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STUDY OF ELECTROMAGNETIC ION-CYCLOTRON INSTABILITY IN A MAGNETOPLASMA

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Abstract—Excitation of electromagnetic ion-cyclotron instability in the magneto plasma in the presence of perpendicular A.C. electric field for a generalized distribution function for background cold electrons and hot ion plasma has been studied. This distribution function is reducible to anisotropic and loss-cone type for different values of spectral index j. The particle trajectories have been estimated and used to find the dispersion relation and growth rate by using the method of characteristic solutions. Temporal electromagnetic ion cyclotron instability for various plasma parameters has been studied. The role of choice of parameters, distribution function and simultaneous presence of A.C. electric field is studied for excitation of electromagnetic ion cyclotron instability. The results have been used to explain the satellite observations of AMPTE/CCE and compared with earlier work done for temperature anisotropy and other types of distribution functions using other techniques.

1. INTRODUCTION

Observations by means of the satellites, especially the Active Magnetospheric Particle Tracer Explorers (AMPTE)/CCE, have indicated the frequent presence of electromagnetic Ion-cyclotron

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(EMIC) waves in the outer magnetosphere beyond $L = 7$ [1–4]. The wave spectral properties and frequency of occurrence were found to be dependent on local time and could be explained on the basis of resonant instability of anisotropic ring current H^+ ions in the background plasma. The waves appear to be generated near the magnetic equator by anisotropic ($T_\perp > T_\parallel$) protons with energy 10 to 50 keV. Low energy (< 20 eV) ions heavier than protons are also found to affect EMIC waves significantly. EMIC waves generated in plasma dominated by hot population have different characteristics from those generated in dense cold plasma. The controlling factors such as temperature anisotropy, $T_\perp$, N_c or N_h are more effective than concentration of He^+ to have the local time correlation with $x_3 = \omega_r/\omega_c$ [1, 2]. Heavy ion concentrations reported in outer magnetosphere are typically small, 5–10% for He^+ and 1–2% for O^+ [5]. The proton cyclotron instability is suppressed in a frequency band just above the heavy ion gyro frequency. This leads to a frequency structure of wave growth ordered by the heavy ion gyro frequencies [6–8]. Recently a complete description of self consistent model of magnetospheric ring current interacting with EMIC wave has been represented on the basis of kinetic equation by Khazanov et al. [9, 10] and Kudrin et al. [43].

In electron/proton plasma for β greater than zero and electron temperature nearly equal to ion temperature, several electromagnetic instabilities may be excited below the proton cyclotron instability when $T_\perp > T_\parallel$ [11, 12]. The anisotropy threshold of ion cyclotron instability is much lower than of other competing instabilities. Now it is well established that the ion-cyclotron instability produces strong pitch angle scattering of ions at relatively low fluctuation levels [13–15]. Denton and Hudson [16] have shown that the growth rate of the loss-cone-driven mode depends strongly on the depth of the loss-cone. Their analytical study explains the scaling of growth rate of the oblique mode with various plasma parameters. Convective instability of EMIC wave in outer magnetosphere have been investigated in detail by Thorne and Horne [3] and Horne and Thorne [4].

Recently, Xue et al. [17, 18] analyzed the dominant ($n = 1$) cyclotron resonance of energetic protons with field aligned as well as oblique left hand mode EMIC waves for a range of parameters representative of the earth's magnetosphere using a Lorentzian Kappa distribution [19–22, 45].

Recently, observational tests of local ion-cyclotron instability in the earth's magnetosphere have been reported [23, 24]. Observations of fluctuating electric fields at magnetospheric height and in shock regions have also been reported along and perpendicular to the magnetic field [25–27]. Earlier GEOS-1 and ISEE mission reported [25, 28, 29] electric field observation and related wave-particle observation in the magnetosphere. The effect of d.c electric field parallel to ambient magnetic field on the growth rate, resonant frequency, resonant energy and marginal stability of EMIC wave with generalized loss-cone distribution function in a low plasma β homogeneous plasma have been studied using the method of particle aspect analysis by Ahirwar et al. [30]. Thus with a view to investigate the role of A.C. electric field along with parameter variations of cold plasma density and hot energy protons on the generation of EMIC waves was sought to explain the AMPTE/CCE data of morning/evening sector in the magnetosphere around $L = 7$. Further, the day time PC-1 waves found to appear in frequency range above the helium gyrofrquency during substroms, are mainly due to increase in energetic proton flux [31, 44].

The aim of this paper is to investigate the generation of EMIC waves in the magnetosphere and to see whether the choice of distribution function and simultaneous presence of perpendicular A.C. field play a vital role and explain the observational details of EMIC waves. This may find further application in active experiment for generating EMIC waves in the magnetosphere. In the present paper the left-hand temporal instability in the presence of proton temperature anisotropy and perpendicular A.C. electric field is studied in the outer magnetosphere. The detail particle trajectories are estimated and the dispersion relation and growth rate are obtained using the method of characteristics for a generalized plasma distribution function [32]. This plasma distribution function could be reduced to a isotropic and an-isotropic Maxwellian and loss-cone type of distribution function.

2. DISPERSION RELATIONS AND GROWTH RATE

The details of method of characteristics and geometry described in the paper [32, 33] have been used. This method is used because it involves the details of unperturbed particle trajectories while obtaining the perturbed distribution for any arbitrary distribution function. In this method we easily get sum and difference of A.C. frequency term and thus Doppler shifted frequency is modified and taken care of nonlinear interaction part so far frequency terms are concerned. Therefore, following geometry and the technique of [32] the unperturbed trajectories of charged particles including electric field and equilibrium distribution f_0 used are written as:

$$
\begin{aligned}
x_0 &= x + \left(\frac{v_y}{\omega_{cs}}\right) + \left(\frac{1}{\omega_{cs}}\right)\left[v_x \sin\omega_{cs}t' - v_y \cos\omega_{cs}t'\right] \\
&\quad + \left(\frac{\Gamma_x}{\omega_{cs}}\right)\left[\frac{\omega_{cs}\sin\nu t' - \nu\sin\omega_{cs}t'}{\omega_{cs}^2 - \nu^2}\right] \\
y_0 &= y + \left(\frac{v_x}{\omega_{cs}}\right) - \left(\frac{1}{\omega_{cs}}\right)\left[v_x \cos\omega_{cs}t' - v_y \sin\omega_{cs}t'\right] \\
&\quad - \left(\frac{\Gamma_x}{\nu\omega_{cs}}\right)\left[1 + \frac{\nu^2\cos\omega_{cs}t' - \omega_{cs}^2\cos\nu t'}{\omega_{cs}^2 - \nu^2}\right] \\
z_0 &= z - v_z t'
\end{aligned} \tag{1}
$$

and the velocities as

$$
\begin{aligned}
v_{x0} &= v_x\cos\omega_{cs}t' - v_y\sin\omega_{cs}t' + \left\{\frac{\nu\Gamma_x(\cos\nu t' - \cos\omega_{cs}t')}{\omega_{cs}^2 - \nu^2}\right\} \\
v_{y0} &= v_x\sin\omega_{cs}t' + v_y\cos\omega_{cs}t' - \left\{\frac{\Gamma_x(\omega_{cs}\sin\nu t' - \nu\sin\omega_{cs}t')}{\omega_{cs}^2 - \nu^2}\right\} \\
v_{z0} &= v_z
\end{aligned} \tag{2}
$$

where $\omega_{cs} = \frac{e_s B_0}{m_s}$ is the cyclotron frequency of species s and $\Gamma_x = \frac{e_s E_0}{m_s}$ and A.C. electric field is varying as $E = E_{0x}\sin\nu t$, ν being the angular a-c frequency. The unperturbed generalized distribution function is written as:

$$
f_0(v) = \frac{n_0 v_\perp^{2j}}{\pi^{3/2}\alpha_{\perp s}^{2(j+1)}\alpha_{\parallel s} j!}\exp\left[-\left(\frac{v_{x0}^2 + v_{y0}^2}{\alpha_{\perp s}^2}\right) - \left(\frac{v_{z0}}{\alpha_{\parallel s}}\right)^2\right] \tag{3}
$$

where s for species, j = loss-cone index; $\alpha_{\perp s}$ and $\alpha_{\parallel s}$ are thermal velocities; for $j = 0$ and it reduces to an-isotropic bi-Maxwellian and further for $\alpha_{\perp s} = \alpha_{\parallel s.}$ it becomes a isotropic Maxwellian. The first order perturbed distribution function f_1 is written as [32]

$$
f_{s1}(r,v,t) = -\frac{e_s}{m_s\omega}\sum_{m,n,p,q}^{\infty}\frac{J_p(\lambda_2)J_m(\lambda_1)J_q(\lambda_3)e^{i(k\cdot r - \omega t)}}{\{\omega - k_\parallel v_\parallel - (n+q)\omega_{cs} + p\nu\}}
$$

$$
\left[E_{1x}J_nJ_p\left\{\left(\frac{n}{\lambda_1}\right)U^* + D_1\left(\frac{p}{\lambda_2}\right)\right\} - iE_{1y}\{J_n'J_pC_1 + J_nJ_p'D_2\} + E_{1z}J_nJ_pW^*\right] \tag{4}
$$

where the Bessel identity

$$
e^{i\lambda\sin\theta} = \sum_{k=-\infty}^{\infty} J_k(\lambda)e^{ik\theta}
$$

has been used, the arguments of the Bessel functions are

$$\lambda_1 = \frac{k_\perp v_\perp}{\omega_{cs}}, \quad \lambda_2 = \frac{k_\perp \Gamma_x \nu}{\omega_{cs}^2 - \nu^2}, \quad \lambda_3 = \frac{k_\perp \Gamma_x \omega_{cs}}{\omega_{cs}^2 - \nu^2}$$

where

$$\begin{aligned}
C_1 &= \frac{1}{v_\perp}\left(\frac{\partial f_0}{\partial v_\perp}\right)(\omega - k_\parallel \cdot v_\parallel) + \left(\frac{\partial f_0}{\partial v_\parallel}\right) k_\parallel \\
U^* &= C_1\left[v_\perp - \left\{\frac{\nu \Gamma_x}{\omega_{cs}^2 - \nu^2}\right\}\right] \\
W^* &= \left[\left(\frac{n\omega_{cs} v_\parallel}{v_\perp}\right)\left(\frac{\partial f_0}{\partial v_\perp}\right) - n\omega_{cs}\left(\frac{\partial f_0}{\partial v_\parallel}\right)\right] \\
&\quad + \left[1 + \left\{\frac{k_\perp \nu \Gamma_x}{\omega_{cs}^2 - \nu^2}\right\}\{(p/\lambda_2) - (n/\lambda_1)\}\right] \\
D_1 &= C_1\left\{\frac{\nu \Gamma_x}{\omega_{cs}^2 - \nu^2}\right\}, \quad D_2 = C_2\left\{\frac{\omega_{cs} \Gamma_x}{\omega_{cs}^2 - \nu^2}\right\} \\
J_n' &= \frac{dJ_n(\lambda_1)}{d\lambda_1}, \quad J_p' = \frac{dJ_p(\lambda_2)}{d\lambda_2}
\end{aligned} \tag{5}$$

After some algebraic manipulations and velocity integrations and for $p = 1$ and $q = 0$, $\sum J_p(\lambda_1) J_p(\lambda_3) = 1$ the resulting dispersion relation for electromagnetic wave is written for parallel propagation as [32].

$$\begin{aligned}
D(k, \omega) \;=\; 1 - \frac{k^2 c^2}{\omega^2} + \sum \frac{8\pi e_s^2 n_0}{m_s \omega^2} \frac{1}{\alpha_{\perp s}^{2(j+1)} j!} \\
\left[X_1 \frac{\omega}{k_\parallel \alpha_{\parallel s}} Z(\xi_s) + X_2 \left(1 + \xi_s Z(\xi_s)\right)\right]
\end{aligned} \tag{6}$$

where

$$\begin{aligned}
X_1 &= \frac{\alpha_{\perp s}^{2(j+1)} j!}{2} - \frac{\nu \Gamma_{xs}}{\omega_c^2 - \nu^2} \frac{\alpha_{\perp s}^{2(j+1)}}{4}\left(j - \frac{1}{2}\right)! \\
X_2 &= \frac{\alpha_{\perp s}^{2(j+1)} j!}{2}\left((j+1)\frac{\alpha_{\perp s}^2}{\alpha_{\parallel s}^2} - 1\right) - \frac{\nu \Gamma_{xs}}{\omega_c^2 - \nu^2} \frac{\alpha_{\perp s}^{2(j+1)}}{4}\left((2j+1)\frac{\alpha_{\perp s}^2}{\alpha_{\parallel s}^2} - 1\right)\left(j - \frac{1}{2}\right)! \\
\xi_s &= \frac{\omega - n\omega_c + \nu}{k_\parallel \alpha_\parallel}
\end{aligned} \tag{7}$$

The above dispersion relation is now approximated in Ion-cyclotron range of frequencies. In this case the electron temperatures are assumed as $T_{e\perp} = T_{\parallel e} = T_e$ and assumed to be magnetized with

$|\omega_r + i\gamma| \ll \omega_{ce}$ while ions are assumed to have $T_{\perp i} > T_{\parallel i}$ and $|k_\parallel \alpha_{\parallel i}| \ll |\omega_r \pm \omega_{ci} + i\gamma|$. The approximated dispersion relation is

$$D(k,\omega_r+i\gamma)=1-\frac{k^2c^2}{(\omega_r+i\gamma)^2}+\frac{1}{\alpha_{\perp e}^{2(j+1)}j!}\left[\frac{2\omega_{pe}^2}{\omega_{ce}^2}-\frac{2\omega_{pe}^2}{(\omega_r+i\gamma)(\pm\omega_{ce})}\right]X_{1e}$$
$$+\frac{1}{\alpha_{\perp i}^{2(j+1)}j!}\frac{2\omega_{pi}^2}{(\omega_r+i\gamma)^2}\left[X_{1i}\frac{\omega_r+i\gamma}{k_\parallel\alpha_{\parallel i}}Z(\xi_i)+X_{2i}\left(1+\xi_i Z(\xi_i)\right)\right] \tag{8}$$

where

$$X_{1i}=\frac{\alpha_{\perp i}^{2(j+1)}j!}{2}-\frac{\nu\Gamma_{xi}}{\omega_c^2-\nu^2}\frac{\alpha_{\perp i}^{2(j+1)}}{4}\left(j-\frac{1}{2}\right)!$$
$$X_{2i}=\frac{\alpha_{\perp i}^{2(j+1)}j!}{2}\left((j+1)\frac{\alpha_{\perp i}^2}{\alpha_{\parallel i}^2}-1\right)-\frac{\nu\Gamma_{xi}}{\omega_c^2-\nu^2}\frac{\alpha_{\perp i}^{2(j+1)}}{4}\left((2j+1)\frac{\alpha_{\perp i}^2}{\alpha_{\parallel i}^2}-1\right)\left(j-\frac{1}{2}\right)! \tag{9}$$
$$X_{1e}=\frac{\alpha_{\perp e}^{2(j+1)}j!}{2}-\frac{\nu\Gamma_{xe}}{\omega_c^2-\nu^2}\frac{\alpha_{\perp e}^{2(j+1)}}{4}\left(j-\frac{1}{2}\right)!$$
$$\omega_{ps}^2=\frac{4\pi e_s^2 n_0}{m_s}$$

Applying the condition $\frac{k^2c^2}{\omega^2} \gg 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2}$ and the charge neutrality condition $\frac{\omega_{pe}^2}{\pm\omega_{ce}} = -\frac{\omega_{pi}^2}{\pm\omega_{ci}}$ the dispersion relation reduces to

$$D(k,\omega_r+i\gamma) = -k_\parallel^2c^2+\frac{2}{\alpha_{\perp s}^{2(j+1)}j!}\left(\frac{\omega_{pe}^2}{\omega_{ce}^2}X_{1e}\right)$$
$$+\frac{2}{\alpha_{\perp s}^{2(j+1)}j!}\left[\omega_{pi}^2X_{1i}\left\{\frac{\omega}{k_\parallel\alpha_{\parallel i}}Z(\xi_i)+\frac{X_{2i}}{X_{1i}}\left(1+\xi_iZ(\xi_i)\right)\right\}\right]$$
$$=-\frac{k_\parallel^2c^2}{\omega_{pi}^2}+\frac{2}{\alpha_{\perp s}^{2(j+1)}j!}\left(\frac{\omega}{\pm\omega_{ci}}X_{1e}\right)$$
$$+\frac{2}{\alpha_{\perp s}^{2(j+1)}j!}X_{1i}\left[\left\{\frac{\omega}{k_\parallel\alpha_{\parallel i}}Z(\xi_i)+\frac{X_{2i}}{X_{1i}}\left(1+\xi_iZ(\xi_i)\right)\right\}\right] \tag{10}$$

Now using the asymptotic expansion of $Z(\xi_i)$ from [34] and for $n = 1$ the real and imaginary parts of dispersion relations are:

$$\mathrm{Im}D(k,\omega) = -\frac{2}{\alpha_{\perp s}^{2(j+1)}j!}X_{1i}\frac{1-X_3+X_4}{k}\left[\frac{X_{2i}}{X_{1i}}-\frac{X_3}{1-X_3+X_4}\right]$$
$$\sqrt{\pi}\,\exp\left[-\left(\frac{1-X_3+X_4}{k}\right)^2\right]$$

where

$$X_3 = \frac{\omega_r}{\pm\omega_{ci}} \qquad X_4 = \frac{-\nu}{\pm\omega_{ci}} \qquad \bar{k} = \frac{k_\parallel \alpha_{\parallel i}}{\pm\omega_{ci}} \tag{11}$$

$$\begin{aligned}\mathrm{Re}D(k,\omega) = &-\frac{1}{\alpha_{\perp s}^{2(j+1)} j!}\frac{X_{1i}\overline{k^2}}{(1-X_3+X_4)^2}\left[\frac{X_{2i}}{X_{1i}} - \frac{X_3}{1-X_3+X_4}\right] \\ &-\frac{2X_3(1-X_3+X_4)}{\overline{k^2}} + \frac{2X_3(1-X_3+X_4)^2 K_1 X_{1e}}{\overline{k^2} X_{1i}}\end{aligned} \tag{12}$$

By using the standard definition of growth rate

$$\gamma/\omega_{ci} = \frac{-\mathrm{Im}D(k,\omega)}{\omega_{ci}\dfrac{\partial \mathrm{Re}D(k,\omega)}{\partial\omega}}$$

$$\frac{\gamma}{\omega_{ci}} = \frac{\dfrac{\sqrt{\pi}}{\bar{k}}\left(\dfrac{X_{2i}}{X_{1i}} - K_4\right) K_3^3 \exp\left(-\left(\dfrac{K_3}{\bar{k}}\right)^2\right)}{1+X_4+\dfrac{(1+X_4)\overline{k^2}}{2K_3^2} - \dfrac{\overline{k^2}}{K_3}\left(\dfrac{X_{2i}}{X_{1i}} - K_4\right) - \dfrac{X_{1e}}{X_{1i}} K_1 K_3^2} \tag{13}$$

where $K_3 = 1 - X_3 + X_4$, $K_3 = \frac{X_3}{1-X_3+X_4}$.

From real part of the dispersion relation (12) set to zero and one gets

$$\begin{aligned} X_3 = \frac{\omega_r}{\omega_c} = \frac{\bar{k}^2}{\beta}&\left[\frac{K_2 X_{1i}(1+X_4)}{X_{1i} - X_{1e}K_1(1+X_4)}\right. \\ &\left.+\frac{X_{2i}}{X_{1i}}\frac{\beta\left(X_{1i}(1+X_4)\right)}{2(1+X_4)^2\left(X_{1i} - X_{1e}K_1(1+X_4)\right)}\right]\end{aligned} \tag{14}$$

$$A_T = \frac{\alpha_\perp^2}{\alpha_\parallel^2} - 1, \quad \beta = \frac{K_B T_\parallel \mu_0 n_0}{B_0^2}, \quad K_2 = \frac{\alpha_\perp^{2(j+1)} j!}{4X_{1i}}, \quad K_1 = \frac{\alpha_{\perp i}^{2(j+1)}}{\alpha_{\perp e}^{2(j+1)}}$$

3. RESULT AND DISCUSSION

Left hand polarized waves two branches, one below the helium gyrofrequency, and other one above cut off frequency which are the most commonly observed EMIC waves in the magnetosphere [5, 23]. The EMIC waves generated by high-energy protons interact with ambient low energetic ions and modified the propagation characteristics of EMIC wave. The cold heavier ions than H^+ of lower energy of (< 20 eV) modified significantly the dispersion relation of left hand frequency above He^+ gyrofrequency. The heavy ion gyrofrquency, some time changes the polarization characteristics also normally EMIC wave above He^+ gyrofrequency are confined to equatorial growth region. The detailed study and effect of different % of He^+ and O^+ have been responsible of thermal ion to EMIC waves in the presence of heavier ion has been reported by Anderson and Fuselier [35]. In this paper emphasis is mainly made on generation of left hand polarized EMIC waves at magnetospheric height, assuming left hand polarization having hot ions and cold electrons. Modification introduced by pitch angle anisotropy and thermal anisotropy on the generation of EMIC wave are discussed. This allows us to explain some low frequency part of spectrum seen by AMPTE at magnetospheric height and by magnetospheres on the ground during different days. The left hand made is guided along the background magnetic latitude. The wave generated above heavy ion gyropequency approach the heavy ion dispersion stop band, cross over to right hand polarization and are reflected at the bi-ion hybrid frequency [5, 35–37]. However, in this paper He^+ and other heavy ions has not been considered.

Numerical solutions of the dispersion equation for EMIC left handed polarized wave propagating along the magnetic field have been made for magnetospheric plasma at $L = 7$ in the equatorial region. The value of plasma parameters adopted paper are as follows.

$$B_0 = 2\times10^{-7}\,\mathrm{T},\ \ n_0 = 3\times10^{6}\,\mathrm{m}^{-3},\ 4\times10^{6}\,\mathrm{m}^{-3},\ 5\times10^{6}\,\mathrm{m}^{-3},$$
$$K_BT_e = 0.3\,\mathrm{eV},\ \ K_BT_{\parallel i} = 3\,\mathrm{keV},\ 4\,\mathrm{keV},\ 5\,\mathrm{keV},\ \ \nu = 0, 4\,\mathrm{Hz},\ 8\,\mathrm{Hz},$$
$$T_{\perp i}/T_{\parallel i} = 1.5,\ 2,\ 2.5,\ \ E_0 = 4\times10^{-3}\,\mathrm{v/m},\ j = 0, 1,\ \ n = 1,\ \ p = 1,\ \ q = 0$$

where all the protons are assumed to be hot and electrons to be cold. The calculations are performed for generalized distribution function reducible to an-isotropic Maxwellian plasma for $j = 0$ and loss cone distribution for $j = 1$ Figure 1 describes variation of the normalized growth rate and real frequency with normalized wave number. Fixed plasma parameters are defined in graph caption. The A.C. frequency affects the growth rate significantly. The maxima shifts towards lower values of $k_\parallel$ as the frequency changes from 4 Hz to 12 Hz for a anisotropic maxwellian plasma at $j = 0$, for $j = 1$ the nature of the variations are similar but the growth rate increases by an order of magnitude and covers wider spectrum of $k_\parallel$. The resonance frequency is influenced by the frequency of the A.C. signal as long as it is less than the proton gyro frequency comparing well with AMPTE/CCE data of 1994 [35]. However wave excitation takes place only when the proton perpendicular and parallel temperature ratio is more than or equal to 1.5.

The magnitude of the wave gain and frequency range for this instability is strongly influenced by the adopted parameters for the energetic proton component. A criterion for the upper frequency is obtained by using method similar to Cuperman and Landau [38, 39] and from the resonant term appearing in denominator and occurring in plasma dispersion function as:

$$\omega_{r,m} = \frac{X_{2i}/X_{1i}}{1 + X_{2i}/X_{1i}} \left(\omega_{ci} - q\omega_{ci} - p\,\nu\right) \tag{15}$$

If we remove the A.C. frequency and put $j = 0$ then it is similar to [31, 40]. The source mechanism for the ion-cyclotron instability is essentially dominated by ion temperature anisotropy. Contribution of A.C. frequency reduces the upper limit of wave number k and normalized real frequency $X_3 = \omega_r/\omega_{ci}$ and increases the growth rate. The A.C. frequency appears only through modification of resonant instability [32, 41].

Figure 2 shows the temporal growth rate with proton temperature ratio of 1.5 to 2.5 for an an-isotropic bi-Maxwellian and loss cone plasma. The results are in agreement with [1, 2, 4, 23, 31]. The maxima wave number do not shifts in this case significantly showing that the excited wave is in a narrow band of k. The real frequency does not change significantly. Cuperman [38, 39] had shown that the relative growth in each band would be influenced by thermal energy of energetic proton component. Thus the effects of thermal velocity for

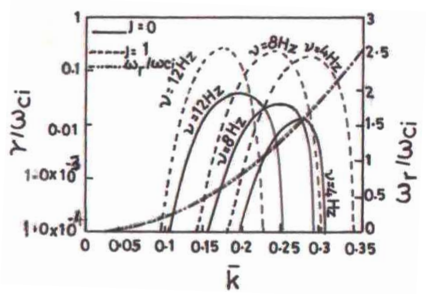

Figure 1. Variation of growth rate γ/ω_c versus $\bar{k}$ for different values of A.C. frequency and others parameters are $B_0 = 2 \times 10^{-7}\,\text{T}$, $n_0 = 5 \times 10^6\,\text{m}^{-3}$, $T_{\perp i}/T_{\parallel i} = 1.5$, $K_B T_{\parallel i} = 5\,\text{keV}$, $K_B T_e = 0.3\,\text{eV}$, $E_0 = 4 \times 10^{-3}\,\text{V/m}$.

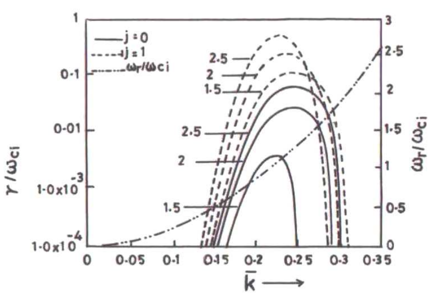

Figure 2. Variation of growth rate γ/ω_c versus $\bar{k}$ for different values of $T_{\perp i}/T_{\parallel i}$ ratio and others parameters are $B_0 = 2 \times 10^{-7}\,\text{T}$, $n_0 = 5 \times 10^6\,\text{m}^{-3}$, $\nu = 8\,\text{Hz}$, $K_B T_{\parallel i} = 5\,\text{keV}$, $K_B T_e = 0.3\,\text{eV}$, $E_0 = 4 \times 10^{-3}\,\text{V/m}$.

an-isotropic Maxwellian and loss-cone distribution have been shown. The loss cone index $j = 1$ is influencing thermal velocity. In order to see correspondence with angular form ω_c Compare our distribution function with that given by Huang et al. [42] for loss-cone distribution and immediately one gets a relation corresponding to loss-cone index $j = 1$ as:

$$\frac{1}{M} = \frac{v_\perp^2}{\alpha_\perp^2}$$

where M in terms of loss-cone angle is written as:

$$M = \frac{1}{\left(1 + \dfrac{\tan^2\theta_c}{A_T + 1}\right)} \tag{16}$$

Thus the angular form in terms of loss-cone angle could be included even through the use of generalized distribution function as taken here. However, the resulting changes and plots for various pitch angle distribution of contour plots shall be dealt in future publication. The loss-cone index j is a measure of the depth of loss-cone distribution of ring current protons. Figure 3 shows the variation of growth rate for different values of the thermal energy of ions and for other fixed parameters written in caption. The dispersive properties of EMIC waves are known to depend sensitively on the density and composition of thermal plasma as well as on properties of resonant energetic ion [2, 4, 17, 18, 23, 31]. Figure 4 shows variation of growth

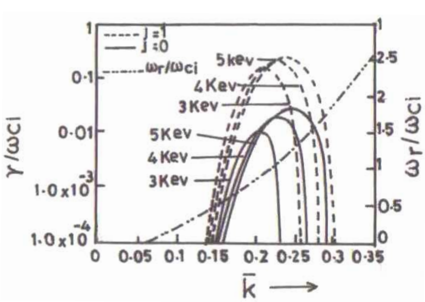

Figure 3. Variation of growth rate γ/ω_c versus $\bar{k}$ for different values of thermal velocity of ions $K_B T_{\parallel i}$ and others parameters $B_0 = 2\times10^{-7}\,\mathrm{T}$, $n_0 = 5\times10^6\,\mathrm{m}^{-3}$, $\nu = 8\,\mathrm{Hz}$, $T_{\perp i}/T_{\parallel i} = 1.5$, $E_0 = 4\times10^{-3}\,\mathrm{V/m}$, $K_B T_e = 0.3\,\mathrm{eV}$.

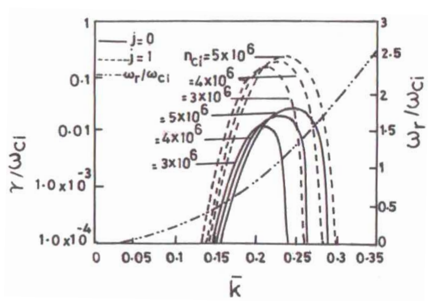

Figure 4. Variation of growth rate γ/ω_c versus $\bar{k}$ for different values of ion density others parameters are $B_0 = 2\times10^{-7}\,\mathrm{T}$, $K_B T_{\parallel i} = 5\,\mathrm{keV}$, $T_{\perp i}/T_{\parallel i} = 1.5$, $\nu = 8\,\mathrm{Hz}$, $K_B T_e = 0.3\,\mathrm{eV}$, $E_0 = 4\times10^{-3}\,\mathrm{V/m}$.

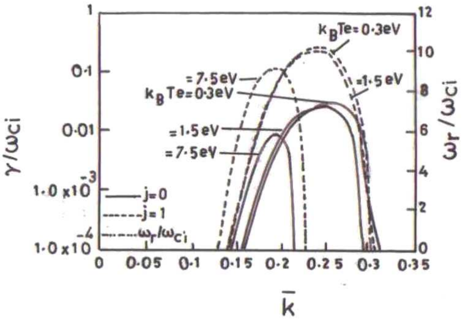

Figure 5. Variation of growth rate γ/ω_c versus $\bar{k}$ for different values of thermal velocity of electrons K_BT_e and others parameters $B_0 = 2 \times 10^{-7}\,\mathrm{T}$, $n_0 = 5 \times 10^6\,\mathrm{m}^{-3}$, $\nu = 8\,\mathrm{Hz}$, $E_0 = 4 \times 10^{-3}\,\mathrm{V/m}$, $K_BT_{\parallel i} = 5\,\mathrm{keV}$, $T_{\perp i}/T_{\parallel i} = 1.5$.

rate for various values of proton density when the thermal velocity is 5 keV and the proton temperature ratio is 0.3. The growth rate increases as the density increases. The maxima shifts to higher values of k, this feature is mostly indicative of the morning sector of the magnetosphere when the density is low. Our results are in agreement with [4] and [3] obtained for convective electromagnetic ion-cyclotron instability as well as to other workers for parameters used for growth rate and ω_r/ω_{ci} [2, 7, 23]. The convective growth rate could be obtained by dividing our expression with the group velocity and solving it for variation with frequency. The instability is quenched for lower values of $k < 0.1.5$ in agreement with [17, 18]. This is due to decrease in velocity comparable to the thermal speed. Figure 5 shows variation of the growth rate with k for various values of cold isotropic Maxwellian electrons for $T_{\perp i}/T_{\parallel i} = 2$ and $K_BT_{\parallel i} = 5\,\mathrm{keV}$ and for other parameters shown in caption. The cold electron temperature is assumed to be 0.3 eV. As the electron temperature increases, not only growth rate is reduced but the maxima shifts drastically towards lower values of k, showing a possibility for wave emission at lower frequencies due to increase in real frequency term and enhanced Landau damping. Thus the choice of cold background electron plasma temperature is also a controlling factor for wave emission.

4. CONCLUSION

The controlling factor for generation of EMIC instability is density, $T_{\parallel i}$ and $T_{\perp i}$ and its ratio. The A.C. frequency reduces the value of $X_3 = \omega_r/\omega_{ci}$ and increases the growth rate. Thus along with parametric effect of the instability the inclusion of A.C. frequency would be of help in explaining the observation of AMPTE/CCE in morning as well as in evening sectors by suitable choice of plasma parameters, so far as observed normalized frequency ω_r/ω_{ci} and growth rate is concerned. As only one set of plasma parameters are taken to indicate generation of EMIC waves, thus detailed comparison with morning or evening sector's data cannot be done here.

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