delta^2}{24}\sqrt{\frac{x_1 + x_2 \pm \sqrt{(x_1+x_2)^2 - 2x_3}}{x_3}}, \tag{17}$$

where the upper (lower) signs are for c (π) mode and m_i and V_{Ji} are the varactor model parameters for the line i ($i = 1, 2$).

As a result, we obtain the following one-soliton solutions specified by A_0:

$$V(x,t) = V_0 + A_0 \ \mathrm{sech}^2\left[\sqrt{\frac{pA_0}{12q}}\left[x - \left(u + \frac{pA_0}{3}\right)t\right]\right], \tag{18}$$

$$W(x,t) = W_0 + RA_0 \ \mathrm{sech}^2\left[\sqrt{\frac{pA_0}{12q}}\left[x - \left(u + \frac{pA_0}{3}\right)t\right]\right], \tag{19}$$

where (p, q, u, R) is set to $(p_{c(\pi)}, q_{c(\pi)}, u_{c(\pi)}, R_{c(\pi)})$ for the $c(\pi)$-mode soliton. Note that A_0 is set positive (negative) for $p > (<)0$.

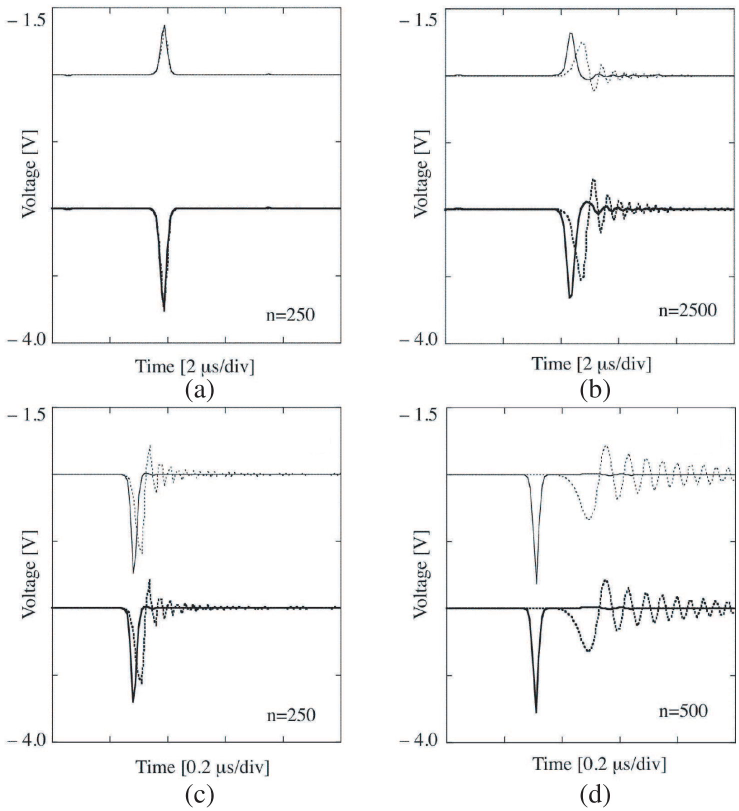

Figure 2. Soliton-like pulses on coupled NLTLs.

We numerically solve Eqs. (2)–(5) using a standard finite-difference time-domain method for a coupled NLTL with Schottky varactors having $C_0 = 60.0\,\text{pF}$, $V_J = 3.0\,\text{V}$, and $m = 2.0$. We set L_1, L_2, C_m, V_0, and W_0 to $4.0\,\mu\text{H}$, $2.0\,\mu\text{H}$, $60.0\,\text{pF}$, $-3\,\text{V}$, and $-2\,\text{V}$, respectively. For these line parameters, the values of R_c, R_π, u_c, and u_π are calculated to be 1.0, −0.5, 1.5×10^8 cell/s, and 5.0×10^7 cell/s, respectively. The c-mode pulse travels three times faster than the π-mode pulse. At $|A_0| = 0.8\,\text{V}$, the temporal width of the c mode nonlinear pulse is estimated to be 11.7 ns, while that of the π mode pulse is 143.3 ns. The former becomes much shorter than the latter. The numerically obtained waveforms are shown in Fig. 2. The input pulses have the one-soliton waveforms given by Eqs. (18) and (19). The bold and solid waveforms show the pulses on lines 1 and 2, respectively. Moreover, the solid and dotted waveforms show nonlinear and linear pulses. To obtain linear pulses, we set the values of $C_1(V)$ and $C_2(W)$ to constants $C_1(V_0)$ and $C_2(W_0)$, respectively. Figs. 2(a) and (b) show the π-mode pulses monitored at $n = 250$ and 2500, respectively, while Figs. 2(c) and (d) show the c-mode pulses monitored at $n = 250$ and 500, respectively. We can see that linear pulses are greatly distorted by the influence of dispersion. On the other hand, nonlinear pulses preserve their original shapes, as shown in Figs. 2(b) and (d). Fig. 3 shows the numerically obtain waveforms with analytically obtained ones. Figs. 3(a) and (b) correspond to the π- and c-mode pulses, respectively. The solid curves show the numerically obtained waveforms monitored at $n = 2500$ with the same

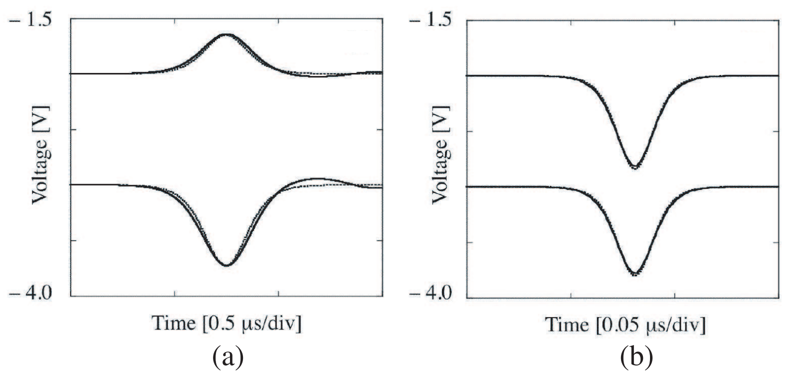

Figure 3. The comparison of numerically obtained waveforms with analytic predictions. (a) The π-mode pulses and (b) c-mode pulses. The solid and dotted curves show the numerical and analytical waveforms.

line parameters used to obtain Fig. 2. The dotted curves show the waveforms obtained analytically by using Eqs. (18) and (19). The parameter A_0 was set to 0.73 and 0.8 V for the π- and c-mode pulses, respectively. The similarity between the solid and dotted waveforms is sufficiently good to mention that the properties of nonlinear pulses are well characterized by Eqs. (18) and (19). The compensation of dispersive distortions by nonlinearity is successfully observed for both the c- and the π-mode pulses.

3. THE METHOD TO DOUBLE REPETITION RATE OF INCIDENT PULSE STREAMS

To operate a coupled NLTL to double repetition rate of pulse stream, one end of each line is terminated with the load resistances matched to the c-mode characteristic impedances, while the other end is matched to the π-mode impedances, as shown in Fig. 4(a). By this arrangement, the multiple reflections of the waves carried by both the c and the π modes are suppressed; therefore, the outputs are free from the distortions caused by the reflections. We consider the case where a T-periodic pulse stream is input to V_{in} as shown in Fig. 4(b). Each pulse is split to the c- and π-mode soliton-like pulses. Because the c-mode soliton-like pulse is generally faster than the π-mode pulse. Thus, the temporal separation between the c-mode and π-mode pulses becomes equal to $T/2$ by setting the length of a coupled NLTL properly. As a result, we succeed in doubling the repetion rate of a pulse stream at V_{out}, although the amplitude has to be halved at the outputs.

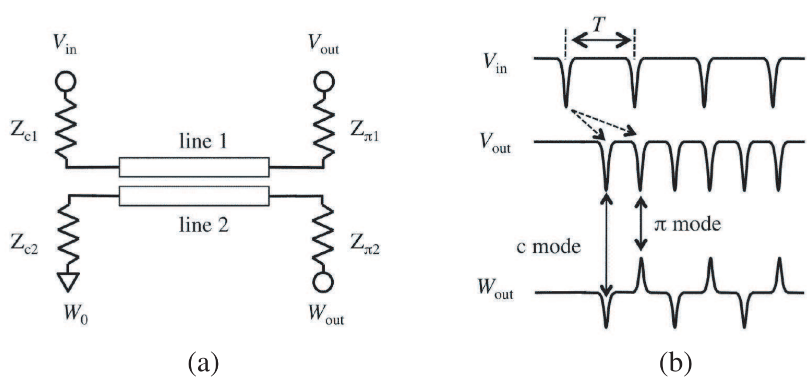

Figure 4. Doubling repetition rate of incident pulse streams. (a) The conguration and (b) the operating principle.

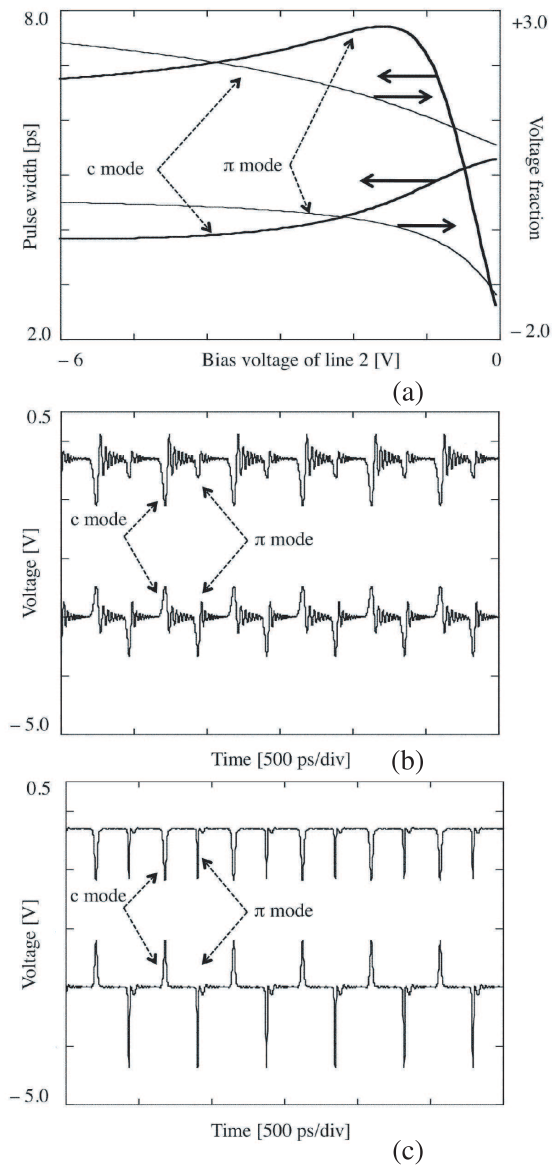

Figure 5. Numerical demonstration of doubling repetition rate of incident pulse streams. (a) The dependence of the pulse widths and voltage fractions on W_0 and the output pulse streams of (b) the linear coupled line and (c) the coupled NLTL.

To demonstrate the doubling method, another calculation is performed by setting T, L_1, L_2, C_m, and V_0 to 500 ps, 0.10 nH, 0.12 nH, 0.02 pF, and -0.3 V, respectively. For Schottky varactors, C_0, V_J, and m are set to 0.1 pF, 1.0 V, and 0.5, respectively. Moreover, the total cell size is set to 300. Fig. 5(a) shows the dependence of the widths of c mode and π mode single solitons (left-vertical) and the voltage fractions $R_{c,\pi}$ (right-vertical) on W_0 is shown, while V_0 is fixed at -0.3 V. In order to obtain nearly coincident pulse amplitude for both modes, we have to set W_0 to satisfy the condition $|R_c| \sim |R_\pi|$. On the other hand, W_0 should be set to the values corresponding to the cross points of two bold curves in order to make the widths of c and π mode pulses coincident. Unfortunately, it is impossible to find the optimal value of W_0 that simultaneously gives nearly coincident widths and amplitudes for c and π pulses, when a pulse stream is injected into the input of one of lines 1 and 2. It is thus required that the amplitude of π-mode pulse has to be greater than that of c-mode pulse at the output in order to make the width of each pulse coincident or the width of π-mode pulse has to be wider than that of c-mode pulse for the coincident amplitudes. Presently, we set W_0 to -3.0 V for matching amplitude, although the widths of c pulse has to be half as long as that of π pulse.

Figures 5(b) and (c) show the calculated waveforms for the linear and nonlinear coupled lines, respectively. The bold and solid curves show the output waveforms of lines 1 and 2, respectively. To obtain linear pulses, we set the values of $C_1(V)$ and $C_2(W)$ to constants $C_1(V_0)$ and $C_2(W_0)$, respectively. Pulses having sech^2 form with 5.6-ps duration were periodically injected into the input of line 1. The dispersion of the linear line greatly distorts both the c and π pulses in Fig. 5(b) because of the wide bandwidth of input pulses. On the other hand, the distortions due to dispersion are well suppressed for both c and π pulses in Fig. 5(c), so that the repetition rate is successfully doubled, although the width differs between the c and π pulses. Moreover, our termination scheme successfully suppresses multiple reflections.

4. CONCLUSION

We describe the method of doubling repetition rate of a pulse stream by using coupled NLTLs. The dispersive distortions are well compensated for by nonlinearity for both c- and π-mode pulses. The properties of nonlinear pulses carried by the c and π modes are quantified based on the reductive perturbation method. The doubled repetition rate is successfully confirmed by the numerical evaluations. Presently, our method has two major drawbacks. One is the halved output amplitude and the other is the amplitude inbalance between the c- and π-mode pulses having the coincident widths. In order to cope with these difficulties, we require some post-processing broadband waveform equalizer such as a traveling-wave field effect transistor [9].

Pulses on nonlinear n-conductor coupled lines can be carried by n different modes. Each has its own velocity; therefore, the repetition rate can be potentially increased by the factor of n at the output of an n-conductor coupled line. However, it becomes difficult to terminate an n-conductor coupled line so as to suppress multiple reflections, because the impedances cannot be generally coincident for two different modes. For increasing the repetition rate by the factor greater than 2, cascading several two-conductor coupled NLTLs is the best way, when the output waveforms of each coupled NLTL are properly equalized.

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