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FULL-WAVE ANALYSIS OF DIELECTRIC RECTANGULAR WAVEGUIDES

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Abstract—In this paper, the characteristic equations of the E^y_{mn} and E^x_{mn} modes of the dielectric rectangular waveguide have been derived using the mode matching technique. No assumptions have been taken in the derivations which have been straight forwardly done. Two ratios have been introduced in the characteristic equations and the new set of characteristic equations thus obtained are then plotted and graphical solutions are obtained for the propagation parameters assuming certain numerical values for the introduced ratios. The results have then been compared to those obtained by Marcatilli and Goell for rectangular dielectric waveguides. The comparisons depicts a good agreement in the three methods at frequencies well above cut-off.

1. INTRODUCTION

A waveguide is a hollow structure in which waves can propagate without any distortion or attenuation along the invariance direction of the guide. Such waves are generally dispersive and their dispersion relation can be obtained by solving self-adjoint eigenvalue problems. The guide is said to be closed if it is transversally bounded, if the cross section is unbounded, one has an open waveguide, for the open waveguide the presence of a continuum of radiating modes which are not guided waves makes all the questions more difficult.

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Since dielectric waveguides of rectangular cross section have no closed-form solution, an exact analytic solution does not exist for the case of wave propagation along a dielectric waveguide of rectangular shape. Eigen-modes of the waveguide have to be found either numerically or using approximate techniques. Theoretical studies on geometrically simple optical and microwave dielectric waveguides have been presented in the past using approximate or numerical methods. The need for approximate techniques to solve the problem associated with rectangular or more general shaped dielectric structures is apparent. Two most common approximate techniques are the Marcatili approach [1] and the circular harmonic point matching technique [2]. Other notable approximate techniques by Schlosser and Unger [3], using rectangular harmonics, by Eyges et al., using the extended boundary condition method [4], and by Shaw et al. [5], using a variational approach have not been much used due to their computational complexity. The approximate methods are represented by an analytical approximation introduced by Marcatili and by the effective index method. The numerical techniques such as the variational methods [6–8], finite element methods [9–13] and integral equation methods [14–16] have been extensively used. These methods have been exclusively applied to two-dimensional problems with most of the existing techniques performing a fine discretization of the cross section. Such discretization introduces many unknowns and strong numerical instabilities. Consequently, an extension of these methods to three-dimensional problems faces many practical limitations and requires special care. The main disadvantages of them are the time and computing resources limitations and impossibility for an analytical analysis of the solution. Of all the methods for analyzing dielectric waveguides which have been well addressed in the literature, the most commonly used is the mode-matching technique [17–20]. With this technique, the transverse plane of the waveguide is divided into different regions such that in each region canonical eigenfunctions can be used to represent the electromagnetic fields. The eigenvalue problem is constructed by enforcing the boundary conditions at the interface of each region.

In this paper, mode matching has been done at all the air dielectric interfaces and thus the characteristic equations have been derived. With the introduction of two ratios the characteristic equations have then been solved graphically to obtain the values of the propagation constant. The results have been compared to those obtained by Marcatili and Goell and a good agreement at high frequencies has been obtained.

2. BOUND MODE ANALYSIS

The rectangular dielectric waveguide provides confinement of fields in two dimensions as compared with the slab guide which can confine the fields only in one dimension. The two-dimensional confinement is necessary not only to guide electromagnetic energy from one point to the other but to a great extent while interconnecting circuit elements. In a rectangular dielectric waveguide, there can exist two independent families of modes One is designated as E^x_{mn} modes having most of its electric field polarized in the x-direction, and the other is designated as E^y_{mn} modes, having most of its electric field in the y-direction. The subscripts n and m represent, the number of extrema along the x and y directions respectively of the field components for this mode. E_y, E_x, E_z, H_x, and H_z, with $H_y = 0$ are the components of the E^y_{mn} modes and E_x, E_z, H_x, H_y, and H_z, with $E_y = 0$ are the components of the E^x_{mn} modes [21].

At very high frequencies the loss factor tends to that for plane waves in the dielectric because of the strong field concentration in the rod. At low frequencies losses are small because the fields are only weakly concentrated in the rod. For this reason the usage of rod waveguides at very low frequencies is not practical.

E^y_{mn} and E^x_{mn} as well as hybrid modes can be supported by the guide. The wave guidance takes place by the total internal reflection at the side walls. The solutions to the rectangular dielectric guide problem can be derived by assuming guided mode propagation (well above cut off) along the dielectric, and exponential decay of fields transverse to the dielectric surface. Thus in the region of confinement, (inside the guide) due to reflections there will be standing wave patterns and when the field goes out of the boundary of the guide, then in the absence of reflection, the field moves away from the guide exponentially i.e. there is an exponential decay of fields transverse to the dielectric surface. The fields are assumed to be approximately (co)sinusoidally distributed inside the waveguide and decaying exponentially outside. That is, we express the field components, as follows:

Assuming propagation along z-axis and in the view of above discussion the wave function can be written as either

$$\begin{aligned}\psi_{even} &= A\cos ux\cos u_1 y e^{-jk_z z} \quad |x|\le a \;\; |y|\le b \text{ (inside the guide)}\\ &= Be^{-vx}\cos u_1 y e^{-jk_z z} \quad |x|\ge a \;\; |y|\le b \text{ (outside the guide)}\\ &= C\cos ux e^{-v_1 y} e^{-jk_z z} \quad |x|\le a \;\; |y|\ge b \text{ (outside the guide)}\end{aligned} \tag{1}$$

The parameters u and u_1 are the transverse propagation constants inside the dielectric guide and v and v_1 are the attenuation constants outside the guide.

or,

$$\begin{aligned}\psi_{odd} &= A\sin ux \sin u_1 y e^{-jk_z z} \;\; |x| \leq a \;\; |y| \leq b \\ &= Be^{-vx} \sin u_1 y e^{-jk_z z} \;\; |x| \geq a \;\; |y| \leq b \\ &= C\sin ux e^{-v_1 y} e^{-jk_z z} \;\; |x| \leq a \;\; |y| \geq b \end{aligned} \tag{2}$$

The regions $x > a$ and $y > b$ have been neglected as fields are very very weak at the corners, this has been done in all the approximate methods [1, 2, 21].

Applying wave equation to Equation (1) the separation parameter equations in each region become

$$u^2 + u_1^2 + k_z^2 = k_d^2 = \omega^2 \in_d \mu_d \text{ (inside the guide)} \tag{3}$$

$$-v^2 - v_1^2 + k_z^2 = 2k_0^2 - k_d^2 = 2\omega^2 \in_0 \mu_0 - \omega^2 \in_d \mu_d \text{ (outside the guide)} \tag{4}$$

Taking the case of ψ_{even}, i.e., ψ an even function of x & y explicitly and then similarly for ψ_{odd} we proceed in analyzing the rectangular dielectric waveguide. The approach is the mode matching at dielectric interfaces in the transverse directions using appropriate boundary Conditions.

2.1. Characteristic Equations for E^y_{mn} Modes

The field components for the TM to y modes are [22]

$$E_x = \frac{1}{\hat{y}} \frac{\delta^2 \psi}{\delta x \delta y} \quad E_y = \frac{1}{\hat{y}} \left(\frac{\delta^2}{\delta^2 y} + k^2 \right) \psi \quad E_z = \frac{1}{\hat{y}} \left(\frac{\delta^2 \psi}{\delta y \delta z} \right) \tag{5}$$

$$H_x = -\frac{\delta \psi}{\delta z} \quad H_y = 0 \quad H_z = \frac{\delta \psi}{\delta x} \tag{6}$$

where $\hat{y} = j\omega\varepsilon d$ corresponding to ψ as an even function of both x and y, the field components inside the guide become

$$\begin{aligned} E_x &= \frac{1}{\hat{y}}(+Auu_1 \sin ux \sin u_1 y e^{-jk_z z}) \\ E_y &= \frac{1}{\hat{y}}(A\cos ux \cos u_1 y e^{-jk_z z})(k_d^2 - u_1^2) \\ E_z &= +\frac{1}{\hat{y}}(Au_1 jkz \cos ux \sin u_1 y e^{-jk_z z}) \\ H_x &= (+jk_z A\cos ux \cos u_1 y e^{-jk_z z}) \\ H_y &= 0 \quad H_z = -Au\sin ux \;\; \cos u_1 y e^{-jk_z z} \end{aligned} \tag{7}$$

Similarly, outside the guide, as the field is symmetric with respect to $x = 0$ and $y = 0$ the components are

For $x \geq a \quad y \leq b$:

$$\begin{aligned} E_x &= \frac{1}{\hat{y}0}(+Bvv_1 e^{-vx} \sin u_1^y e^{-jk_z z}) \\ E_y &= \frac{-1}{\hat{y}0}(k_0^2 - u_1^2)B(e^{-vx} \cos u_1^y e^{-jk_z z}) \\ E_z &= \frac{1}{\hat{y}0}(+Bjkzu_1 e^{-vx} \cos u_1 y e^{-jk_z z}) \\ H_x &= (Bjk_z e^{-vx} \cos u_1 y e^{-jk_z z}) \\ H_y &= 0 \quad H_z = -Bve^{-vx} \cos u_1 y e^{-jk_z z} \end{aligned} \tag{8}$$

For $x \leq a \quad y \geq b$:

$$\begin{aligned} E_x &= \frac{1}{\hat{y}0}(+Cuv_1 \sin uxe^{-v_1 y} e^{-jk_z z}) \\ E_y &= \frac{1}{\hat{y}0}C(k_0^2 - v_1^2)(\cos uxe^{-v_1 y} e^{-jk_z z}) \\ E_z &= \frac{1}{\hat{y}0}(Cjkzv_1 \cos uxe^{-v_1 y} e^{-jk_z z}) \\ H_x &= (Cjk_z \cos uxe^{-v_1 y} e^{-jk_z z}) \\ H_y &= 0 \quad H_z = -Cu_1 \sin uxe^{-v_1 y} e^{-jk_z z} \end{aligned} \tag{9}$$

Referring to Figure 1, at the interface $x = a$ in the y-z plane, the tangential components of E & H should be continuous (E_z, E_y, H_z, H_y). As $H_y = 0$, let us match the strongest field components, thus taking the continuity of E_y & H_z we have from Equations (7) and (8)

$$\frac{-1}{\hat{y}_0}(k_0^2 - u_1^2)B(e^{-va} \cos u_1^y e^{-jk_z z}) = \frac{1}{\hat{y}}(A \cos ua \cos u_1 y e^{-jk_z z})(k_d^2 - u_1^2) \tag{10}$$

$$-Bve^{-va} \cos u_1 y e^{-jk_z z} = -Au \sin ua \ \cos u_1 y e^{-jk_z z} \tag{11}$$

The above set of equations reduces to

$$\frac{-1}{\in_0}(k_0^2 - u_1^2)B(e^{-va} \cos u_1^y e^{-jk_z z}) = \frac{1}{\in_d}(A \cos ua \cos u_1 y e^{-jk_z z})(k_d^2 - u_1^2) \tag{12}$$

$$-Bve^{-va} \cos u_1 y e^{-jk_z z} = -Au \sin ua \cos u_1 y e^{-jk_z z} \tag{13}$$

Dividing (13) by (12) gives

$$u \tan ua = \frac{\in_0 \ (k_d^2 - u_1^2)}{\in_d \ (k_0^2 - u_1^2)} v \quad \text{or} \quad ua \tan ua = \frac{\in 0(k_d^2 - u_1^2)}{\in d(k_0^2 - u_1^2)} va \tag{14}$$

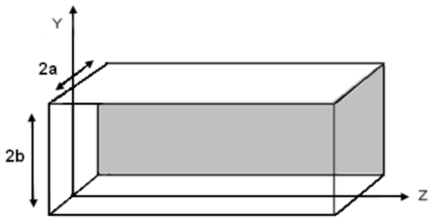

Figure 1. The $x = a$ interface.

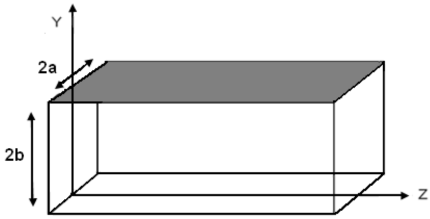

Figure 2. The $y = b$ interface.

Similarly, applying the continuity condition of E & $H(E_z, H_x)$ at $y = b$ interface (Figure 2) by using (7) and (9), we get

$$+\frac{1}{\hat{y}}(Au_1 jkz\cos ux\sin u_1 be^{-jk_z z}) = \frac{1}{\hat{y}_0}(Cjkzv_1\cos uxe^{-v_1 b}e^{-jk_z z}) \quad (15)$$

$$\& \quad (+jk_z A\cos ux\cos u_1 be^{-jk_z z}) = (Cjk_z\cos uxe^{-v_1 b}e^{-jk_z z}) \quad (16)$$

Dividing (15) by (16), we get

$$u_1\tan u_1 b = \frac{\in d}{\in_0}v1 \quad \text{or} \quad u_1 b\tan u_1 b = \frac{\in d}{\in_0}v1b \quad (17)$$

Equations (14) and (17) coupled with Equations (3) & (4) give the characteristic equations for determining k_z of the even E^y_{mn} modes.

Similarly, for the wave function ψ as an odd function of x & y both, Evaluating the field components tangential to the air-dielectric interface at $x = a$ gives

For $x \geq a \quad y \leq b$:

$$E_y = \frac{-1}{\hat{y}_0}(k_0^2 - u_1^2)B(e^{-va}\sin u_1^y e^{-jk_z z}) \quad H_z = -Bve^{-va}\sin u_1 ye^{-jk_z z} \quad (18)$$

Imposing the continuity of these fields at the interface, we get

$$\frac{1}{\hat{y}}(A\sin ua\sin u_1 ye^{-jk_z z})(k_d^2 - u_1^2) = \frac{-1}{\hat{y}_0}(k_0^2 - u_1^2)B(e^{-va}\sin u_1 ye^{-jk_z z}) \quad (19)$$

$$\& \quad Au\cos ua\sin u_1 ye^{-jk_z z} = -Bve^{-va}\sin u_1 ye^{-jk_z z} \quad (20)$$

Dividing (20) by (19) we get

$$u\cot ua = \frac{-v(k_d^2 - u_1^2)\in_0}{(k_0^2 - u_1^2)\in_d} \quad \text{or} \quad ua\cot ua = \frac{-v(k_d^2 - u_1^2)\in_0 a}{(k_0^2 - u_1^2)\in_d} \quad (21)$$

Similarly for the $y = b$ interface

$$u_1 b\cot u_1 b = -v1(\in_d / \in_0) \quad (22)$$

Equations (21) and (22) are the characteristic equations for the odd E^y_{mn} modes.

2.2. Characteristic Equations for E^x_{mn} Modes

When the E^y_{mn} mode is changed to E^x_{mn}, the field components are given by

$$
\begin{aligned}
E_x &= \frac{-\delta\psi}{\delta z} \quad H_x = \frac{1}{\hat{z}}\left(\frac{\delta^2\psi}{\delta y\delta x}\right) \\
E_y &= 0 \quad H_y = \frac{1}{\hat{z}}\left(\frac{\delta^2}{\delta y^2} + k^2\right)\psi \\
E_z &= \frac{\delta\psi}{\delta x} \quad H_z = \frac{1}{\hat{z}}\left(\frac{\delta^2\psi}{\delta y\delta z}\right)
\end{aligned}
\tag{23}
$$

Proceeding in the manner similar to done for the E^x_{mn} mode we get the characteristic equations as
for ψ_{even}

$$ua\tan ua = va \quad \& \quad u_1 b\tan u_1 b = \frac{(k_d^2 - u_1^2)}{(k_0^2 + v_1^2)}v_1 b \tag{24}$$

for ψ_{odd}

$$-ua\cot ua = va \quad \& \quad -u_1 b\cot u_1 b = \frac{(k_d^2 - u_1^2)}{(k_0^2 + v_1^2)}v_1 b \tag{25}$$

The u and u_1 and v and v_1 still satisfy Equation (3) and (4). The even wave functions generating these E^x_{mn} modes are those of Equation (1) and the odd wave functions generating the E^x_{mn} modes are those of Equation (2).

2.3. Determination of K_z at any Frequency above Cut-off

Let us take the case for the E^y_{mn} mode propagation considering even wave functions.The four equations that are governing the propagation are:

$$u^2 + u_1^2 + k_z^2 = k_d^2 = \omega^2\mu_d \in_d \tag{26}$$

$$-v^2 - v_1^2 + k_z^2 = 2k_0^2 - k_d^2 = 2\omega^2 \in_0 \mu_0 - \omega^2 \in_d \mu_d \tag{27}$$

$$ua\tan ua = \frac{\in_0 (k_d^2 - u_1^2)}{\in_d (k_0^2 - u_1^2)}va \tag{28}$$

$$u_1 b\tan u_1 b = \frac{\in_d}{\in_0}v_1 b \tag{29}$$

Equations (28) and (29) can be rewritten as

$$\tan\left(ua - \frac{m\pi}{2}\right) = \frac{\in_d (k_0^2 - u_1^2)u}{\in_0 (k_d^2 - u_1^2)v} \quad \text{and} \quad \tan\left(u_1 b - \frac{n\pi}{2}\right) = \frac{\in_0 u_1}{\in_d v_1} \tag{30}$$

where m and n are arbitrary integers characterizing the order of the propagating mode.

To obtain the propagation constant we have a set of four transcendental Equations (26)–(29) and five unknowns (u, u_1, v, v_1, k_z). In order to obtain solutions we introduce two ratios c_1 and c_2 such that

$$c1 = u/u1 \quad \text{and} \quad c2 = v/v_1 \tag{31}$$

Using (31), Equations (26)–(29) reduce to

$$u_1^2 + (c_1 u_1)^2 + k_z^2 \;=\; k_d^2 \;=\; \omega^2 \mu_d \in_d \tag{32}$$

$$-v_1^2 - (c_2 v_1)^2 + k_z^2 \;=\; 2k_0^2 - k_d^2 = \; 2\omega^2 \in_0 \mu_0 - \omega^2 \in_d \mu_d \tag{33}$$

$$c_1 u_1 a \tan c_1 u_1 a \;=\; \frac{\in_0\, (k_d^2 - u_1^2)}{\in_d\, (k_0^2 - u_1^2)} c_2 v_2 a \tag{34}$$

$$(u_1) b \tan (u_1) b \;=\; \frac{\in_0}{\in_d} v_1 b \tag{35}$$

From the above equations the values of v_1 can be obtained as

$$v_1 \;=\; \sqrt{\frac{k_d^2 - k_0^2 - u_1^2 \left(1 + c_1^2\right)}{(1 + c_2^2)}} \tag{36}$$

$$v_1 \;=\; \frac{\in_d\, (k_0^2 - u_1^2) u_1 c_1}{\in_0\, (k_d^2 - u_1^2) c_2} \tan\left(u_1 c_1 a\right) \tag{37}$$

$$v_1 \;=\; \frac{\in_0}{\in_d} u_1 \tan\left(u_1 c_1 a\right) \tag{38}$$

For given values of c_1 and c_2, the above equations can be solved graphically. To obtain the values of u_1 and v_1 Equation (37) represents a modified tan function and Equation (36) represents an ellipse. A crossing of the two curves in the upper half of the graph is a solution, i.e., a surface wave. The dominant mode corresponds to the point where the ellipse crosses the first modified tan function and the solutions for the higher order modes are at the crossing of ellipse with the next modified tan functions as depicted in the Figure 3.

Similarly when Equations (36) and (38) are plotted in the u-v plane, the crossing of the two curves provides solution for u (u_1/c_1) and v (v_1/c_2). knowing the values of the four transverse propagation constants, the propagation constant along z direction, i.e., K_z can be evaluated as

$$k_z = \sqrt{2k_d^2 - k_0^2 + v_1^2 + v^2} = \sqrt{k_d^2 - u_1^2 - u^2} \tag{39}$$

The case where $c_1 = c_2 = 0$ the Equations (32) to (35) reduce to that for the dielectric slab guide where direct solution for k_z can be obtained graphically. The graphical solution of the E^x_{mn} modes can be obtained in a similar manner.

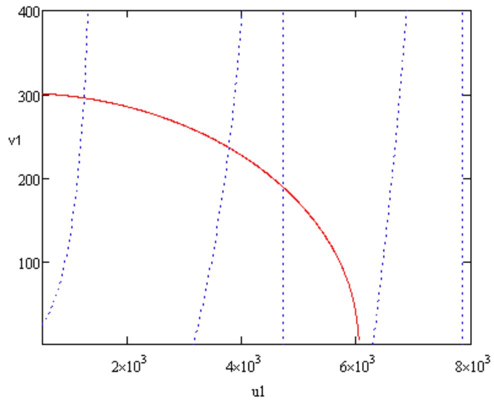

Figure 3. Plot of v_1 versus u_1.

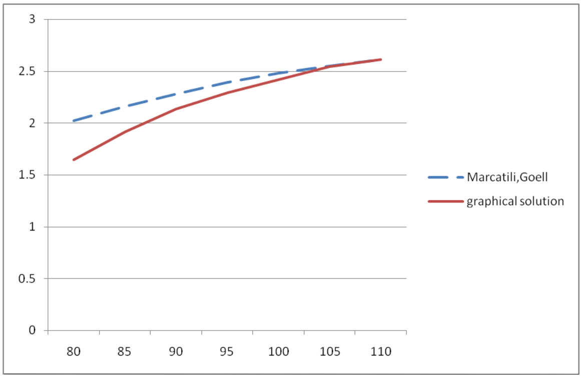

Figure 4. Normalized propagation constant (k_z/k_0) versus frequency (GHz).

3. NUMERICAL RESULTS AND DISCUSSION

The comparison of results of the calculations of the propagation factor k_z/k_0 using the graphical method introduced in this paper to Marcatili's and Goell's methods for a silicon dielectric waveguide with $0.5 \times 1.0\,\mathrm{mm}^2$ cross section, E_{11}^y mode have been depicted in Figure 3. The values of c_1 and c_2 have been optimized empirically at $c_1 = (f/63)$ and $c_2 = c_1 * 40$ where f is the operating frequency in Giga hertz. It can be seen from Figure 4 that in spite of its simplicity the graphical method works quite well for high frequencies that is when the wave is well guided the results agree very well with the Marcatili's and Goell's method. At lower frequencies near cut-off, accurate calculations are more complicated.

4. CONCLUSION

This paper introduces a new method of solving the propagation constant for the bound modes in the Dielectric Rectangular Waveguides. This method provides a graphical solution of the characteristic equations obtained for a specific mode family (E^x_{mn} or E^y_{mn}) which have been obtained by complete mode matching at the interfaces of the guide without using any approximations. As the characteristic equations are general the graphical solutions can be obtained for a Dielectric Rectangular Waveguide of any dimension, for single mode or multimode operations with a careful choice of the values of of c_1 and c_2.

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