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A ROBUST BEAMFORMER BASED ON WEIGHTED SPARSE CONSTRAINT

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Abstract—Applying a sparse constraint on the beam pattern has been suggested to suppress the sidelobe level of a minimum variance distortionless response (MVDR) beamformer. In this letter, we introduce a weighted sparse constraint in the beamformer design to provide a lower sidelobe level and deeper nulls for interference avoidance, as compared with a conventional MVDR beamformer. The proposed beamformer also shows improved robustness against the mismatch between the steering angle and the direction of arrival (DOA) of the desired signal, caused by imperfect estimation of DOA.

1. INTRODUCTION

Multiple-antenna systems have received considerable attention from both the wireless industry and academia, because of their strong potential in realizing high data rate wireless communications in next generation wireless networks. A beamformer is a versatile form of spatial filtering, using multiple antenna systems to separate signals that have overlapping frequency spectra but originate from different spatial locations. Beamforming has become a key technique in current and future wireless communications, radar, sonar, etc [1].

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The minimum variance distortionless response (MVDR) beamformer has been considered as a popular method for enhancing the signal from the desired direction while suppressing all signals arriving from other directions as well as the background noise [1], but its relatively high sidelobe level would lead to significant performance degradation, especially with unexpected increase in interference or background noise [2]. In order to provide sidelobe suppression for an MVDR beamformer, a sparse constraint on the beam pattern was recently proposed in [3], but this sparse constraint was simply weighted equally in every direction, resulting in limited improvement in sidelobe suppression. At the same time, possible mismatch between the steering angle of the beamformer and the direction of arrival (DOA) of the signal of interest (SOI) may further degrade the beamforming performance [4].

In this letter, we take into account the DOAs of interfering signals in the design of a robust beamformer. More specifically, we incorporate the a posteriori DOA distribution of interfering signals, which can be coarsely estimated through a correlation operation at the beamformer, into the sparse constraint, so that the beam pattern is weighted in different directions according to the spatial distribution of interference. Numerical evaluations will show that the proposed beamformer achieves a much lower sidelobe level, deeper nulls for interference avoidance, and stronger robustness against DOA estimation errors, as compared with existing designs of beamformers.

2. MVDR BEAMFORMER

The signal received by a uniform linear array (ULA) with M antennas can be represented by an M-by-1 vector, $\mathbf{x}(k)$, the expression of which is given by

$$\mathbf{x}(k) = s(k)\mathbf{a}(\theta_0) + \sum_{j=1}^{J} \beta_j(k)\mathbf{a}(\theta_j) + \mathbf{n}(k) \tag{1}$$

where k is the index of time, J is the number of interference sources, $s(k)$ and $\beta_i(k)$ (for $j = 1, \ldots, J$) are the amplitudes of the SOI and interfering signals at time instant k, respectively, θ_l (for $l = 0, 1, \ldots, J$) are the DOAs of the SOI and interfering signals, $\varphi_l = (2\pi d/\lambda)\sin\theta_l$, with d being the distance between two adjacent antennas and λ being the operating wavelength [8–10], i.e., the wavelength of the SOI, and $\mathbf{n}(k)$ is the additive white Gaussian noise (AWGN) vector at time instant k.

The output of a beamformer for the time instant k is then given by

$$y(k)=\mathbf{w}^H\mathbf{x}(k)=s(k)\mathbf{w}^H\mathbf{a}(\theta_0)+\sum_{j=1}^{J}\beta_j(k)\mathbf{w}^H\mathbf{a}(\theta_j)+\mathbf{w}^H\mathbf{n}(k) \quad (2)$$

where $\mathbf{w}$ is the M-by-1 complex-valued weighting vector of the beamformer.

The MVDR beamformer is designed to minimize the total array output energy, subject to a linear distortionless constraint on the SOI. The weighting vector of the MVDR beamformer [1] is given by

$$\mathbf{w}_{MVDR}=\arg\min_{\mathbf{w}}\left(\mathbf{w}^H\mathbf{R}_x\mathbf{w}\right),\quad \text{s.t.}\mathbf{w}^H\mathbf{a}(\theta_0)=1 \quad (3)$$

where $\mathbf{R}_x$ is the M-by-M covariance matrix of the received signal vector $\mathbf{x}(k)$, and $\mathbf{w}^H\mathbf{a}(\theta_0)=1$ is the distortionless constraint applied on the SOI.

3. WEIGHTED SPARSE CONSTRAINT BEAMFORMER

In the perspective of the beam pattern, it is observed from (3) that there is only an explicit constraint on the desired DOA, while no constraint is put onto the interference. To repair this drawback, we propose the following cost function with a regularization term, which forces sparsity of the beam pattern with respect to the potential interference DOA [3]. Accordingly, the weighting vector of the improved MVDR beamformer based on a sparse constraint is given by

$$\mathbf{w}_{SC}=\arg\min_{\mathbf{w}}\left(\mathbf{w}^H\mathbf{R}_x\mathbf{w}+\gamma\left\|\mathbf{w}^H\mathbf{A}\right\|_p^p\right),\quad \text{s.t.}\mathbf{w}^H\mathbf{a}(\theta_0)=1 \quad (4)$$

where γ is the factor that controls the tradeoff between the minimum variance constraint on the total array output energy and the sparse constraint on the beam pattern, the M-by-N matrix $\mathbf{A}$ consists of N steering vectors for all possible interference with DOA in the range of $[-90°,\theta_0)\cup(\theta_0,90°]$, with θ_0 being the DOA of the SOI as defined in (1), i.e.,

$$\mathbf{A}=\begin{bmatrix}1 & 1 & \cdots & 1\\ \exp(j\varphi_1) & \exp(j\varphi_2) & \cdots & \exp(j\varphi_N)\\ \vdots & \vdots & \ddots & \vdots\\ \exp(j(M-1)\varphi_1) & \exp(j(M-1)\varphi_2) & \cdots & \exp(j(M-1)\varphi_N)\end{bmatrix} \quad (5)$$

$$\varphi_l=\frac{2\pi d}{\lambda}\sin\theta_l,\quad \text{for } l=1,\ldots,N \quad (6)$$

and $\|\mathbf{x}\|_p = (\sum_i |x_i|^p)^{1/p}$ is the $\mathscr{C}_p$ norm of a vector $\mathbf{x}$. When $0 \le p \le 1$, the $\mathscr{C}_p$ norm provides a measurement of sparsity for $\mathbf{x}$. The smaller the value of $\|\mathbf{x}\|_p^p$ is, the sparser the vector $\mathbf{x}$ is, meaning that the number of trivial entries in $\mathbf{x}$ is larger [3]. As the product $\mathbf{w}^H\mathbf{A}$ indicates sidelobe levels of the beam pattern [3], a smaller value of $\|\mathbf{w}^H\mathbf{A}\|_p^p$ would imply lower sidelobe levels. The $\mathscr{C}_p$ norm can also be explained as a diversity measure [5]. When p is fixed, some principles of choosing a proper γ, such as L-curve, have been introduced in [5], but how to choose a proper γ requires further investigation.

We consider the M-by-1 vector, $\mathbf{x}(k)$, as a snapshot of the received signal at time instant k. If we collect the snapshots of K ($K \ge 1$) different time instants in a matrix, then we can have an M-by-K data matrix as

$$\mathbf{X} = [\ \mathbf{x}(1) \quad \mathbf{x}(2) \quad \ldots \quad \mathbf{x}(K)\ ] \tag{7}$$

It has been shown that the cross-correlation of the steering matrix $\mathbf{A}$ with the received data matrix $\mathbf{X}$ coarsely represents the a posteriori spatial distribution of interfering signals [7]. We use this property to define a weighted sparse constraint for further suppressing sidelobe levels of the beam pattern. As a result, the weighting vector of the beamformer with a weighted sparse constraint is given by

$$\mathbf{w}_{WSC} = \arg\min_{\mathbf{w}} \left(\mathbf{w}^H\mathbf{R}_x\mathbf{w} + \gamma \left\|\mathbf{w}^H\mathbf{A}\mathbf{Q}\right\|_p^p\right), \quad \text{s.t.}\mathbf{w}^H\mathbf{a}(\theta_0) = 1 \tag{8}$$

where the N-by-N matrix $\mathbf{Q} = \text{diag}\left[\text{SNM}(\mathbf{A}^H\mathbf{X})\right]$ serves as a weighting matrix, and $\text{SNM}(\mathbf{A}^H\mathbf{X})$ is an N-by-1 vector containing as elements the squared normalized mean value of each row of the N-by-K matrix $\mathbf{A}^H\mathbf{X}$ [7]. Comparing (8) with (4), we can see that the matrix $\mathbf{Q}$ in (8) provides additional weighting on the sparse constraint, in accordance with the DOA distribution of interfering signals. More specifically, the larger the probability of interference arriving in a certain direction, the larger the weight applied on the sparse constraint in the corresponding direction.

The optimal weighting vector indicated by (8) can be found by using an adaptive iteration algorithm [3, 5]. When $p = 1$, a simpler method, called basis pursuit [6], can be used to solve (8) efficiently. We also observe that (4) can be considered as a special case of (8), in terms that (8) reduces to (4) when $\mathbf{Q} = \mathbf{I}$, corresponding to the case of equal weighting in every direction.

In both (4) and (8), the beamformer weighting vector is subject to a distortionless constraint applied on the SOI. The robust minimum variance beamforming (RMVB) [10] introduced a modified constraint on the SOI, for enhancing the beamformer's robustness against mismatch between the steering angle and the DOA of the SOI. Specifically, the RMVB weighting vector minimizes the total weighted power output of the array, subject to the constraint that the array gain exceeds unity for all array responses within a pre-specified ellipsoid [10], i.e.,

$$\mathbf{w}_{RMVB} = \arg\min_{\mathbf{w}} \left(\mathbf{w}^H \mathbf{R}_x \mathbf{w}\right), \quad \text{s.t. real}\left(\mathbf{w}^H \mathbf{a}(\theta)\right) \geq 1, \ \ \forall \mathbf{a}(\theta) \in \xi \quad (9)$$

where real (.) denotes taking the real part, ξ is the ellipsoid that covers all possible values of the estimated $\mathbf{a}(\theta_0)$, including variations caused by uncertainty in the array manifold and imprecise knowledge of the DOA of the SOI [10], and $\mathbf{a}(\theta)$ represents any steering vector that falls within the ellipsoid. By comparing (9) with (3), we can see that the RMVB scheme provides an improvement over the MVDR beamformer, because it explicitly takes into account variation or uncertainty in the array response, so as to improve the robustness of the beamformer against mismatch in steering angle [1, 10].

To improve the robustness against imperfectly estimated steering angle for our proposed beamformer in (8), we integrate the RMVB as shown in (9) with the weighted sparse constraint used in (8), and obtain a robust beamformer as follows

$$\begin{aligned} \mathbf{w}_{RWSC} &= \arg\min_{\mathbf{w}} \left(\mathbf{w}^H \mathbf{R}_x \mathbf{w} + \gamma \left\|\mathbf{w}^H \mathbf{A}\mathbf{Q}\right\|_p^p\right), \\ &\text{s.t. real}\left(\mathbf{w}^H \mathbf{a}(\theta)\right) \geq 1, \quad \forall \mathbf{a}(\theta) \in \xi \end{aligned} \quad (10)$$

for which the technique of Lagrange multipliers [1] can be used to obtain the updating formula of $\mathbf{w}_{RWSC}$ [3]. The proposed beamformer in (10) thus includes both a weighted sparse constraint based on a coarse estimation of interference DOA distribution for sidelobe suppression and an improved constraint on the SOI that enhances the beamformer's robustness against DOA estimation errors.

4. SIMULATION RESULTS

In the simulations, an ULA with 8 half-wavelength spaced antennas is considered. The AWGN noise at each sensor is assumed spatially uncorrelated. The DOA of the SOI is set to be 0°, and DOAs of three interfering signals are set to be −30°, 30°, and 70°, respectively. The signal to noise ratio (SNR) is set at 10 dB, and the interference to noise ratio (INR) is assumed to be 20 dB, 20 dB, and 40 dB in −30°, 30°, and 70°, respectively. 100 snapshots were used for each simulation. Without loss of generality, p is set to be 1 and γ is set to be 2. The matrix $\mathbf{A}$ consists of all steering vectors in the DOA range of $[-90^\circ, 0^\circ) \cup (0^\circ, 90^\circ]$ with a sampling interval of 1°.

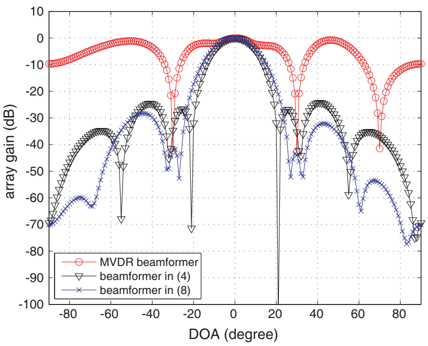

Figure 1. Normalized beam patterns of the MVDR beamformer and beamformers in (4) and (8), without mismatch between the steering angle and the DOA of the SOI.

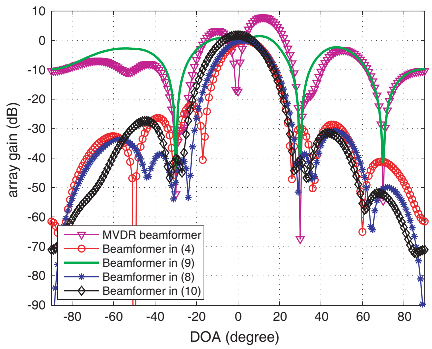

Figure 2. Normalized beam patterns of the MVDR beamformer and beamformers in (4), (8), (9) and (10), each having a 3 mismatch between the steering angle and the DOA of the SOI.

Figure 1 shows beam patterns of an MVDR beamformer, the sparse-constraint beamformer in (4), and the weighted-sparse-constraint beamformer in (8). It is obvious that the best sidelobe suppression performance is achieved by the beamformer in (8), because the weighted sparse constraint on the beam pattern explores information about the DOA distribution of interfering signals. Among the three beamformers, the beamformer in (8) also provides the deepest nulls in the directions of interference, i.e., -30°, 30° and 70°, especially in 70° where the strongest interference lies.

Figure 2 shows beam patterns of all beamformers that we have discussed, with each beamformer having a 3° mismatch between the steering angle and the DOA of the SOI. We can see that the MVDR beamformer has a deep notch in 0°, which however is the DOA of the SOI. It can be explained by using the fact that the MVDR beamformer is designed to minimize the total array output energy subject to a distortionless constraint on the SOI, so when the steering angle is in 3°, instead of 0°, the MVDR beam pattern maintains distortionless in 3° while resulting in a deep null in 0°. This observation shows the high sensitivity of the MVDR beamformer to steering angle mismatch. Comparing beam patterns of beamformers defined in (4), (8), (9) and (10), we can see that the weighted sparse constraints used in (8) and (10) are able to further suppress sidelobe levels and deepen the nulls for interference avoidance, especially in 70°, where the strongest interference locates, as compared with the other two beamformers.

A comparison between the two beamformers in (8) and (10) shows that the 3° mismatch in the steering angle leads to an approximate 3° shift of the beam pattern for the beamformer in (8), while the beamformer in (10) can still accurately steer the main lobe of the beam patter in 0°, which is the DOA of the SOI. Therefore, our proposed beamformer in (10) provides significant improvements in terms of sidelobe suppression, nulling for interference avoidance, and robustness against DOA estimation errors, with respect to existing beamformers.

5. CONCLUSION

In this letter, we have proposed a robust beamformer based on a weighted sparse constraint, which leads to a desirable beam pattern with low sidelobe levels, deep nulls in the directions of strong interference, and an accurate steering direction in spite of possible estimation error in the DOA of the desired signal. Numerical experiments have demonstrated that the proposed beamformer significantly outperforms existing beamformers in terms of sidelobe suppression, interference avoidance, and robustness against steering angle mismatch.

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