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CALCULATION OF THE NONLINEAR ABSORPTION COEFFICIENT OF A STRONG ELECTROMAGNETIC WAVE BY CONFINED ELECTRONS IN DOPING SUPERLATICES

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Abstract—Analytic expressions for the nonlinear absorption coefficient (nonlinear absorption coefficient = NAC) of a strong electromagnetic wave (laser radiation) caused by confined electrons for the case of electron — optical phonon scattering in doping superlattices (doping superlattices = DSLs) are calculated by using the quantum kinetic equation for electrons. The problem is also considered for both the absence and the presence of an external magnetic field. The dependence of the NAC on the intensity E_0 and the energy $\hbar\Omega$ of the external strong electromagnetic wave (electromagnetic wave = EMW), the temperature T of the system, the doping concentration n_D and the cyclotron frequency Ω_B for case of an external magnetic field is obtained. Two cases for the absorption: Close to the absorption threshold $|k\hbar\Omega - \hbar\omega_0| \ll \bar{\varepsilon}$ and far away from the absorption threshold $|k\hbar\Omega - \hbar\omega_0| \gg \bar{\varepsilon}$ ($k = 0, \pm 1, \pm 2 \ldots$, $\hbar\omega_0$ and $\bar{\varepsilon}$ are the frequency of optical phonon and the average energy of electrons, respectively) are considered. The analytic expressions are numerically evaluated, plotted, and discussed for a specific DSLs n-GaAs/p-GaAs. The computations show that the NAC in DSLs in case presence of an external magnetic field is much more greater than to it is absence of an external magnetic field. The appearance of an external magnetic field causes surprising changes in the nonlinear absorption. All the results for the presence of an external magnetic field are compared with those for the absence of an external magnetic field to show the difference.

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1. INTRODUCTION

In recent times, there has been more and more interest in studying and discovering the behavior of low-dimensional system, in particular, two-dimensional systems, such as compositional superlattices, quantum wells and DSLs. The confinement of electrons in these systems considerably enhances the electron mobility and leads to their unusual behaviors under external stimuli. As a result, the properties of low-dimensional systems, especially the optical properties, are very different in comparison with those of normal bulk semiconductors [1–5]. The problems of the optical properties in normal bulk semiconductors [6–10], as well as two-dimensional systems [11–20], or one-dimensional systems [21], or zero-dimensional systems [22] have ever been enthusiastically investigated. The nonlinear absorption of a strong electromagnetic wave by free electrons in normal bulk semiconductors has been studied by using the quantum kinetic equation method [8] while the linear absorption of a weak electromagnetic wave by confined electrons in low-dimensional systems has been investigated by using the Kubo-Mori method [18, 19].

However, the nonlinear absorption problem of an EMW, which has strong intensity and high frequency, in DSLs is still open for study. In this paper, we study the nonlinear absorption of a strong EMW by confined electrons in DSLs. The electron-optical phonon scattering mechanism are considered. The NAC is calculated for a magnetic field applied perpendicular to its barriers and for no magnetic field by using the quantum kinetic equation [14] for electrons in a DSLs. Then, we estimate numerical values for the specific DSLs n-GaAs/p-GaAs to clarify our results.

2. NONLINEAR ABSORPTION COEFFICIENT IN THE CASE ABSENCE OF AN EXTERNAL MAGNETIC FIELD

It is well known that the monition of an electron in DSLs is confined and that its energy spectrum is quantized into discrete levels. We assume that the quantization direction is the z direction. The Hamiltonian of the electron-optical phonon system in a DSLs in the second quantization representation can be written as.

$$\begin{aligned} H = & \sum_{n,\vec{k}_\perp} \varepsilon_n \left(\vec{k}_\perp - \frac{e}{\hbar c} \vec{A}(t) \right) a^+_{n,\vec{k}_\perp} a_{n,\vec{k}_\perp} + \sum_{\vec{q}} \hbar\omega_{\vec{q}} b^+_{\vec{q}} b_{\vec{q}} \\ & + \sum_{n,n',\vec{q},\vec{k}_\perp} C_{\vec{q}} I_{n,n'}(q_z)\, a^+_{n',\vec{k}_\perp+\vec{q}_\perp} a_{n,\vec{k}_\perp} \left(b^+_{-\vec{q}} + b_{\vec{q}} \right) \end{aligned} \tag{1}$$

where n $(n = 1, 2, 3, \ldots)$ denotes the quantization of the energy spectrum in the z direction, $(n, \vec{k}_\perp)$ and $(n', \vec{k}_\perp + \vec{q}_\perp)$ are electron states before and after scattering, respectively. $\vec{k}_\perp$ $(\vec{q}_\perp)$ is the in-plane (x, y) wave vector of the electron (phonon), $a^+_{n,\vec{k}_\perp}$ and $a_{n,\vec{k}_\perp}$ ($b^+_{\vec{q}}$ and $b_{\vec{q}}$) are the creation and the annihilation operators of the electron (phonon), respectively, $\vec{q} = (\vec{q}_z + \vec{q}_\perp)$, $\vec{A}(t)$ is the vector potential open external electromagnetic wave $\vec{A}(t) = \frac{e}{\Omega}\vec{E}_0 \sin(\Omega t)$ and $\hbar\omega_q$ is the energy of the optical phonon. $C_{\vec{q}}$ is a constant in the case of electron-optical phonon interaction [8]:

$$\left|C_{\vec{q}}\right|^2 = \frac{1}{V}\frac{2\pi e^2\hbar\omega_0}{\varepsilon_0\left(q_\perp^2 + q_z^2\right)}\left(\frac{1}{\chi_\infty} - \frac{1}{\chi_0}\right) \tag{2}$$

here, V, e and ε_0 are the normalization volume, the effective charge and the electronic constant (often $V = 1$). χ_0 and χ_∞ are the static and the high-frequency dielectric constants, respectively. The electron form factor, $I_{n,n'}(q_z)$ are written as:

$$I_{n,n'}(q_z) = \sum_{j=1}^{N_1}\int_0^d \psi_n(z - jd)\,\psi_{n'}(z - jd)\,e^{iq_z d}dz \tag{3}$$

With $\psi_n(z)$ is the wave function of the n-th state in one of the one-dimensional potential wells which compose the DSLs potential, d is the DSLs period, N_1 is the number of DSLs period.

In DSLs, the electron energy takes the simple form:

$$\varepsilon_{n,\vec{k}_\perp} = \frac{\hbar^2 k_\perp^2}{2m} + \hbar\omega_p\left(n + \frac{1}{2}\right) \tag{4}$$

here, $\omega_p = \hbar\left(\frac{4\pi e^2 n_D}{\varepsilon_0 m}\right)^{1/2}$, n_D is the doping concentration, m is the effective mass.

In order to establish the quantum kinetic equations for electrons in DSLs, we use general quantum equations for the particle number operator (or electron distribution function) $n_{n,\vec{k}_\perp}(t) = \left\langle a_{n,\vec{k}_\perp} + a_{n,\vec{k}_\perp}\right\rangle_t$ [8]

$$i\hbar\frac{\partial n_{n,\vec{k}_\perp}(t)}{\partial t} = \left\langle a^+_{n',\vec{k}_\perp} a_{n,\vec{k}_\perp}, H\right\rangle_t. \tag{5}$$

where $\langle\psi\rangle_t$ denotes a statistical average value at the moment t, and $\langle\psi\rangle_t = Tr(\hat{W}\hat{\psi})$ ($\hat{W}$ being the density matrix operator). Starting from Hamiltonian (1) and using the commutative relations of the creation and the annihilation operators, we obtain the quantum kinetic equation for electrons in DSLs:

$$\frac{\partial n_{n,\vec{k}_\perp}(t)}{\partial t} = -\frac{1}{\hbar^2}\sum_{s,l=-\infty}^{\infty} J_s\left(\frac{\lambda}{\Omega}\right) J_l\left(\frac{\lambda}{\Omega}\right)\exp\left[-i\left(s-l\right)\Omega t\right]$$

$$\sum_{\vec{q}_\perp,n}\left|C_{\vec{q}}\right|^2\left|I_{n,n'}\left(q_z\right)\right|^2\int_{-\infty}^{t} dt_1\times\Big\{\Big[n_{n,\vec{k}_\perp}\left(t_1\right)N_{\vec{q}}-n_{n',\vec{k}_\perp+\vec{q}_\perp}\left(t_1\right)\left(N_{\vec{q}}+1\right)\Big]$$

$$\exp\left[\frac{i}{\hbar}\left(\varepsilon_{n'}\left(\vec{k}_\perp+\vec{q}_\perp\right)-\varepsilon_n\left(\vec{k}_\perp\right)-\hbar\omega_0-l\hbar\Omega+i\delta\right)\left(t-t_1\right)\right]$$

$$+\left[n_{n,\vec{k}_\perp}\left(t_1\right)\left(N_{\vec{q}}+1\right)-n_{n',\vec{k}_\perp+\vec{q}_\perp}\left(t_1\right)N_{\vec{q}}\right]$$

$$\exp\left[\frac{i}{\hbar}\left(\varepsilon_{n'}\left(\vec{k}_\perp+\vec{q}_\perp\right)-\varepsilon_n\left(\vec{k}_\perp\right)+\hbar\omega_0-l\hbar\Omega+i\delta\right)\left(t-t_1\right)\right]$$

$$-\left[n_{n',\vec{k}_\perp-\vec{q}_\perp}\left(t_1\right)\left(N_{\vec{q}}+1\right)-n_{n,\vec{k}_\perp}\left(t_1\right)N_{\vec{q}}\right]$$

$$\exp\left[\frac{i}{\hbar}\left(\varepsilon_n\left(\vec{k}_\perp\right)-\varepsilon_{n'}\left(\vec{k}_\perp-\vec{q}_\perp\right)-\hbar\omega_0-l\hbar\Omega+i\delta\right)\left(t-t_1\right)\right]$$

$$-\left[n_{n',\vec{k}_\perp-\vec{q}_\perp}\left(t_1\right)\left(N_{\vec{q}}+1\right)-n_{n,\vec{k}_\perp}\left(t_1\right)N_{\vec{q}}\right]$$

$$\exp\left[\frac{i}{\hbar}\left(\varepsilon_n\left(\vec{k}_\perp\right)-\varepsilon_{n'}\left(\vec{k}_\perp-\vec{q}_\perp\right)+\hbar\omega_0-l\hbar\Omega+i\delta\right)\left(t-t_1\right)\right]\Big\} \quad (6)$$

It is well known that to obtain the explicit solutions from Eq. (6) is very difficult. In this paper, we use the first-order tautology approximation method to solve this equation. In detain, in Eq. (6), we use the approximation

$$n_{n,\vec{k}_\perp}(t)\approx\bar{n}_{n,\vec{k}_\perp};\quad n_{n,\vec{k}_\perp+\vec{q}_\perp}(t)\approx\bar{n}_{n,\vec{k}_\perp+\vec{q}_\perp};\quad n_{n,\vec{k}_\perp-\vec{q}_\perp}(t)\approx\bar{n}_{n,\vec{k}_\perp-\vec{q}_\perp}$$

where $\bar{n}_{n,\vec{k}_\perp}$ is the time-independent component of the electron distribution function. The approximation is also applied for a similar exercise in bulk semiconductors [7]. We perform the integral with respect to t_1; next, we perform the integral with respect to t of Eq. (6). The expression for the electron distribution can be written as

$$n_{n,\vec{k}_\perp}(t) = -\frac{1}{\hbar^2}\sum_{\vec{q},n'}\left|I_{n,n'}\left(q_z\right)\right|^2\left|C_{\vec{q}}\right|^2\sum_{k,l=-\infty}^{+\infty} J_k\left(\frac{e\vec{E}_0\vec{q}_\perp}{m\Omega^2}\right)J_{l+k}\left(\frac{e\vec{E}_0\vec{q}_\perp}{m\Omega^2}\right)\frac{\hbar}{l\Omega}\exp\left(-ik\Omega t\right)$$

$$\times\left\{\frac{\bar{n}_{n',\vec{k}_\perp-\vec{q}_\perp}N_{\vec{q}}-\bar{n}_{n,\vec{k}_\perp}\left(N_{\vec{q}}+1\right)}{\varepsilon_n\left(\vec{k}_\perp\right)-\varepsilon_{n'}\left(\vec{k}_\perp-\vec{q}_\perp\right)-\hbar\omega_0-l\hbar\Omega+i\delta\hbar}\right.$$
$$+\frac{\bar{n}_{n',\vec{k}_\perp-\vec{q}_\perp}\left(N_{\vec{q}}+1\right)-\bar{n}_{n,\vec{k}_\perp}N_{\vec{q}}}{\varepsilon_n\left(\vec{k}_\perp\right)-\varepsilon_{n'}\left(\vec{k}_\perp-\vec{q}_\perp\right)+\hbar\omega_0-l\hbar\Omega+i\delta\hbar}$$
$$-\frac{\bar{n}_{n,\vec{k}_\perp}N_{\vec{q}}-\bar{n}_{n',\vec{k}_\perp+\vec{q}_\perp}\left(N_{\vec{q}}+1\right)}{\varepsilon_{n'}\left(\vec{k}_\perp+\vec{q}_\perp\right)-\varepsilon_n\left(\vec{k}_\perp\right)-\hbar\omega_0-l\hbar\Omega+i\delta\hbar}$$
$$\left.-\frac{\bar{n}_{n,\vec{k}_\perp}\left(N_{\vec{q}}+1\right)-\bar{n}_{n',\vec{k}_\perp+\vec{q}_\perp}N_{\vec{q}}}{\varepsilon_{n'}\left(\vec{k}_\perp+\vec{q}_\perp\right)-\varepsilon_n\left(\vec{k}_\perp\right)+\hbar\omega_0-l\hbar\Omega+i\delta\hbar}\right\} \tag{7}$$

where, $\bar{n}_{n,\vec{k}_\perp}$ ($N_{\vec{q}}\equiv N_{\vec{q}_\perp}$) is the time-independent component of the electron (phonon) distribution function, $\vec{E}_0$, and Ω are the intensity and frequency of electromagnetic wave; $J_k(x)$ is the Bessel function. The carrier current density formula in DSLs takes the form

$$\vec{J}_\perp(t)=\frac{e\hbar}{m}\sum_{n,\vec{k}_\perp}\left(\vec{k}_\perp-\frac{e}{\hbar c}\vec{A}(t)\right)n_{n,\vec{k}_\perp}(t) \tag{8}$$

Because the motion of electrons is confined along the z direction in a DSLs, we only consider the in plane (x,y) current density vector of electron, $\vec{j}_\perp(t)$. Using Eq. (7), we find the expression for current density vector:

$$\vec{J}_\perp(t)=-\frac{e^2}{mc}\sum_{n,\vec{k}_\perp}\vec{A}(t)\,n_{n,\vec{k}_\perp}(t)+\sum_{l=1}^{\infty}\vec{J}_l\sin(l\Omega t). \tag{9}$$

The NAC of a strong EMW by confined electrons in the DSLs takes the simple form [18]:

$$\alpha=\frac{8\pi}{c\sqrt{\chi_\infty}E_0^2}\left\langle\vec{J}_\perp(t)\,\vec{E}_0\sin\Omega t\right\rangle_t \tag{10}$$

By using the electron-optical phonon interaction factor $C_{\vec{q}}$ in Eq. (2) and using nature of the Bessel function, from the expression for the current density vector we establish the nonlinear absorption coefficient of the electromagnetic wave in DSLs:

$$\alpha=\frac{32\pi^3e^2\Omega k_BT}{e_0c\sqrt{\chi_\infty}E_0^2}\left(\frac{1}{\chi_\infty}-\frac{1}{\chi_0}\right)\sum_{n,n'}\sum_{\vec{k}_\perp,\vec{q}_\perp}\sum_{l=1}^{\infty}\left|I_{n,n'}\right|^2\frac{l}{q^2}J_l^2\left(\frac{\lambda}{\Omega}\right)$$
$$\times\bar{n}_{n,\vec{k}_\perp}\delta\left(\varepsilon_{n'}\left(\vec{k}_\perp+\vec{q}_\perp\right)-\varepsilon_n\left(\vec{k}_\perp\right)+\hbar\omega_0-\hbar\Omega\right) \tag{11}$$

Eq. (11) is the general expression for the nonlinear absorption of a strong electromagnetic wave in a DSLs. In this paper, we will consider two limiting cases for the absorption, close to the absorption threshold and far away from the absorption threshold, to find the explicit formula for the absorption coefficient α.

2.1. The Absorption Close to Threshold

In this case, the condition $|k\hbar\Omega - \hbar\omega_0| \ll \bar{\varepsilon}$ is needed. Therefore, we can't ignore the presence of the vector $\vec{k}_\perp$ in the formula of the δ function. This also means that the calculation depends on the form of the electron distribution function $\bar{n}_{n,\vec{k}_\perp}$. We restrict the problem to the case of one photon absorption ($|k\hbar\Omega - \hbar\omega_0| \ll \bar{\varepsilon}$) and consider the electron gas to be non-degenerate:

$$\bar{n}_{n,\vec{k}_\perp} = n_0^* \exp\left(\frac{\varepsilon_{n,\vec{k}_\perp}}{kT}\right) \text{ with } n_0^* = \frac{n_0 e^{\frac{3}{2}} \pi^{\frac{3}{2}} \hbar^3}{V m^{\frac{3}{2}} (kT)^{\frac{3}{2}}}$$

Using the electron distribution function $\bar{n}_{n,\vec{k}_\perp}$, we do the addition following the vectors $\vec{k}_\perp$ and $\vec{q}_\perp$ in Eq. (10). Finally, the expression for the absorption coefficient of a strong EMW in a DSLs for the case of absorption close to its threshold is obtained:

$$\alpha = \frac{\sqrt{2}\pi e^4 n_0^* (k_B T)^2}{8c\sqrt{m\chi_\infty}\Omega^3\hbar^3}\left(\frac{1}{\chi_\infty} - \frac{1}{\chi_0}\right)\sum_{n,n',\vec{q}_z}\left|I_{n,n'}(q_z)\right|^2 \exp\left(-\frac{\hbar\omega_p\left(n+\frac{1}{2}\right)+\frac{\xi}{2}}{k_B T}\right)$$
$$\times e^{-2\sqrt{\rho\sigma}}\left(\frac{\rho}{|\xi|\,\sigma}\right)^{\frac{1}{2}}\left\{1+\frac{3}{16\sqrt{\rho\sigma}}+\frac{3e^2E_0^2}{32m^2\Omega^4}\left(\frac{\rho}{\sigma}\right)^{\frac{1}{2}}\left[1+\frac{1}{\sqrt{\rho\sigma}}+\frac{1}{16\rho\sigma}\right]\right\} \quad (12)$$

Here, $\xi = \hbar\omega_p(n-n') + \hbar\omega_0 - \hbar\Omega$; $a = \frac{3}{8}\left(\frac{eE_0}{2m\Omega^2}\right)^2$; $\rho = \frac{m\xi^2}{2\hbar^2 k_B T}$; $\sigma = \frac{\hbar^2}{8mk_B T}$.

In bulk materials, there is a strong dispersion when the phonon energy is close to the optic phonon energy. However, in a DSLs, we will see an increase in the absorption coefficient of electromagnetic wave (see the numerical calculation and the discussion sections). This is due to the surprising changes in the electron spectrum and the wave function in DSLs. This also results in significant proper for low-dimensional materials.

2.2. The Absorption Far Away from Threshold

In this case, for the absorption of a strong EMW in a DSLs the condition $|k\hbar\Omega - \hbar\omega_0| \gg \bar{\varepsilon}$ must be satisfied. Here, $\bar{\varepsilon}$ is the average energy of an electron in a DSLs. If: $k\hbar\Omega \gg \bar{\varepsilon}$ the equation is $\sum_{n,\vec{p}_\perp} n_{n,\vec{p}_\perp} = n_0$ (n_0 being the electron density in a DSLs). This means that the result for the current density vector and the NAC do not depend on the type of electron distribution function, $n_{n,\vec{k}_\perp}(t)$. The approximation is also applied for a similar exercise in bulk semiconductors [7]. Because the δ function does not depend on the wave vector $\vec{k}_\perp$ and we can easily perform the addition following vector $\vec{k}_\perp$ and $\vec{q}_\perp$ in Eq. (11), we have the explicit formula for the NAC of a strong EMW in a DSLs for the case of absorption far away from its threshold:

$$\alpha = \frac{\pi^2 e^4 k_B T n_0}{e_0 c\sqrt{\chi_\infty} m\hbar^2\Omega^3}\left(\frac{1}{\chi_\infty}-\frac{1}{\chi_0}\right)\sum_{n,n',q_z}\left|I_{n,n'}(q_z)\right|^2 \left[\frac{2m}{\hbar}\omega_p\left(n-n'\right)+\frac{2m}{\hbar}\left(\Omega-\omega_0\right)\right]^{\frac{1}{2}} \times\left[1+\frac{3}{32}\frac{e^2E_0^2}{m^2\Omega^4}\left(\frac{2m}{\hbar}\omega_p\left(n-n'\right)+\frac{2m\left(\Omega-\omega_0\right)}{\hbar}\right)^{\frac{1}{2}}\right] \quad (13)$$

The term in proportion to quadratic intensity of a strong EMW tend toward zero, the nonlinear result in (12) and (13), will turn back to the linear case which was calculated by another method-the Kubo-Mori [19].

3. NONLINEAR ABSORPTION COEFFICIENT IN THE CASE OF THE PRESENCE OF AN EXTERNAL MAGNETIC FIELD

We assume that the quantization direction is the z direction. As well as, we consider a DSLs with a magnetic field $\vec{B}$ applied perpendicular to it is barriers. The Hamiltonian of the electron-optical phonon system in DSLs in the presence of an external magnetic field $\vec{B}$ in second quantization representation can be written as:

$$H_{\vec{B}} = \sum_{n,N,\vec{k}_\perp}\varepsilon^H_{n,N}\left(\vec{k}_\perp-\frac{e}{\hbar c}\vec{A}(t)\right)a^+_{n,N,\vec{k}_\perp}a_{n,N,\vec{k}_\perp}+\sum_{\vec{q}}\hbar\omega_{\vec{q}}b^+_{\vec{q}}b_{\vec{q}} + \sum_{n,N,n',N'}\sum_{\vec{q},\vec{k}_\perp}C_{\vec{q}}I_{n,n'}(q_z)J_{N,N'}(u)a^+_{n',N',\vec{k}_\perp+\vec{q}_\perp}a_{n,N,\vec{k}_\perp}\left(b^+_{-\vec{q}}+b_{\vec{q}}\right) \quad (14)$$

where, N ($N = 1, 2, \ldots$) is the Landau level index, n denotes the quantization of the energy spectrum in the z direction ($n = 1, 2, \ldots$),

$(n,\ N,\ \vec{k}_\perp)$ and $(n',\ N',\ \vec{k}_\perp + \vec{q}_\perp)$ are electron states before and after scattering, respectively. $a^+_{n,N,\vec{k}_\perp}$ and $a_{n,N,\vec{k}_\perp}$ ($b^+_{\vec{q}_\perp}$ and $b_{\vec{q}_\perp}$) are the creation and the annihilation operators of the electron (phonon), respectively, $(\vec{q} = (\vec{q}_\perp, q_z))$.

In DSLs, in the presence of an external magnetic field, the electron energy takes the simple form

$$\varepsilon^H_{n,N}\left(\vec{k}\right) = \hbar\omega_p\left(n + \frac{1}{2}\right) + \hbar\Omega_B\left(N + \frac{1}{2}\right) \tag{15}$$

With, Ω_B is the cyclotron frequency $\Omega_B = \frac{eB}{mc}$. In DSLs, the $C_{\vec{q}}$ and the electron form factor similar to Eqs. (2) and (3) respectively:

$$J_{N,N'}(u) = \int_{-\infty}^{\infty} e^{i\vec{q}_\perp \overrightarrow{r}_\perp} dr \varphi_{N'}\left(\overrightarrow{r}_\perp - a_c^2 \overrightarrow{p}_\perp - a_c^2 \vec{q}_\perp\right) \varphi_N\left(\overrightarrow{r}_\perp - a_c^2 \overrightarrow{p}_\perp\right) \tag{16}$$

where $\overrightarrow{r}_\perp$ is the position vector of electron, a_c is the orbit radius in the x-y plane, $a_c^2 = \frac{c\hbar}{eB}$, $u = \frac{a_c^2 q_\perp^2}{2}$ and φ_N represents the harmonic wave function.

Starting from Hamiltonian (14) and realizing operator algebraic calculations. We obtain the quantum kinetic equation for the case of the presence of an external magnetic field. Then, we use the first order tautology approximation method [7] to solve this equation. The expression of the electron distribution function can be written as:

$$\begin{aligned}
n_{n,N,\vec{k}_\perp}(t) = &-\frac{1}{\hbar^2} \sum_{\vec{q},N',n'} \left|I_{n,n'}(q_z)\right|^2 \left|C_{\vec{q}}\right|^2 \left|J_{N,N'}(u)\right|^2 \\
&\sum_{k,l=-\infty}^{+\infty} \frac{\hbar}{l\Omega} J_l\left(\frac{\lambda}{\Omega}\right) J_{l+k}\left(\frac{\lambda}{\Omega}\right) \exp(-i\Omega t) \\
&\times\left\{ \frac{\bar{n}_{n',N',\vec{k}_\perp - \vec{q}_\perp} N_{\vec{q}} - \bar{n}_{n,N,\vec{k}_\perp}\left(N_{\vec{q}} + 1\right)}{\varepsilon^H_{n,N}\left(\vec{k}_\perp\right) - \varepsilon^H_{n',N'}\left(\vec{k}_\perp - \vec{q}_\perp\right) - \hbar\omega_0 - l\hbar\Omega + i\delta\hbar} \right. \\
&+ \frac{\bar{n}_{n',N',\vec{k}_\perp - \vec{q}_\perp}\left(N_{\vec{q}} + 1\right) - \bar{n}_{n,N,\vec{k}_\perp} N_{\vec{q}}}{\varepsilon^H_{n,N}\left(\vec{k}_\perp\right) - \varepsilon^H_{n',N'}\left(\vec{k}_\perp - \vec{q}_\perp\right) + \hbar\omega_0 - l\hbar\Omega + i\delta\hbar} \\
&- \frac{\bar{n}_{n,N,\vec{k}_\perp} N_{\vec{q}} - \bar{n}_{n',N',\vec{k}_\perp + \vec{q}_\perp}\left(N_{\vec{q}} + 1\right)}{\varepsilon^H_{n',N'}\left(\vec{k}_\perp + \vec{q}_\perp\right) - \varepsilon^H_{n,N}\left(\vec{k}_\perp\right) - \hbar\omega_0 - l\hbar\Omega + i\delta\hbar}
\end{aligned}$$

$$-\frac{\bar{n}_{n,N,\vec{k}_\perp}\left(N_{\vec{q}}+1\right)-\bar{n}_{n',N',\vec{k}_\perp+\vec{q}_\perp}N_{\vec{q}}}{\varepsilon^H_{n',N'}\left(\vec{k}_\perp+\vec{q}_\perp\right)-\varepsilon^H_{n,N}\left(\vec{k}_\perp\right)+\hbar\omega_0-l\hbar\Omega+i\delta\hbar}\Bigg\}, \tag{17}$$

Solving the quantum kinetic equation, which is established for electrons confined in DSLs in the presence of an external magnetic field, by using a method similar to that used to solve Eq. (6), we obtain the absorption coefficient

$$\alpha=\frac{e^4 n_o^* k_B T\Omega_B}{4\pi e_0 mc\sqrt{x_\infty}a_c^2\Omega^3}\left(\frac{1}{x_\infty}-\frac{1}{x_0}\right)\sum_{n,n',N,N'}\left[1+\left(N+N'+1\right)+\frac{3e^2E_0^2}{16m^2a_c^2\Omega^4}\right]$$
$$\times\frac{\left\{\exp\left[-\frac{1}{k_BT}\left(\hbar\omega_p(n+\frac{1}{2})+\hbar\Omega_B(N+\frac{1}{2})\right)\right]-\exp\left[-\frac{1}{k_BT}\left(\hbar\Omega_B(N'+\frac{1}{2})+\hbar\omega_p(n'+\frac{1}{2})\right)\right]\right\}}{|N-N'|\left[n\left(\Omega-\omega_0\right)+n\Omega_B\left(N-N'\right)+n\omega_p\left(n-n'\right)\right]^2+n^2A^*}$$
$$\times\frac{1}{\pi}e^{-2\left(\frac{N_1 d}{a_1}\right)^2}\left(\frac{N_1 d}{a_1}\right)^2\left[1+\frac{2}{3}\left(\frac{N_1 d}{a_1}\right)^2\right]^2 n\sqrt{A^*|N-N'|} \tag{18}$$

Here,

$$A^*=\frac{N_0 e^2\omega_0}{2\hbar}\left(\frac{1}{\chi_\infty}-\frac{1}{\chi_0}\right)e^{-2\left(\frac{N_1 d}{a_1}\right)^2}\left(\frac{N_1 d}{a_1}\right)^2\left[1+\frac{2}{3}\left(\frac{N_1 d}{a_1}\right)^2\right]^2;$$
$$a_1=\frac{\hbar}{m\omega_p};\quad N_0=\frac{k_BT}{\hbar\omega_0}$$

The term in proportion to quadratic intensity of a strong EMW tend toward zero, the nonlinear result in (18), will turn back to the linear case which was calculated by another method-the Kubo-Mori [19].

4. NUMERICAL RESULTS AND DISCUSSIONS

In order to clarify the mechanism for the nonlinear absorption of a strong electromagnetic wave in a DSLs, in this section, we will evaluate, plot and discuss the expression of the NAC for the case of a specific DSLs n-GaAs/p-GaAs. We use some results for linear absorption in [19] to make the comparison between for both the case: the absence and the presence of an external magnetic field. The parameters used in the calculations are as follows [19]: $\chi_\infty = 10 : 9$; $\chi_0 = 12 : 9$; $n_0 = 10^{23}\,\mathrm{m}^{-3}$; $e_0 = \frac{10^{-9}}{36\pi}$, $m = 0 : 067m_0$ (m_0 being the mass, of free electron) $d = 800\,\mathring{A}$, being the DSLs period, $\hbar\omega_0 = 36.25\,\mathrm{meV}$;

$\Omega = 5.5 \cdot 10^{13}\,\mathrm{s}^{-1}$, $T = 300\,\mathrm{K}$, $e = 2.07e_0$, $e_0 = 1.6 \cdot 10^{-19}\mathrm{C}$, $E_0 = 3.5.10^{14}\,\mathrm{V/cm}$.

Figures 1 and 2 show the dependence of nonlinear absorption coefficient α in DSLs on the intensity, E_0, of the electromagnetic wave is strong for the two cases present and absent magnetic field. Graph shows, the absorption coefficient dependency strong and nonlinear on the intensity E of electromagnetic waves. When the intensity of E increases, the absorption coefficient also increased rapidly. However, for the case present external magnetic field, the value of the nonlinear coefficient absorption larger than when absent external magnetic field.

Figures 3 and 4 are the results from the survey of absorption coefficient on temperature and concentration of concentration doped. Graph shows absorption coefficient is also dependence strong and nonlinear on temperature T and concentration doped n_D. For both cases present and absent magnetic field, nonlinear absorption coefficient increased, when the temperature and the concentration doped of the system increases. Causes of this phenomenon: when the temperature and the concentration doped of system increases, leading to increased particle density, increased stimulation resulting vibrations in the material particles. So that absorption coefficient of electromagnetic waves increased.

Figures 5 and 6 show the dependence of the nonlinear absorption coefficient α on the energy $\hbar\Omega$ in DSLs of an external strong electromagnetic wave for the cases two absent and present magnetic field. Graphs show clearly the difference between them. In

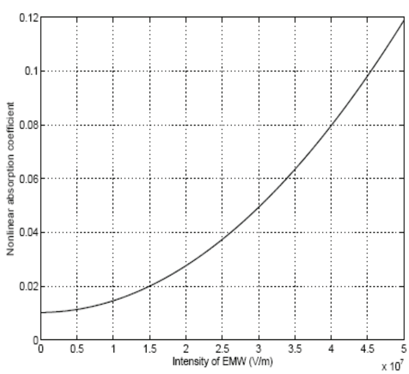

Figure 1. The dependence of α on the E_0 (Absence of an external magnetic field).

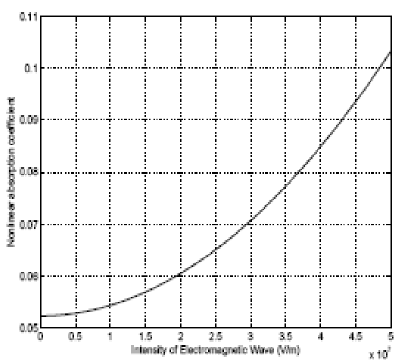

Figure 2. The dependence of α on the E_0 (Presence of an external magnetic field).

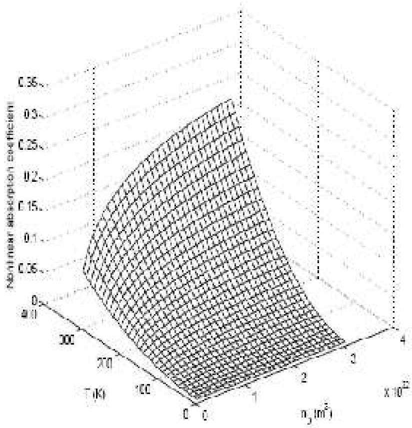

Figure 3. The dependence of α on the n_D (Absence of an external magnetic field).

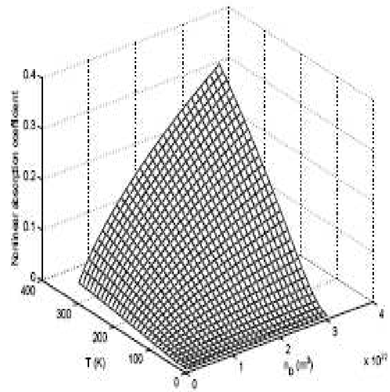

Figure 4. The dependence of α on the n_D (Presence of an external magnetic field).

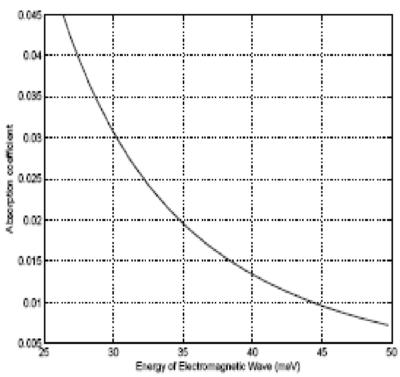

Figure 5. The dependence of α on the $\hbar\Omega$ (Absence of an external magnetic field).

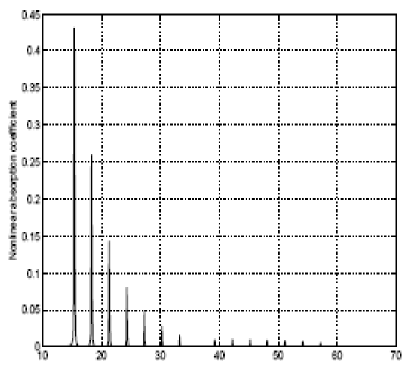

Figure 6. The dependence of α on the $\hbar\Omega$ (Presence of an external magnetic field).

Figure 5, energy absorption spectrum is continuous, in Figure 6, energy absorption spectrum is interrupted, and in the case presence of magnetic field, the absorption coefficient strong increases more in the case the absence of magnetic field. The line spectrum formation of absorption coefficient in Figure 6 clearly shows the influence of magnetic field on the absorption coefficient of strong external electromagnetic waves. This difference can be explained as follows:

In the presence of a magnetic field, the energy spectrum of electronic is interruptions. In addition, the Landau level that electrons can reach must be defined. In other word, the index of the absorption process must satisfy the condition

$$\hbar\omega_p \left(n' - n\right) + \hbar\Omega_B \left(N' - N\right) + \hbar\omega_0 - \hbar\Omega = 0 \tag{19}$$

in function Delta-Dirac. This is different from that for case absent a magnetic field (index of the landau level that electrons can reach after the absorption process is arbitrary), therefore, the dependence of the absorption coefficient α on $\hbar\Omega$ is not continuous.

5. CONCLUSION

In this paper, we analytically investigated the nonlinear absorption of a strong electromagnetic wave by electrons confined in DSLs. We obtained a quantum kinetic equations for electrons in DSLs. By using the tautology approximation methods, we solved this equation to find function. Thus, found we received the formula of the nonlinear absorption coefficient in DSLs.

We numerically calculated and graphed the nonlinear absorption coefficient for n-GaAs/p-GaAs DSLs to clarify the theoretical results. Numerical results for n-GaAs/p-GaAs DSLs present clearly the dependence of the absorption coefficient on the parameters of the external field: the intensity E_0, the temperature T of the system, the doping concentration n_D and the energy $\hbar\Omega$ of the strong electromagnetic wave for both the absence and the presence of an external magnetic field.

In the case of a magnetic field applied perpendicular to the barriers, the analytical expressions indicate a complicated dependence of the absorption coefficient on the intensity E_0, the temperature T of the system, the doping concentration n_D and the energy $\hbar\Omega$ of the strong electromagnetic wave. These dependences differ from those for the case absent magnetic field. The absorption coefficient of a strong external magnetic field depends strongly on the condition in Eq. (18) and the index of the Landau sub-band to which the electron can reach after the absorption process in defined by that condition.

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