# CALCULATION OF ANTENNA MODE SCATTERING BASED ON METHOD OF MOMENTS

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Abstract—The formation process of antenna mode scattering is analyzed, and a new prediction method to calculate the antenna mode scattering is proposed. Since the antenna mode scattering is essentially the reradiation of reflected energy, this reflected energy comes from the incident wave received by the antenna and reflects on the mismatched point of the feed network. Thus the calculation of the antenna mode scattering can be divided into three steps: 1. calculation of the antenna received energy; 2. calculation of the reflected energy from the feed network; 3. reradiation of this reflected energy. The numerical results of antenna mode scattering from a patch antenna and a dipole array are proposed to verify this approach.

## 1. INTRODUCTION

With the development of stealth technology and detection technology, the scattering characteristics of a target have received more and more attention. As a complex scatterer, the antenna greatly affects the scattering properties of its carrier. Different from general scatterers, the antenna scattering is mainly made up of two contributors: structural mode scattering and antenna mode scattering [1]. The structural mode scattering is similar with the scattering of general objects, which is related to the antenna structure, shape and material And the antenna mode scattering is essentially the reradiation of antenna which is associated with the antenna radiation performance and feed network. The calculation for the two kinds of scattered fields is not only beneficial to the analysis of antenna scattering, but also help for providing a reference for radar cross section (RCS) reduction

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Extensive work has been performed for predicting the antenna scattering characteristics. The scattering of a microstrip patch has been discussed in [2–4]. The RCS of a dipole antenna and a monopole array have been studied in [5–7]. Detailed analysis of the scattered field about a horn antenna and reflector antenna has also been performed [8,9]. For the more complex system, approximate equations have been presented for the RCS of a phased array antenna with the series and parallel feed beam forming networks [10, 11]. However, all of these analyses and calculations are mainly related to the total scattered field, the structural mode scattering and antenna mode scattering have been less considered. Due to the formation of structural mode scattering from an antenna is similar to the scattering form a general target (like a sphere or a plane), thus, it can be calculated by some mature technologies, such as physical optics (PO), uniform theory of diffraction (UTD), finite element method (FEM), and so on. However, there are few available literatures about the calculation of the antenna mode scattering. The common calculation method for the antenna mode scattering is open/short circuit method [12]. Whereas, in practice, this approach is complex and the calculation precision of the antenna mode scattering largely depends on the insertion loss of the feed line.

Considering that the essence of antenna mode scattering is the reradiation of antenna reflected energy, therefore, this paper presents a precise method for the calculation of antenna mode scattering. The calculation process is in accordance with the process of energy propagation, which is the reception of incident energy, the reflection of received energy, the reradiation of reflected energy. The detailed analysis and discussion are described in Section 2. The comparison results of the antenna mode RCS from a patch antenna and dipole array in monostatic or bistatic case are presented in Section 3.

### 2. THEORY

### 2.1. Antenna Mode Scattered Field

For most antennas, the structural mode and antenna mode are difficult to identify, in particular when the antenna is installed on a platform. The relationship between the two modes is given by

$$\sigma^{total} = \left| \sqrt{\sigma^{st}} + \sqrt{\sigma^{an}} e^{j\varphi} \right|^2 \tag{1}$$

where,  $\sigma$  is the antenna RCS, superscript 'st' and 'an' denote the structural mode and antenna mode respectively.  $\varphi$  is the phase difference between the two modes, the determination of which is

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difficult. However, the open/short circuit method proposed by Liu [12] can be adopted to identify the two modes. When an antenna is terminated with short circuit and open circuit loads, the scattered fields obtained can be denote by  $\mathbf{E}^{s}(0)$  and  $\mathbf{E}^{s}(\infty)$ , respectively. Thus the total scattered field  $\mathbf{E}^{s}$  with arbitrary load  $Z_{L}$  can be given by

$$\mathbf{E}^{s}(Z_{L}) = \frac{(1+\Gamma_{A})\mathbf{E}^{s}(0) + (1-\Gamma_{A})\mathbf{E}^{s}(\infty)}{2} + \frac{\Gamma_{L}}{1-\Gamma_{L}\Gamma_{A}}\frac{1-\Gamma_{A}^{2}}{2}[\mathbf{E}^{s}(\infty) - \mathbf{E}^{s}(0)]$$
(2)

where,  $\Gamma_A$  is the antenna reflection coefficient,  $\Gamma_L$  is the load reflection coefficient. In (2), the first term is the structural mode scattered field, and the second term is the antenna mode scattered field, which can be denoted as

$$\begin{cases} \mathbf{E}^{st} = \frac{(1+\Gamma_A)\mathbf{E}^s(0) + (1-\Gamma_A)\mathbf{E}^s(\infty)}{2} \\ \mathbf{E}^{an} = \frac{\Gamma_L}{1-\Gamma_L\Gamma_A} \frac{1-\Gamma_A^2}{2} [\mathbf{E}^s(\infty) - \mathbf{E}^s(0)] \end{cases}$$
(3)

According to the scattering matrix method [1], the antenna mode scattered field also can be written as [12]

$$\mathbf{E}^{an} = a_1 \times \frac{\Gamma_L}{1 - \Gamma_L \Gamma_A} \times \mathbf{E}_0 \tag{4}$$

where,  $a_1$  is the antenna received amplitude when an antenna is terminated with matched load,  $E_0$  is the radiated field when antenna is excited by a unit current excitation. Eq. (4) shows the essence of the antenna mode scattering: Under the illumination of an incident wave, firstly, as a receiving device, the antenna receives the incident energy (correspond to the first term in (4); then, some energy is reflected on the mismatch point of the feed network (correspond to the second term in (4); finally, the antenna as a radiation device is excited by this reflected energy and transmits this energy to the free space (correspond to the third term in (4). Thus the generation of the antenna mode scattering is a process of reception-reflection-reradiation.

### 2.2. MoM Calculation for Antenna Mode Scattering

According to the above analyses, we can obtain the antenna mode scattering by computing the received signal, reflected signal and radiation pattern. The method of moments (MoM) is adopted to calculate these antenna parameters in this paper. The electric field integral equation (EFIE), RWG basis function [13] and Galerkin's method are used in proposed MoM.

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#### 2.2.1. Calculation of the Received Energy

In the MoM process, the most important step is solving matrix equations (5).

$$\begin{cases} [Z^S][I^S] = [V^S], & \text{for scattering} \\ [Z^R][I^R] = [V^R], & \text{for radiation} \end{cases}$$
(5)

where, [Z] is the impedance matrix, which is determined by the antenna structure and electrical size. [I] is the unknown current vector, and [V] is the voltage vector. Superscript 'R' and 'S' denote the radiation and scattering calculation respectively. Let the total number of the MoM common edges be M, when an incident wave illuminates an antenna, the *m*th element in [V] can be given as [13]

$$v_m^S = l_m (\mathbf{E}_m^+ \cdot \boldsymbol{\rho}_m^{c+} / 2 + \mathbf{E}_m^- \cdot \boldsymbol{\rho}_m^{c-}), \quad \mathbf{E}_m^{\pm} = \mathbf{E}^{inc} (\mathbf{r}_m^{\pm}), \quad m = 1, \dots, M$$
(6)

where,  $l_m$  is the length of the *m*th edge,  $\rho$  is the basis function vector.  $\mathbf{E}^{inc}(\mathbf{r}_m)$  is the incident *E*-field on the location  $\mathbf{r}_m$ . The superscript '±' denotes the positive/negative triangle pair, the superscript 'c' denotes the center of the triangle pair. Thus, the unknown current vector  $[I^S]$ can be solved. Further more, the received voltage  $U_a$  and current  $I_a$ in the antenna output port can be written as

$$I_a = I_n l_n, U_a = I_a Z_{IN} \tag{7}$$

where,  $I_n$  and  $l_n$  denote the unknown current coefficient and the length of the *n*th edge (feed edge),  $n \in [1, M]$ ,  $Z_{IN}$  is the input impedance of the antenna.

#### 2.2.2. Calculation of the Reflected Energy in Feed Network

Antenna mode scattering is related to the matching state of an antenna. Assuming that the transmission line is lossless and the terminal load impedance is  $Z_L$ . In the antenna input port, the reflected voltage  $U_b$ is

$$|U_b| = \Gamma_L |U_a| \tag{8}$$

where,  $\Gamma_L$  is the load reflection coefficient, which can be expressed as  $\Gamma_L = (Z_L - Z_0)/(Z_L + Z_0)$ .  $Z_0$  is the characteristic impedance of the transmission line. The port voltage  $U_b$  is just the antenna excitation voltage in the antenna reradiation process.

#### 2.2.3. Calculation of the Reradiated Energy in Free Space

When the antenna is excited by a constant voltage source  $U_b$ , we can calculate the *E*-field in free space by solving Eq. (5). The element  $v_m^R$ 

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in the excitation vector  $[V^R]$  can be obtained as

$$v_m^R = \begin{cases} l_m U_b, & m = n\\ 0, & m \neq n \end{cases}$$
(9)

where, n is the numbering of feed edge in MoM. The unknown reradiation current coefficient  $[I^R]$  can be solved by Eq. (5). The calculation for E-field is the same as that for radiation field. In this paper, the dipole model method with analytical expression is used to calculate the E-field [13, 14]. It means that the total E-field can be equal to the summation of the respective E-field which is produced by each equivalent infinitesimal dipole. The contribution of themth infinitesimal dipole  $\mathbf{M}_m$  can be defined as follows

$$\mathbf{M}_m = \int\limits_{T_m^+ + T_m^-} I_m \mathbf{f}_m(\mathbf{r}) ds = l_m I_m (\mathbf{r}_m^{c-} - \mathbf{r}_m^{c+})$$
(10)

where,  $\mathbf{f}_m$  is the basis function,  $\mathbf{r}$  is the observation point vector. Then, the *E*-field of the *m*th edge at observation point  $\mathbf{r}$  can be calculated as

$$\mathbf{E}_{m}(\mathbf{r}) = \frac{\eta}{4\pi} \left\{ \left[ \left( \frac{(\mathbf{r} \cdot \mathbf{M}_{m})\mathbf{r}}{r^{2}} \right) - \mathbf{M}_{m} \right] \left[ \frac{jk}{r} + \frac{1}{r^{2}} \left( 1 + \frac{1}{jkr} \right) \right] + \left( \frac{(\mathbf{r} \cdot \mathbf{M}_{m})\mathbf{r}}{r^{2}} \right) \cdot \frac{2}{r^{2}} \left( 1 + \frac{1}{jkr} \right) \right\} e^{-jkr}$$
(11)

where,  $\eta$  is the wave impedance in free space,  $k = 2\pi/\lambda$ ,  $\lambda$  is the wavelength. So the total antenna mode scattered field can be summed to obtain

$$\mathbf{E}^{an}(\mathbf{r}) = \sum_{m=1}^{M} \mathbf{E}_m(\mathbf{r})$$
(12)

#### **3. NUMERICAL RESULTS**

# **3.1.** Example1: Antenna Mode Scattering from a Patch Antenna

Consider a patch antenna, which is shown in Fig. 1. This antenna consists of a 57 mm × 42 mm patch and a 75 mm × 60 mm ground plane. A coaxial probe passes through a 2-mm-thick air substrate and connects the ground plane with the patch. The distance between the coaxial probe and the edge of the ground plane is 22.3 mm. The radiation performance of this antenna has been reported in [15]. When a 3.2 GHz  $\varphi$ -polarized plane wave illuminates this antenna perpendicularly, the bistatic antenna mode RCS (in X-Z plane) calculated by the proposed method are shown in Fig. 2.



Figure 1. Sketch of patch antenna.



Figure 2. Antenna mode RCS of a patch antenna.

Figure 2 also depicts the antenna mode RCS calculated by ansoft HFSS based on the method in [12], which are used to verify the proposed method. In this example, three kinds of terminal load of  $Z_L = 0$ , 200 and 100  $\Omega$  are considered, and the corresponding load reflection coefficients  $\Gamma_L$  are -1.0, 0.60 and 0.33, respectively. It can be seen that the RCS calculated by the proposed method are in agreement with the results obtained by the method in [12]. Due to the insertion loss of the transmission line, the RCS calculated by HFSS seem slightly less than the RCS obtained by the proposed method. Around the observation direction of  $\theta = \pm 90^{\circ}$ , the differences increase. This is because Liu's method in [12] need to calculate the scattering in short and open circuit cases, which is implemented through the transmission line with different length. In practical applications, the scattering influence from the transmission line is inevitably added to the antenna scattered field. When observation angle  $\theta$  is  $\pm 90^{\circ}$ , the transmission line is perpendicular to the incident wave, this influence is the greatest. But fortunately, this difference is so small that hardly influence the analysis to antenna scattering.

# **3.2.** Example2: Antenna Mode Scattering from a Dipole Array

Consider an 8-element half-wave dipole array [16], which is shown in Fig. 3. The total length of the diplole arm is 150 mm, and the spacing between the neighboring element is d. In this example  $d = 0.5\lambda$ ,  $\lambda = \lambda_i = 300$  mm, where  $\lambda$  and  $\lambda_i$  are the operating wavelength and the incident wavelength, respectively When a  $\varphi$ -polarized incident wave illuminates this array and each array element is terminated with a short-circuit load, the bistatic antenna mode RCS (in Y-Z plane) calculated by the proposed method are shown in Fig. 4.



Figure 3. Sketch of dipole array.

Figures 4(a), (b) and (c) depict the antenna mode RCS at incident angles of  $\theta_i = 0^\circ$ ,  $\theta_i = 30^\circ$  and  $\theta_i = 45^\circ$ , respectively. It can be seen that the RCS calculated by the two methods are almost the same. For the incident waves at  $\theta_i = 30^\circ$  and  $\theta_i = 45^\circ$ , the bistatic RCS peaks of antenna mode appear in the mirror observation direction of  $\theta = -30^\circ$ and  $\theta = -45^\circ$ , respectively.

The monostatic scattering can be seen as a special bistatic scattering with incident angle  $\theta_i$  = observation angle  $\theta$ . Fig. 5 shows the monostatic antenna mode RCS of a 12-element dipole array. The array structure can also be depicted by Fig. 3, but in this example, the element spacing d is  $0.4\lambda$  (operating wavelength  $\lambda$  and incident wavelength  $\lambda_i$  are both 300 mm). When a  $\varphi$ -polarized incident wave illuminates this array, the RCS curves calculated by the two methods have almost the same trend. It proves the feasibility of the proposed method for calculating the antenna mode scattering.

However, the difference between the two results increases when  $|\theta| > 35^{\circ}$ . The RCS curve calculated by the proposed method is smoother than that obtained by HFSS. That's because the calculation based on HFSS adopts the open/short circuit method [12], which requires taking into account the short circuit scattering and the open circuit scattering simultaneously. Due to the short and open circuits need to be achieved by using the different length transmission line, when the incident angle  $\theta_i$  is small, the transmission line will be shadowed by the antenna. With the increase of  $\theta_i$ , the transmission line is gradually exposed. In this case, the short or open circuit



**Figure 4.** Bistatic antenna mode RCS of an 8-element dipole array at different incident angles  $\theta_i$ . (a)  $\theta_i = 0^\circ$ ; (b)  $\theta_i = 30^\circ$ ; (c)  $\theta_i = 45^\circ$ .



Figure 5. Monostatic antenna mode RCS of a 12-element dipole array.

scattering not only contains the antenna scattering, but also includes the scattering influence from the transmission line. According to Liu's method [12], the final antenna mode scattering can be obtained by the weighted sum of the open and short circuit scattering. When the scattering peaks of open and short circuit are inconsistent, the final result will be not smooth. In practical application, due to the influence of the transmission line, this case is difficult to avoid for adopting the open/short circuit method. However, the proposed method calculates the reflected energy by adopting the S parameter model, therefore, the final result can avoid being influenced by the transmission line, and further the curve obtained is more accurate and smoother.

# 4. CONCLUSION

This paper analyzes the origin of the antenna mode scattering and indicates that the generation of antenna mode scattering is a process of reception-reflection-reradiation. According to this principle, the calculation method based on MoM is proposed. This numerical method can be used to calculate the antenna mode RCS more accurately. The calculation examples of a microstrip patch and dipole array for bistatic or monostatic RCS are presented. The calculation results obtained by the proposed method are in good agreement with the simulated results Thus this method can be used to analyze the composition of antenna scattering or provide the necessary numerical results in antenna RCS reduction.

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