# A NEW DETECTION ALGORITHM BASED ON CFAR FOR RADAR IMAGE WITH HOMOGENEOUS BACK-**GROUND**

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Abstract—A new detection algorithm based on constant false alarm rate (CFAR) algorithm, which is applicable to radar image with homogeneous background, is proposed in this paper. This algorithm firstly estimates the parameters of the probability model of background accurately. Then a conventional global CFAR is done using the results of estimation. In estimating the parameters of background, a novel iterative algorithm, which is self-adaptive, is given. The simulation results demonstrate that the performance of the proposed detection algorithm is very close to the theoretical optimum value, and better than CA-CFAR, GO-CFAR and SO-CFAR.

## 1. INTRODUCTION

In the field of radar image detection, constant false alarm rate (CFAR) algorithm can set the threshold adaptively according to the level of background intensity [1–9]. "Target" in this paper indicates significant object with strong reflection and extremely small geographic region. "Background" means all the pixels in the image which are not targets. The threshold in a CFAR detector is calculated using the initial false alarm rate and the information of the background.

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Several well known examples of CFAR processor are "cell averaging" (CA) CFAR, "greatest of" (GO) CFAR, and "smallest of" CFAR [2, 3]. In these well known processors, the CA-CFAR processor is acknowledged as the optimum CFAR processor in a homogeneous background because it can maximize the detection probability under the same false alarm probability [2–4]. CA-CFAR assumes that the background pixels obey an exponential distribution, and estimates the distribution parameters using the pixels in a reference window around the test pixel. These pixels are named reference pixels. As the size of the reference window increases, the detection probability also increases. However, it can not approach the theoretical optimum value even though the window is as large as the whole image because the targets are wrongly included in the reference pixels.

We propose a new CFAR algorithm in this paper. In Section 2, the theoretical basis and some assumptions are introduced. In Section 3, the estimation algorithm, which uses a novel iterative method, is given in detail. Section 4 describes the proposed detection algorithm. The numerical results of the experiments are shown in Section 5.

The radar image mentioned in this paper is the single-look intensity image with homogeneous background.

## 2. INTRODUCTION OF THEORETICAL BASIS AND BASIC ASSUMPTIONS

Let x denote a pixel in the image, the CFAR detection solution is  $[4]$ 

$$
H_0: x < T \quad H_1: x \ge T \tag{1}
$$

where  $H_0$  is the null hypothesis of not being a target, namely, background,  $H_1$  is the alternative hypothesis of being a target, and T is the threshold determined by a fixed false alarm rate before detection.

The performance of a detection algorithm is usually described by false alarm probability  $P_{fa}$  and detection probability  $P_d$ .

Let  $p_b(x, \Theta_b)$  denote the probability density function (PDF) of the background, where  $\Theta_b$  is the collection of the background parameters. Then the false alarm probability  $P_{fa}$  is given by [2]

$$
P_{fa} = \int_T^{\infty} p_b(x, \Theta_b) dx
$$
 (2)

Similarly, the detection probability  $P_d$  is given by

$$
P_d = \int_T^{\infty} p_t(x, \Theta_t) dx
$$
 (3)

where  $p_t(x, \Theta_t)$  is the PDF of the target, and  $\Theta_t$  is the collection of the target parameters.

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In order to analyze the detection performance of a CFAR processor in homogenous background, we assume that the pixel values of the intensity image are exponentially distributed [2, 3, 7], with PDF

$$
p(x) = \frac{1}{\mu} \exp\left(-\frac{x}{\mu}\right), \quad x \ge 0 \tag{4}
$$

Under the null hypothesis  $H_0$  of not being a target,  $\mu$  is the average intensity of the background, namely, the mean value of the background pixels in the image. It will be represented by  $\mu_b$  hereinafter. Under the alternative hypothesis  $H_1$  of being a target,  $\mu$  is the average intensity of the target, namely, the mean value of the target pixels in the image. It will be represented by  $\mu_t$  hereinafter. A pixel belongs to target or background. The background and the target are mutual independent.

Under these assumptions, the false alarm probability  $P_{fa}$  is given by  $\mathbf{r}$  $\mathbf{r}$ 

$$
P_{fa} = \int_{T_{opt}}^{\infty} \frac{1}{\mu_b} \exp\left(-\frac{x}{\mu_b}\right) dx = \exp\left(-\frac{T_{opt}}{\mu_b}\right) \tag{5}
$$

where  $T_{opt}$  denotes the optimum threshold. Similarly, the optimum detection probability  $P_{d,opt}$  is given by

$$
P_{d,opt} = \int_{T_{opt}}^{\infty} \frac{1}{\mu_t} \exp\left(-\frac{x}{\mu_t}\right) dx = \exp\left(-\frac{T_{opt}}{\mu_t}\right) \tag{6}
$$

Substituting (5) into (6) we get

$$
P_{d,opt} = P_{fa}^{\frac{1}{r}} \tag{7}
$$

where  $r = \frac{\mu_t}{\mu_t}$  $\frac{\mu_t}{\mu_b}$  denotes the ratio of the average intensity of target to background, usually being named SCR (Signal Clutter Ratio).

From Equation (5), we get

$$
T_{opt} = -\mu_b \ln P_{fa} \tag{8}
$$

Obviously,  $\mu_b$  is significant for determining  $T_{opt}$ , because  $P_{fa}$  is fixed before detection in CFAR algorithm. In other words, we will get the threshold  $T_{opt}$  as accurate as we can estimate  $\mu_b$ .

## 3. ESTIMATION OF  $\mu_b$

#### 3.1. Principle of Estimation and Correction

As estimating  $\mu_b$  with the mean value of all pixels in the image will increase the estimation result because the target pixels are wrongly included, an improvement is firstly determining a threshold  $T$  in some way. Then the mean value of those pixels whose value is lower than T



Figure 1. Estimation error before correction; "A" denotes the real average value  $\mu_b$ ; "B" denotes the estimation result.

is considered as  $\mu_b$ . However, there is also much error because some target pixels are mistakenly included and some background pixels are mistakenly excluded.

Figure 1 shows the estimation error of the method described in last paragraph.

Our proposed estimation algorithm firstly gets a threshold T using the conventional CFAR algorithm under the assumption of the whole image being exponentially distributed. The mean value of the pixels, which are lower than  $T$ , is the preliminary estimation result. We denote it as  $\hat{\mu}_b$ . Then  $\hat{\mu}_b$  is corrected as follows.

The entire image contains background and target pixels, and the background and the target are mutual independent, so the PDF of the whole image can be considered as weighted sum of the PDFs of background and target:

$$
p(x) = \lambda \frac{1}{\mu_b} e^{-\frac{x}{\mu_b}} + (1 - \lambda) \frac{1}{\mu_t} e^{-\frac{x}{\mu_t}}, \quad x \ge 0
$$
 (9)

where  $\lambda = n/N$  is the ratio of the number of real background pixels n to the number of pixels in the whole image N.

The mean value of all the pixels in the whole image is

$$
\mu = E[x] = \int_{-\infty}^{\infty} p(x) \, x \, dx
$$

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$$
= \int_0^\infty \left[ \lambda \frac{1}{\mu_b} e^{-\frac{x}{\mu_b}} + (1 - \lambda) \frac{1}{\mu_t} e^{-\frac{x}{\mu_t}} \right] x dx = \lambda \mu_b + (1 - \lambda) \mu_t \quad (10)
$$

where  $\mu =$  $\frac{N}{\sqrt{2}}$  $i=1$  $x_i$ N,  $x_i$   $(i = 1, 2, \dots, N)$  is the value of a pixel in the

image.

The pixels whose value is lower than threshold  $T$  are regarded as background in CFAR detection algorithm. Then we have h i

$$
\mu_{bT} = \frac{\int_0^T p(x) \, x dx}{\lambda_T} = \frac{\int_0^T \left[ \lambda \frac{1}{\mu_b} e^{-\frac{x}{\mu_b}} + (1 - \lambda) \frac{1}{\mu_t} e^{-\frac{x}{\mu_t}} \right] x dx}{\lambda_T}
$$
\n
$$
= \frac{\lambda \left( -Te^{-\frac{T}{\mu_b}} - \mu_b e^{-\frac{T}{\mu_b}} + \mu_b \right) + (1 - \lambda) \left( -Te^{-\frac{T}{\mu_t}} - \mu_t e^{-\frac{T}{\mu_t}} + \mu_t \right)}{\lambda_T} \tag{11}
$$

where  $\mu_{bT} =$  $\frac{n_T}{2}$  $i=1$  $x_i/n_T$ ,  $x_i$   $(i = 1, 2, \dots, n_T)$  is the value of a pixel which is lower than  $T$ ,  $n<sub>T</sub>$  is the number of pixels which are lower than T,  $\lambda_T$  is the ratio of the number of pixels which are lower than T to the number of pixels in the whole image.

In Equations (10) and (11), there are three unknown parameters, namely,  $\mu_b$ ,  $\mu_t$  and  $\lambda$ .

From the theory of CFAR, we know that the number of points which are lower than  $T$  is equal to the number of real background pixels, when  $T$  is appropriate to make the number of false alarm pixels be equal to the number of miss alarm pixels. That is

$$
\lambda = \frac{n}{N} = \frac{n_T}{N} \tag{12}
$$

when

$$
\int_{T}^{\infty} \lambda \frac{1}{\mu_b} e^{-\frac{x}{\mu_b}} dx = \int_{0}^{T} (1 - \lambda) \frac{1}{\mu_t} e^{-\frac{x}{\mu_t}} dx \tag{13}
$$

Equations  $(10)$ ,  $(11)$  and  $(13)$  are not easy to be solved. So we adopt an iterative method to calculate  $\mu_b$ ,  $\mu_t$  and  $\lambda$ . The detail steps are described in Section 3.2.

#### 3.2. Implementation

The procedures of estimation and correction are as follows:

Step 1: Calculate the mean value of all the pixels in image

$$
\mu = \sum_{i=1}^{N} x_i / N, \quad x_i (i = 1, 2, \cdots, N) \tag{14}
$$

where  $N$  is the number of the pixels in the whole image, and  $x_i$  is the value of a pixel in the image. From Equation (8), we get the initial threshold  $T = -\mu \ln \alpha$  with the fixed false alarm rate  $\alpha$ .  $\alpha$  can range from 0.1 to 0.000001. Usually we set it 0.001 or 0.0001.

Step 2: Test whether the pixel value is lower than T pixel by pixel. Suppose  $\{x_i | j = 1, 2, \dots, n_T\}$  is the collection of the pixels which are lower than  $T$ . Then the mean value of the background pixels according to  $T$  is  $n_{\rm}$ 

$$
\mu_{bT} = \frac{\sum_{j=1}^{n_T} x_j}{n_T} \tag{15}
$$

Correspondingly, the mean value of the target pixels according to  $T$  is

$$
\mu_{tT} = \frac{\sum_{i=1}^{N} x_i - \sum_{j=1}^{n} x_j}{N - n_T}
$$
\n(16)

The estimation value of  $\lambda$  is

$$
\hat{\lambda} = \frac{n_T}{N} \tag{17}
$$

Step 3: Substituting (14), (15), (17) and  $\lambda = \hat{\lambda}$  into Equations (10) and  $(11)$ , we can solve the simultaneous equations of  $(10)$  and  $(11)$ through the fixed-point iteration method to get the preliminary correction results of  $\mu_b$  and  $\mu_t$ .

Step 4: Save the current threshold as  $T_{old}$ . Substituting  $\hat{\lambda}$  obtained in step 2,  $\mu_b$  and  $\mu_t$  obtained in step 3 into Equation (13), we can get the new threshold T through the fixed-point iteration method.

Step 5: Repeat from step 2 to 4 until the relative error of  $T$  is small enough.

Then the final  $\mu_b$  after step 5 is the estimation result of our proposed algorithm.

### 4. PROPOSED DETECTION ALGORITHM

Because the background is homogeneous, it is reasonable to adopt a global threshold which is adaptive to the whole image.

We can get the global detection threshold  $T_q$  from (5) with  $\hat{\mu}_b$ estimated in Section 3 and  $P_{fa}$  fixed before detection.

$$
T_g = -\hat{\mu}_b \ln P_{fa} \tag{18}
$$

The pixels in the image are tested one by one. One pixel is classified into target if its value is larger than  $T_q$ .

### 5. NUMERICAL RESULTS AND DISCUSSION

### 5.1. Performance of Estimating  $\mu_b$

In order to verify the effectiveness of the proposed estimation algorithm for  $\mu_b$ , we have resorted to the mathematical software MATLAB to generate the test images. Firstly, we created an background image with  $4000 \times 4000$  pixels which obeyed an exponential distribution. Then 40,000 target points which also obeyed an exponential distribution were embedded in the background image with intervals of 20 pixels. We controlled the parameters of PDFs (background and target) to generate the test images with different SCR.

The formulation calculating the relative error is as follow:

$$
\delta = \frac{|\hat{\mu}_b - \mu_b|}{\mu_b} \times 100\%
$$
\n(19)

where  $\hat{\mu}_b$  is the result of estimation.

The relative error related to different SCR and  $\alpha$  is shown in Figure 2. The SCR ranges from 10 dB to 30 dB, which is the most common range for radar image. We can see that even the maximum relative error is less than 0.03%. Moreover, the proposed estimation algorithm can get accurate result self-adaptively even when the SCRs of the images are different.

The fixed false alarm rate  $\alpha$  in Section 3.2 is the only initial parameter in the proposed estimation algorithm. The results shown



**Figure 2.** Relative error of estimating  $\mu_b$  according to different SCR and the initial parameter  $\alpha$ .

in Figure 2 indicated that when  $\alpha$  ranged from  $10^{-1}$  to  $10^{-6}$ , the estimation results were unaltered.

These experiments indicate that the proposed estimation algorithm is effective and self-adaptive, furthermore, not dependent on the selection of initial parameters.

#### 5.2. Performance of the Proposed Detection Algorithm

Figure 3 shows the detection probability of the proposed detection algorithm in Section 4 comparing with CA-CFAR, GO-CFAR, SO-CFAR and the theoretical optimum value, when  $P_{fa}$  is 10<sup>-6</sup>. It is clear that the detection probability of the proposed algorithm approaches to the optimum one and is higher than CA-CFAR, GO-CFAR and SO-CFAR steadily despite the change of SCR.

We can see that the performance of CA-CFAR is best during CA-CFAR, GO-CFAR and SO-CFAR, and we know that the detection performance of CA-CFAR will become better when the number of reference points N increases. However, it will not approach the optimum line, because once the size of the reference window is so large as to include in target pixels, the mean value of reference pixels will not be equal to the average intensity of background. The error brought by including target pixels is analyzed as follows.

Suppose *n* target pixels are included, and the total number of the reference pixels is  $N$ . Then the mean value calculated according



Figure 3. Detection probability comparison of different detection algorithms;  $P_{fa}$  is 10<sup>-6</sup> and the numbers of the reference pixels in CA-CFAR, GO-CFAR, and SO-CFAR are all 16.

Table 1. Computational times comparison of different detection algorithms; the numbers of the reference pixels in CA-CFAR, GO-CFAR, and SO-CFAR are all 16; the SCRs of three test images are all 13 dB.

Image Size	Proposed	$CA-CFAR$	GO-CFAR	SO-CFAR
$1000 \times 4000$	1.029 s	11.176 s	11.136 s	18.336 s
$2000 \times 4000$	2.173 s	21.170 s	24.716 s	21.581 s
$3000 \times 4000$	2.710 s	31.515 s	39.096 s	34.520 s
$4000 \times 4000$	3.566 s	40.839 s	48.380 s	49.261 s

to the reference pixels is  $\left[n\mu_t + (N-n)\mu_b\right]/N$ . Substituting  $\hat{\mu}_b$  =  $[n\mu_t + (N - n)\mu_b]/N$  into (19), we get the relative error

$$
\delta = \frac{|[n\mu_t + (N - n)\mu_b]/N - \mu_b|}{\mu_b} \tag{20}
$$

Substituting  $r = \frac{\mu_t}{\mu_t}$  $\frac{\mu_t}{\mu_b}$  and  $\lambda = \frac{n}{N}$  $\frac{n}{N}$  into (20), we get

$$
\delta = \lambda (r - 1) \tag{21}
$$

The relative error will increase with r and  $\lambda$ .

In addition, the threshold in the proposed algorithm is calculated only once. Correspondingly, CA-CFAR, GO-CFAR and SO-CFAR calculate the thresholds pixel by pixel, so their computational times are much larger than the proposed algorithm. The detailed results are shown in Table 1 below. The CPU used is Intel Pentium Processor  $1500 \text{ MHz}$ , and the program language used is  $C_{++}$ .

### 6. CONCLUSION

In this work, a new CFAR detection algorithm for radar image with homogeneous background is given. In order to validate our algorithm, simulation experiments have been carried out. Better performance comparing with CA-CFAR, GO-CFAR and SO-CFAR has been observed. And the theoretical optimum value has been approached by our algorithm. In addition to the good performance on the probability of detection, our algorithm offers many other advantages, such as self-adaptation, not depending on the selection of the initial parameter, and low computing complexity.

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