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GENERAL DESIGN OF N-WAY MULTI-FREQUENCY UNEQUAL SPLIT WILKINSON POWER DIVIDER USING TRANSMISSION LINE TRANSFORMERS

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Abstract—In this paper, a new N-way multi-frequency unequal split Wilkinson power divider (WPD) is proposed. The dividers are composed of multi-section transmission line transformers (TLT) and isolation resistors, which provide high isolation and very good input/output ports matching simultaneously at arbitrary design frequencies. To verify the validity of the design, several multi-frequency power dividers are designed and simulated. Specifically, a 3-way unequal split dual-frequency WPD operating at 900 and 1800 MHz, a 3-way unequal split triple-frequency WPD operating at 1, 2, 3 GHz, and a 4-way equal split quad-frequency WPD operating at 1, 2, 3, 4 GHz, are designed.

1. INTRODUCTION

The new technologies of mobile wireless communications, especially the design of multi-frequency antenna arrays attracted much attention and interest in multi-frequency power dividers. In the case of three or more antenna elements, N-way multi-frequency power dividers are used to feed antenna elements. In this paper, a simple technique is used to design N-way multi-frequency unequal split Wilkinson power dividers (WPDs). The proposed technique is based on the equivalent 2-way model [1, 2]. This model is analyzed using the even-odd mode analysis [3]. The most important characteristics of the conventional Wilkinson power dividers are achieved; high isolation between output ports, and input/output ports matching at the desired frequencies.

Very recently, a couple of papers have been published [4, 5] in which the analysis was carried to design N-way dual-frequency equal

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split WPD. It should be emphasized that the proposed WPDs in [4, 5] were equal split and dual-frequency ones. In our paper, we consider the general design of unequal split, multi-frequency (dual, triple, and quad) WPDs. Another interesting paper [6] discussed the design of dual-frequency unequal split N-way WPD with planar isolation resistors. Several structures have been also proposed to design three-way Wilkinson power dividers. Jui [7] proposed the use of coupled line impedance transformers in the middle section of the divider to couple the input signal and to provide impedance transformation function to the output signals. Equal power split was only achieved. Also, as the number of the output ports increases, the complexity of the analysis increases, and multi-frequency operations were not considered. Another planar dual-frequency three-way WPD was proposed in [8]. The structure was a modified version of Nagai hybrid power divider [9]. Each transmission line section was replaced by two transmission line sections to account for dual-frequency operations. Unfortunately, the isolation performance between the output ports was not considered, and the output ports matching conditions were neglected. Based on the same concept, Wang [10] proposed the use of an RLC circuit instead of using a single resistor to enhance the isolation performance between the output ports. However, the problem of output ports matching was not solved in [10]. In [11], Cheng presented the design of a planar six-way power divider using folded and hybrid expanded coupled lines. Multi-frequency operation and unequal power split were not discussed.

Moreover, many researchers investigated the design of 2-way dual-frequency Wilkinson power divider with equal [12, 13] and unequal [14–18] output power split ratio. To our knowledge, none of these topologies have been extended to N-way unequal split Wilkinson power dividers. The simplicity of our proposed structure makes it applicable to design and implement N-way multi-frequency unequal split WPD. The proposed structure is similar to the original Wilkinson divider [19]. The multi-frequency operation is accomplished by replacing the quarter-wave branches of the conventional N-way Wilkinson power divider by transmission line transformers consisting of two sections (in the case of dual-frequency), three sections (in the case of triple-frequency divider) or four sections (in the case of quad-frequency divider).

2. ANALYSIS OF CONVENTIONAL 2-WAY WPD

The even-odd mode analysis presented in this section is similar to that presented in [3] which was used to design and analyze 2-way unequal split Wilkinson power dividers. This analysis will be briefly presented here since it is widely used throughout this paper after reducing an

N-way WPD to its equivalent 2-way model. The 2-way unequal split WPD is shown in Figure 1, where $k^2 = P_3/P_2$ [3].

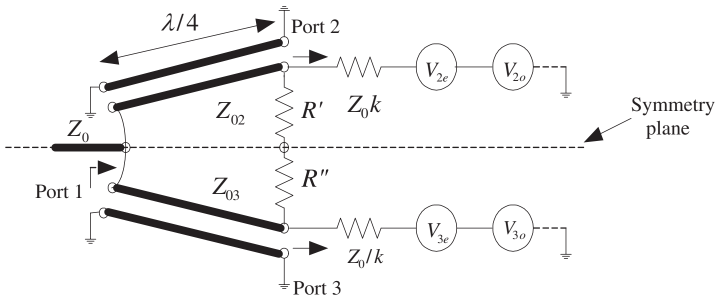

Figure 1. 2-way unequal split WPD.

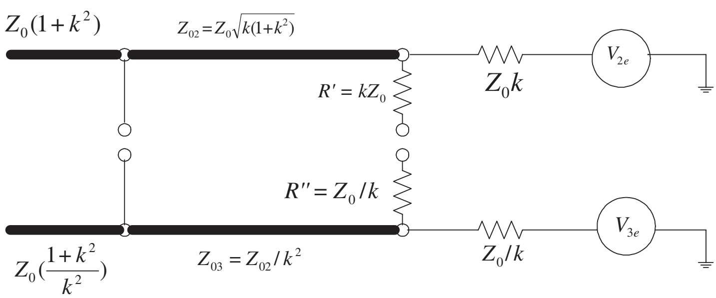

Figure 2. Even mode excitation equivalent circuit.

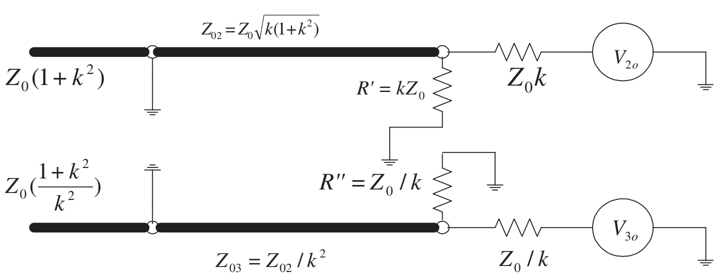

Figure 3. Odd mode excitation equivalent circuit.

2.1. Even Mode Excitation

For the even-mode excitation, $V_{2e} = V_{3e}$, and $V_{2o} = V_{3o} = 0$. Thus, there is no current flow through the resistors (R', R'') or the short circuit between the inputs of the two transmission lines at port 1. So, the network of Figure 1 can be bisected with open circuits at these points to obtain the equivalent network shown in Figure 2.

2.2. Odd Mode Excitation

For the odd-mode excitation, $V_{3o} = -V_{2o}$, and $V_{2e} = V_{3e} = 0$. This results in a voltage null along the middle of Figure 1. Thus, this circuit can be bisected by grounding it at two points on its midplane to give the equivalent network shown in Figure 3.

Using the above analysis the design of conventional 2-way unequal split WPD is obtained. Furthermore, equal split can be obtained by substituting $k^2 = 1$.

3. DESIGN OF N-WAY SINGLE-FREQUENCY UNEQUAL SPLIT WPD

For an N-way WPD, the above even-odd mode analysis can be used after reducing the N-way WPD to its equivalent 2-way model similar to the analysis of equal split N-way divider presented in [1, 2]. In this equivalent 2-way model, the first branch represents the nth-branch and the second branch represents the combination of the remaining $N - 1$ branches. The analysis of an N-way WPD starts by assigning the power split ratio to each output port, then applying the combining technique to find the parameters of each nth-branch. The equivalent 2-way WPD consists of the nth-branch with its pre-assigned power split ratio, and another branch, which is the sum of the power ratios of the rest $N - 1$ branches. The nth-branch parameters are the ones of interest, while the combined branch parameters are not of interest. This process is repeated until the parameters of all the N original branches are found. The case is much easier for equal split WPD since the power split ratio between all the output ports are equal, and thus the equivalent 2-way model have to be done only once.

To clarify the above idea; a 3-way single frequency unequal split WPD shown in Figure 4 is designed and simulated. The design frequency is assumed as $f = 0.9\,\text{GHz}$, and the system impedance $Z_o = 50\,\Omega$. The input power is divided as follows: 50% goes to port 2, 25% goes to port 3, and 25% goes to port 4.

The parameters of branch 1 (Z_{01} and l_1) are found by combining ports 3 and 4 (branch 2 and branch 3). The equivalent 2-way model is shown in Figure 5. This 2-way model is equal split, i.e., $k^2 = 1$, (P% branch 2 + P% branch 3 = 50%). Using the analysis described in Section 2, branch 1 parameters are as follows: $Z_{01} = Z_0\sqrt{k(1+k^2)} = 70.71\,\Omega$, $l_1 = \lambda/4$, and $R_1 = kZ_0 = 50\,\Omega$. Port 2 impedance is $50\,\Omega$ too.

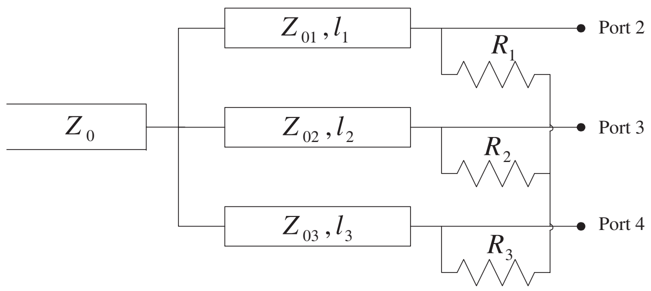

Figure 4. 3-way single-frequency unequal split WPD.

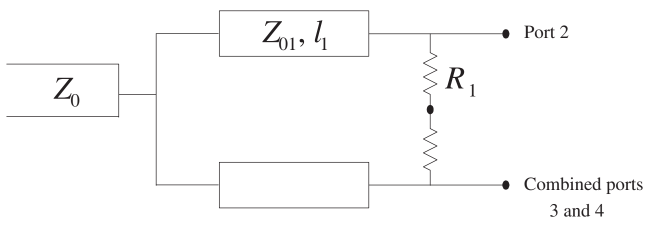

Figure 5. Equivalent model to derive the parameters of branch 1.

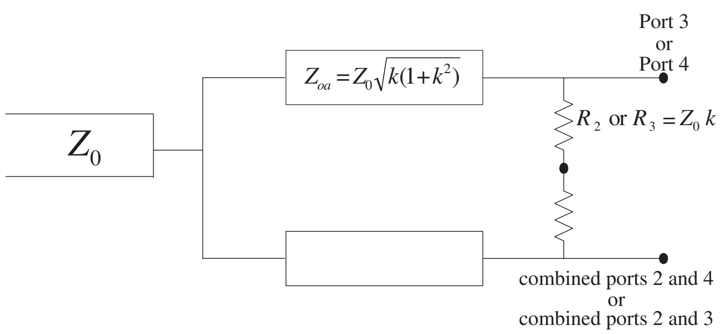

Figure 6. Equivalent model to derive the parameters of branch 2 (or branch 3).

The parameters of branch 2 (Z_{02} and l_2) are found by combining ports 2 and 4 (branch 1 and branch 3). Since ports 3 and 4 have the same power ratio, the same results are achieved by combining ports 2 and 3 to find the parameters of branch 3 (Z_{03} and l_3). In this case, the equivalent 2-way model is shown in Figure 6. The power split ratio in this divider is $k^2 = 3$, since the combined ports take 75% of the input power.

Thus, the parameters of branches 2 and 3 are given as follows: $Z_{02} = Z_{03} = Z_0\sqrt{k(1+k^2)} = 131.61\,\Omega$, $R_2 = R_3 = Z_0 k = 86.60\,\Omega$, $R_{L2} = R_{L3} = Z_0 k = 86.60\,\Omega$ and $l_2 = l_3 = \lambda/4$, where R_{L2} and R_{L3} represent ports 3 and 4 impedances, respectively. Using the above parameters, a microstrip 3-way WPD is designed and simulated using Ansoft Designer [2]. FR-4 substrate with $\varepsilon_r = 4.4$ and height $h = 1.5\,\text{mm}$ is used.

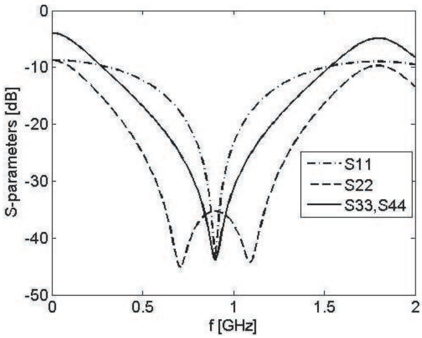

Figure 7. 3-way single-frequency WPD matching S-parameters.

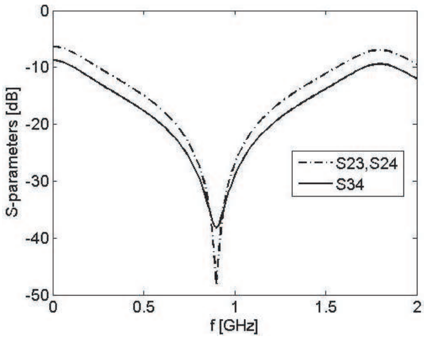

Figure 8. 3-way single-frequency WPD isolation S-parameters.

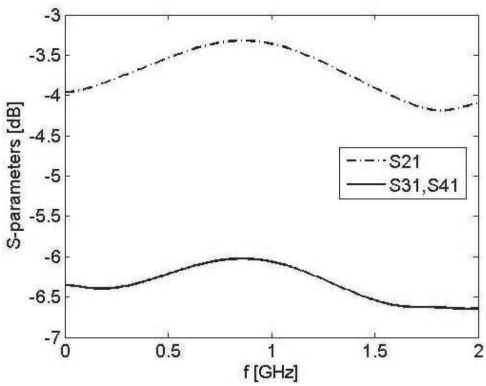

Figure 9. 3-way single-frequency WPD transmission S-parameters.

Figures 7, 8, and 9 show the simulated results of the designed 3-way single frequency WPD. It is observed that very good matching at all ports and very good isolation between the output ports are obtained at the design frequency $f = 0.9\,\mathrm{GHz}$. The power dividing ratios of this WPD are ideally equal to 3 dB for port 2, and 6 dB for ports 3 and 4. Slight deviation is observed in Figure 9 due to conductor and dielectric losses considered in the simulation.

4. DESIGN OF N-WAY UNEQUAL SPLIT MULTI-FREQUENCY WPD

As stated earlier, to achieve multi-frequency operation, the quarter-wave branches of the conventional N-way Wilkinson power divider are replaced by transmission line transformers (TLTs) consisting of two sections (in the case of dual-frequency divider) [21], three sections (in the case of triple-frequency divider) [22], or four sections (in the case of quad-frequency divider) [23]. This gives very good input/output ports matching at the design frequencies. Isolation resistors are placed at each end of the transmission line transformers to achieve good isolation performance between the output ports. Figure 10(a) shows

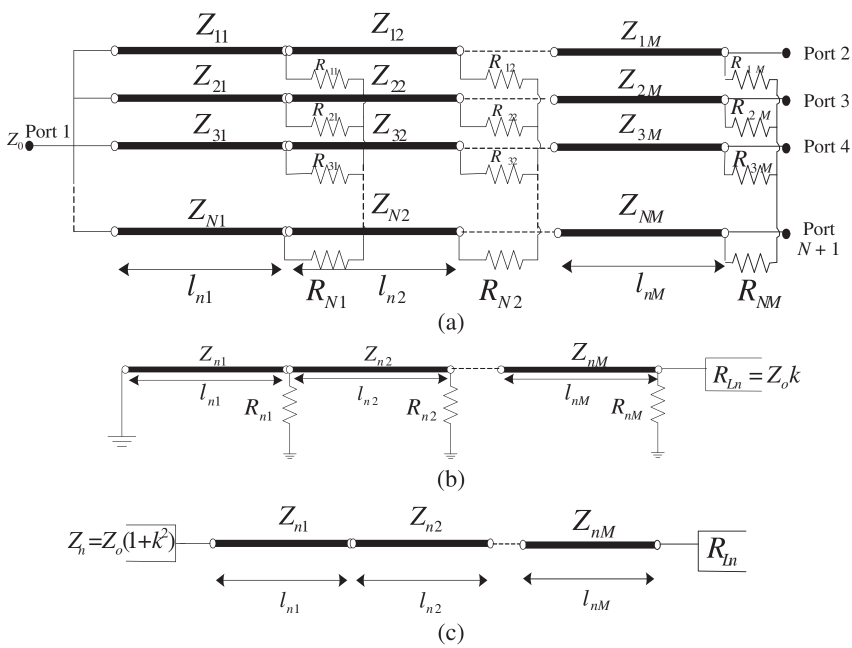

Figure 10. (a) N-way M-frequency unequal split WPD. (b) Odd-mode excitation. (c) Even-mode excitation.

the general schematic of the proposed N-way multi-frequency (M-frequency) unequal split WPD. The main steps in the design procedure of N-way dual-frequency, tri-frequency, and quad-frequency WPDs are briefly described below.

4.1. Dual-Frequency Design

In the dual-frequency unequal split N-way WPD, $M = 2$ is used in Figure 10(a). Using the combining technique (Section 3) and even-odd mode analysis (Section 2), the branches parameters (Z_{n1}, l_{n1}, Z_{n2}, l_{n2}), and isolation resistors (R_{n1}, R_{n2}) can be found, where $n = 1, 2, 3, \ldots, N$. To get the parameters of the nth branch, a 2-way model is first developed. One of the branches of this 2-way model is the nth branch itself, while the other one is the combined $N - 1$ branches. The split ratio of this equivalent 2-way model is obtained as described before.

In the even mode excitation of the equivalent 2-way model, the branches can be thought of as two dual-frequency TLTs for which closed form design equations were derived in [21]. The characteristic impedances and line lengths for the two sections of branch "n" (shown in Figure 10(c) with $M = 2$) can be directly calculated using the equations presented in [21, 24]. In the odd mode excitation, the 2-way equivalent model is reduced to the circuit shown in Figure 10(b). By imposing the matching condition at the output ports, the resistors values can be obtained using the closed form design equations presented in [12, 24]. It should be mentioned that to ensure that the values of these resistors are real and positive, a condition on the ratio between the two design frequencies (f_2/f_1) was derived in [4]. The same condition applies here too.

4.2. TRI-Frequency Design

The tri-frequency ($M = 3$) unequal split WPD is obtained by using three sections of transmission line transformers in the WPD of Figure 10(a). Similar to the dual-frequency WPD, the tri-frequency divider can be designed and analyzed using the above techniques. In the even mode excitation of the equivalent 2-way model, the branches can be thought of as two tri-frequency TLTs for which closed form design equations were derived in [22]. The characteristic impedances and line lengths for the three sections of branch "n" (shown in Figure 10(c) with $M = 3$) can be directly calculated using equations presented in [22, 24]. In the odd mode excitation, the 2-way equivalent model is reduced to the circuit shown in Figure 10(b). By imposing the matching condition at the output ports at the three design frequencies, the isolation resistors can be found using an optimization technique [22, 24]. Here, the Particle Swarm Optimization method [25] is used to find the values of the resistors.

4.3. Quad-Frequency Design

Quad-frequency unequal split WPD is obtained when $M = 4$ in Figure 10. Four isolation resistors are used in each branch to enhance the isolation between the output ports. Four sections of transmission line transformer are used to match the output ports at 4 arbitrary frequencies. In the even mode excitation of the equivalent 2-way model, the branches can be thought of as two quad-frequency TLTs for which closed form design equations were derived in [23]. The characteristic impedances and line lengths for the four sections of branch "n" (shown in Figure 10(c) with $M = 4$) can be directly calculated using equations presented in [23, 24]. In the odd mode excitation, the 2-way equivalent model is reduced to the circuit shown in Figure 10(b). By imposing the matching condition at the output ports at the four design frequencies, the isolation resistors can be found using the PSO technique [24].

5. NUMERICAL EXAMPLES

In this section, a 3-way dual-frequency unequal split WPD, a 3-way tri-frequency unequal split WPD, and a 4-way quad-frequency equal split WPD are designed and simulated to verify the proposed analysis. The simulations were carried using Ansoft Designer software [20]. The simulation is based on microstrip lines on FR-4 substrate including loss effects. The reference impedance Z_0 is chosen to be $50\,\Omega$.

5.1. Example 1: 3-Way Dual-Frequency Unequal Split WPD

Power division is assumed as follows: 50% to port 2, and 25% to each of ports 3 and 4. The design frequencies are $f_1 = 0.9\,\text{GHz}$, and $f_2 = 1.8\,\text{GHz}$. The parameters of the WPD calculated using the above dual-frequency analysis are as follows: $Z_{11} = 79.29\,\Omega$, $Z_{12} = 63.06\,\Omega$, $Z_{21} = 151.04\,\Omega$, $Z_{22} = 114.68\,\Omega$, $Z_{31} = 151.04\,\Omega$, $Z_{32} = 114.68\,\Omega$, $l_{n1} = l_{n2} = \lambda/6$ at $f_1 = 0.9\,\text{GHz}$ $(n = 1, 2, 3)$ $R_{11} = 55\,\Omega$, $R_{12} = 101.5\,\Omega$, $R_{21} = 151\,\Omega$, $R_{22} = 170.5\,\Omega$, $R_{31} = 151\,\Omega$, $R_{32} = 170.5\,\Omega$. Moreover, the impedances of the output ports are as follows: $R_{L2} = 50\,\Omega$, $R_{L3} = 86.60\,\Omega$, and $R_{L4} = 86.60\,\Omega$. The simulation results of the S-parameters are presented in Figures 11, 12, and 13. Figure 11 shows that very good matching is obtained at all ports at the design frequencies. The isolation parameters S_{23}

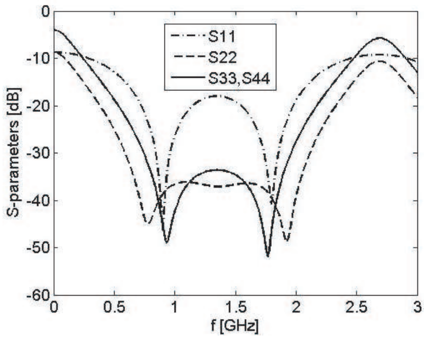

Figure 11. 3-way dual-frequency WPD matching S-parameters.

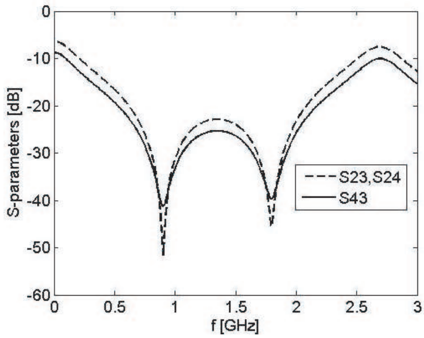

Figure 12. 3-way dual-frequency WPD isolation S-parameters.

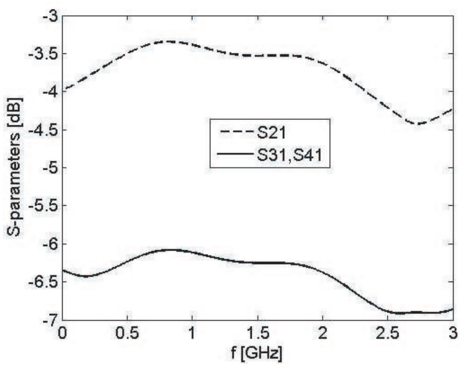

Figure 13. 3-way dual-frequency WPD transmission S-parameters.

and S_{24} show the same performance (better than $-45\,\text{dB}$), because the power split ratio between these ports are the same, while the isolation parameter S_{43} is around $-40\,\text{dB}$ at the center frequencies. Insertion losses S_{31} and S_{41} are around the ideal value of $-6\,\text{dB}$ at the center frequencies with deviation of $-0.1\,\text{dB}$ for the first frequency and $-0.27\,\text{dB}$ for the second frequency. The insertion loss S_{21} is around the ideal value of $-3\,\text{dB}$, since 50% of the input power goes to port (2). Deviations of $-0.35\,\text{dB}$ at f_1 and $-0.54\,\text{dB}$ at f_2 are encountered due to losses.

5.2. Example 2: 3-Way Tri-Frequency Unequal Split WPD

Power division is assumed as follows: 35% for each of ports 2 and 3 and 30% to port 4. The operating frequencies are $f_1 = 1\,\text{GHz}$, $f_2 = 2\,\text{GHz}$, and $f_3 = 3\,\text{GHz}$. The parameters of this WPD calculated using the above tri-frequency analysis are as follows: for the first two branches ($n = 1, 2$): $Z_{n1} = 118.64\,\Omega$, $Z_{n2} = 98.74\,\Omega$, $Z_{n3} = 82.18\,\Omega$, $l_{n1} = l_{n2} = l_{n3} = \lambda/8$ at $f_1 = 1\,\text{GHz}$, $R_{Ln} = 68.19\,\Omega$, $R_{n1} = 80.14\,\Omega$, $R_{n2} = 147.34\,\Omega$, $R_{n3} = 208.41\,\Omega$. Moreover, for the third branch: $Z_{31} = 136.75\,\Omega$, $Z_{32} = 112.92\,\Omega$, $Z_{33} = 93.25\,\Omega$, $R_{L3} = 76.32\,\Omega$, $R_{31} = 92.4\,\Omega$, $R_{32} = 164.92\,\Omega$, $R_{33} = 228.29\,\Omega$, and $l_{31} = l_{32} = l_{33} = \lambda/8$ at $f_1 = 1\,\text{GHz}$. The simulation results of the S-parameters are presented in Figures 14, 15, and 16. From Figure 14, it can be seen that very good matching is obtained at all ports at the four design frequencies. Insertion losses S_{21} and S_{31} are around the ideal value of $-4.55\,\text{dB}$ with deviations of $-0.25\,\text{dB}$ at f_1, $-0.45\,\text{dB}$ at f_2, and $-0.65\,\text{dB}$ at f_3. The insertion loss S_{41} is around the ideal value of $-5.23\,\text{dB}$ with deviations of $-0.17\,\text{dB}$ at f_1, $-0.36\,\text{dB}$ at f_2, and $-0.56\,\text{dB}$ at f_3. Isolation parameters S_{23}, S_{24}, and S_{34} are very similar and show performance better than $-40\,\text{dB}$ at the design frequencies f_1, f_2, and f_3.

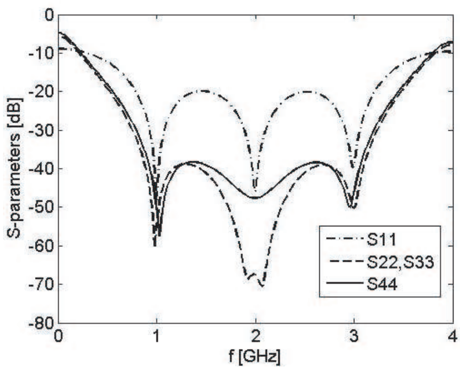

Figure 14. 3-way tri-frequency WPD matching S-parameters.

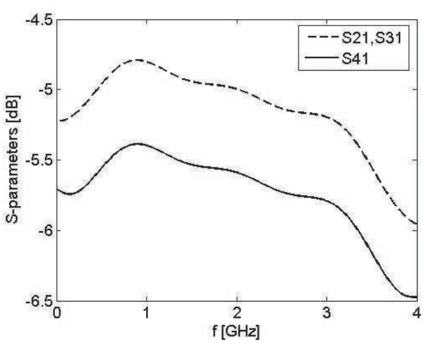

Figure 15. 3-way tri-frequency WPD transmission S-parameters.

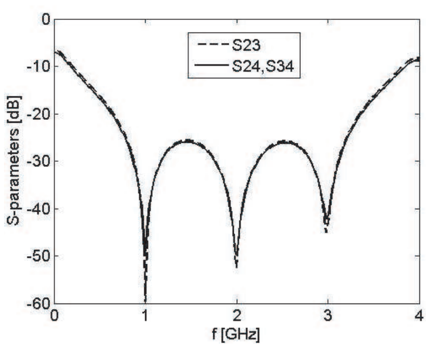

Figure 16. 3-way tri-frequency WPD isolation S-parameters.

5.3. Example 3: 4-Way Quad-Frequency Equal Split WPD

In this example, the input power is assumed to be equally divided between the output ports i.e., 25% goes to each output port. The operating frequencies are chosen as $f_1 = 1\,\text{GHz}$, $f_2 = 2\,\text{GHz}$, $f_3 = 3\,\text{GHz}$, and $f_4 = 4\,\text{GHz}$. The parameters of the WPD are calculated using the above quad-frequency analysis giving: $Z_{n1} = 168.77\,\Omega$, $Z_{n2} = 142.92\,\Omega$, $Z_{n3} = 121.18\,\Omega$, $Z_{n4} = 102.62\,\Omega$, $l_{n1} = l_{n2} = l_{n3} = l_{n4} = \lambda/10$ at $f_1 = 1\,\text{GHz}$, $R_{Ln} = 86.60\,\Omega$, $R_{n1} = 110.99\,\Omega$, $R_{n2} = 202.64\,\Omega$, $R_{n3} = 277.13\,\Omega$, and $R_{n4} = 342.64\,\Omega$ (for all branches, i.e., $n = 1, 2, 3, 4$). The simulation results of the S-parameters are presented in Figures 17 and 18. Insertion losses S_{21}, S_{31}, S_{41}, and S_{51} are around the ideal value of $-6\,\text{dB}$ with deviations of $-0.4\,\text{dB}$ at f_1, $-0.5\,\text{dB}$ at f_2, $-0.71\,\text{dB}$ at f_3, and $-0.92\,\text{dB}$ at f_4. Isolation parameters S_{23}, S_{24}, S_{25}, S_{34}, S_{45}, and S_{35} are all the same and less than $-40\,\text{dB}$ at the design frequencies. Figure 18 shows that very good matching is obtained at all ports. In fact, this WPD can be used in a wide band that extends from 0.5–5 GHz.

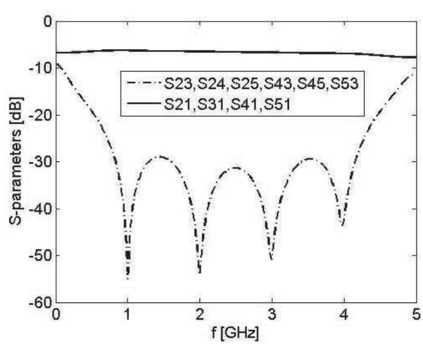

Figure 17. 4-way quad-frequency WPD isolation and transmission S-paramete

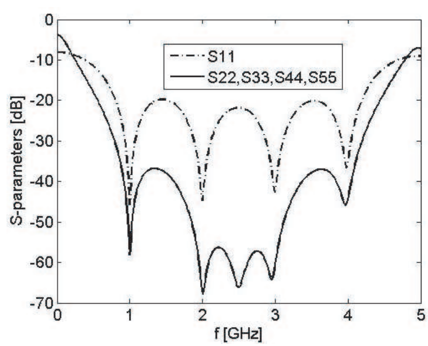

Figure 18. 4-way quad-frequency WPD matching S-parameters.

6. CONCLUSIONS AND FINAL REMARKS

This paper has presented the analysis and design of a general N-way multi-frequency equal/unequal split Wilkinson power divider without any significant modification to the original Wilkinson structure (without transmission line stubs or reactive components), which simplifies the mathematical analysis of the WPD. The proposed structure and the analytical design method are verified through several simulated power dividers with different design frequencies and different power split ratios. The simulated results of the designed Wilkinson powers dividers showed the validity of the proposed design procedure and proved the multi-frequency nature of the proposed WPD.

Unfortunately, we can not fabricate and measure such devices due to the lack of needed equipment. However, a dual-frequency equal split 3-way WPD, having a structure similar to the one proposed here, was fabricated and measured in [4]. The experimental results were very close to the theoretical ones, which makes us believe that our proposed unequal split WPDs are practically realizable. Indeed, one drawback of our proposed WPD is the fact that they are not planar. Having to connect the isolation resistors at a common node could make the implementation of these WPDs with large N somewhat difficult. At the present time, we are working on developing a similar theory to design planar N-way unequal split WPDs.

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