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COMPARISON OF SPECTRAL AND SUBSPACE ALGORITHMS FOR FM SOURCE ESTIMATION

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Abstract—In this paper, direction of arrival (DOA) algorithms for Frequency Modulated (FM) point source have been implemented over a real time system. The source was a commercial FM radio station broadcasting at 89 MHz. Experiments were carried out in order to determine the location of a FM transmitter using spectral and subspace algorithms. The complete Radio Frequency (RF) front end and Uniform Linear Array (ULA) of dipole antennas were designed at 98 MHz having bandwidth of 20 MHz covering the complete FM band. The estimated DOAs are in close agreement to each other.

1. INTRODUCTION

DOA has been an area of great research over the last few decades [1–5]. DOA estimation is important in many sensor systems such as radar, sonar, Electronic Surveillance Measure (ESM), etc. Several algorithms for DOA estimation have been proposed over the years such as Capon [1], the maximum entropy method [2], Multiple Signal Classification (MUSIC) [3, 4] and Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) [5] etc. In this paper, vector type algorithms that rely both on magnitude and phase have been employed to determine the DOA of a point FM source (broadcasting within frequency band of 88–108 MHz). These methods include conventional beamforming, Capon's method, MUSIC and ESPRIT.

Experiments were carried out to determine DOA for a commercial FM radio broadcasting at 89 MHz. A complete hardware setup that

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included antenna and RF front end was designed and is described in the next sections.

Problem statement is given in Section 2. Details of experimental setup are discussed in Section 3. Algorithms for DOA estimation and their results are explained in Section 4, and finally conclusion is given in Section 5.

2. PROBLEM STATEMENT

Consider a FM point source P which is broadcasting in all directions. Let θ be an angle that our source subtends at Uniform Linear Array (ULA). ULA consists of dipole antenna spaced at 0.5λ designed at center frequency 98 MHz. Our objective is to find the angle θ between ULA and point source P which is our DOA. Point source and ULA is shown in Fig. 1.

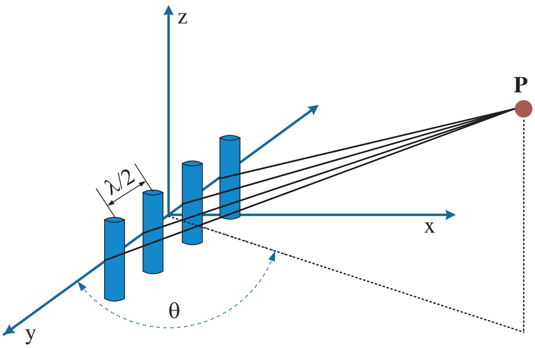

Figure 1. FM point source and ULA.

3. EXPERIMENTAL SETUP

Experimental setup used in DOA estimation is explained below.

3.1. Antenna Selection

Dipole antenna was selected due to a number of reasons such as wider bandwidth in FM band as compared to other antennas. Dipole antenna has bandwidth of 20 MHz while other antennas have bandwidth of about 6 MHz. So, it covers the FM band completely, which is 88–108 MHz. It is centered at 98 MHz and gives a bandwidth of 20 MHz. An ULA of four elements was designed with an equal spacing of 0.5λ. The radiation pattern of the ULA is shown in Fig. 2.

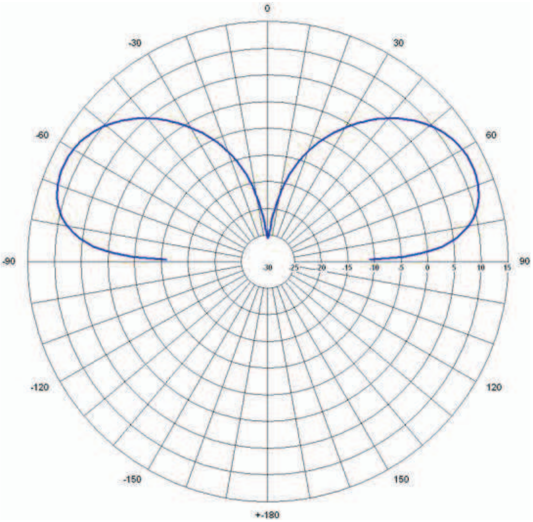

Figure 2. Far-field radiation pattern of ULA.

3.2. RF Front End

Constructional details of RF front end of FM direction finding system are discussed in this section. At first, the signal was demodulated and then down converted to 2 MHz. This signal was then passed through a bandpass filter of 88–108 MHz. After filtering signal was passed through series of Low Noise Amplifiers (LNAs). Output of LNA stage is then forwarded to data acquisition and signal processing unit. Specification of RF components is shown in Table 1. Block diagram of RF front end and hardware setup is shown in Fig. 3(a) and Fig. 3(b) respectively.

Table 1. Specification of RF components.

Device	**Specifications**
Dipole Antenna	[Locally designed, BW = 88–108 MHz, f_c = 98 MHz]
LNA	ASL19W, Gain = 72 dB, NF = 1.2 dB
Mixer	RF2713, IF = 100 kHz–250 MHz
Limiter	SMP1330, IL = 0.3 dB, Gain = 8.8 dB
Attenuator	[Locally designed, Gain = 3 dB]
Filter	[Locally designed, Gain = 3 dB, BW = 88–108 MHz]
AGC	AD8367

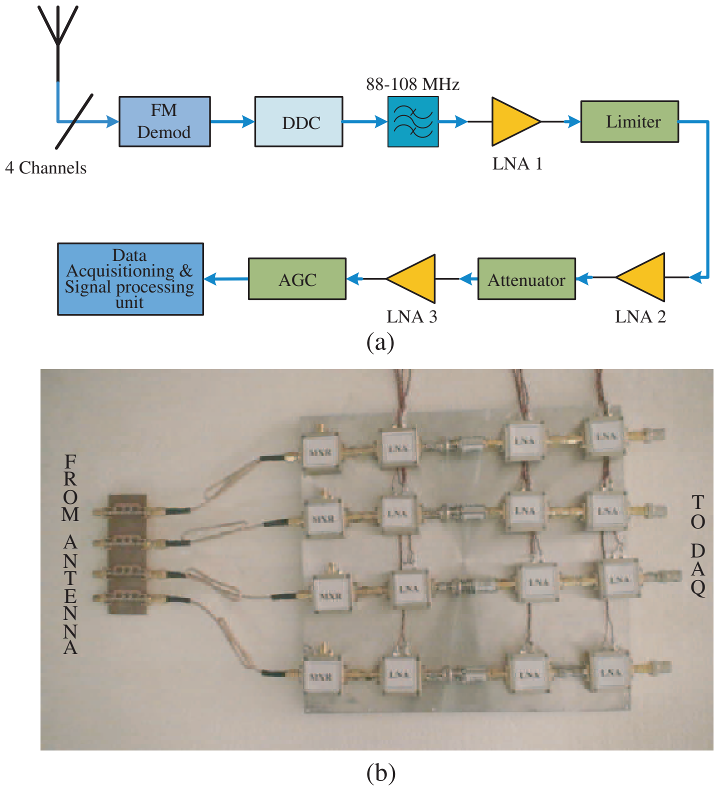

Figure 3. RF front end. (a) Block diagram; (b) Hardware setup.

3.3. Data Acquisition

Intermediate frequency signal of 2 MHz is digitized using data acquisition card. The complex samples of the baseband signal u_i $(i = 1, 2, \ldots, M)$ are filtered in digital domain and applied to the signal processor for DOA estimations. The acquired data for four channels are shown in Fig. 4(a). FFT response of channel 1 after down conversion is shown in Fig. 4(b).

4. DOA ESTIMATION

The signal incident over ULA of dipoles can be modeled as,

$$\mathbf{x}(t) = \mathbf{A}(\theta)\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

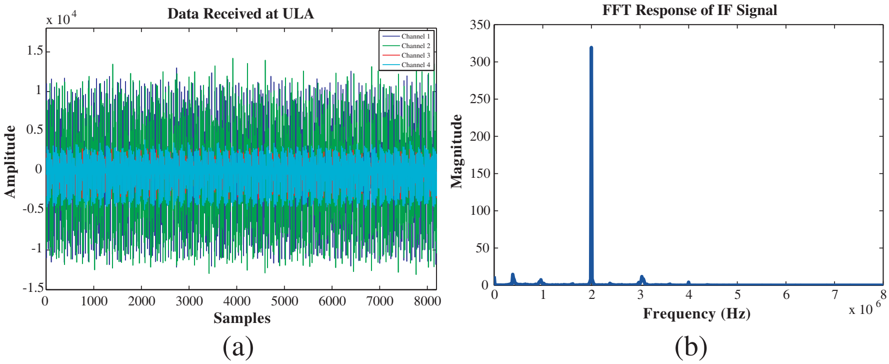

Figure 4. Data acquisition. (a) Acquired data of 4 channels; (b) FFT of channel 1.

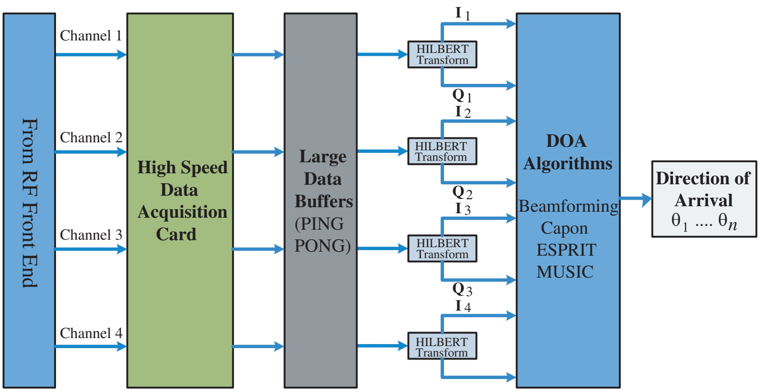

Figure 5. Signal processing block diagram.

where $\mathbf{A}(\theta)$ is a steering vector $(m \times p)$ (p uniform plane waves incident on array of m elements). $\mathbf{s}(t)$ is a signal vector $(p \times 1)$, and $\mathbf{n}(t)$ is a noise vector $(m \times 1)$. DOAs are contained in $(p \times 1)$ parameter vector θ. Also, signal received at array elements is stored in $\mathbf{x}(t)$ and given by Equation (2).

$$\mathbf{x}(\mathbf{t}) = \begin{bmatrix} x_1(t_1) & x_1(t_2) & \dots & x_1(t_{samples}) \\ x_2(t_1) & x_2(t_2) & \dots & x_2(t_{samples}) \\ \dots & \dots & \dots & \dots \\ x_m(t_1) & x_m(t_2) & \dots & x_m(t_{samples}) \end{bmatrix} \tag{2}$$

where, $t_1, t_2, \dots, t_{samples}$ are the instances at which signal is sampled. Further, it is assumed that $m > p$ (more antennas than signals). DOA was estimated using different algorithms both based on spectral estimation techniques and subspace methods. Block diagram of signal processing for FM DOA estimation is shown in Fig. 5. DOA algorithms can basically be divided into two main categories [6].

(i) Spectral and
(ii) Subspace estimations.

4.1. Spectral Estimation

Spectral based methods rely on calculating a spatial spectrum, and DOAs are obtained as locations of peaks in the spectrum [7]. The spectral methods are conceptually simple but offer modest or poor performance in terms of resolution [8]. One of the main advantages of spectral estimation techniques is that they can be used in situations where we lack information about properties of signal since they do not assume anything about the statistical properties of data [7]. Two of these techniques (naming conventional beamforming and Capon's method) are discussed next.

4.1.1. Conventional Beamforming

Output power is maximized when the beam is scanned over the angular region ($-90°$, $+90°$) in discrete steps, and is calculated as a function of Angle Of Arrival (AOA). The received energy is focused on one direction at a time. This can be expressed as,

$$\mathbf{y}(t) = \mathbf{w}^H \mathbf{x}(t) \tag{3}$$

where the weighing vector $\mathbf{w}$ is a spatial filter that emphasizes one particular direction [7]. Let the samples be $y(1)$, $y(2)$, ..., $y(N)$, then total output power P of the conventional beamformer is:

$$P = \frac{1}{N}\sum_{t=1}^{N} |y(t)|^2 = \frac{1}{N}\sum_{t=1}^{N} \mathbf{w}^H \mathbf{x}(t)\mathbf{x}^H(t)\mathbf{w} = \mathbf{w}^H R' \mathbf{w} \tag{4}$$

if weighting factor is chosen to maximize the received power in a certain direction θ.

$$\mathbf{w}_{BF} = \frac{\mathbf{a}(\theta)}{\sqrt{\mathbf{a}^H(\theta)\mathbf{a}(\theta)}} \tag{5}$$

where $\mathbf{a}(\theta)$ is samples for $A(\theta)$. The classical spatial spectrum is given as,

$$P_{BF}(\theta) = \frac{\mathbf{a}^H(\theta)\mathbf{R}'\mathbf{a}(\theta)}{\mathbf{a}^H(\theta)\mathbf{a}(\theta)} \tag{6}$$

where $\mathbf{R}'$ is autocorrelation matrix. The results of beamforming algorithm are shown in Fig. 6(a).

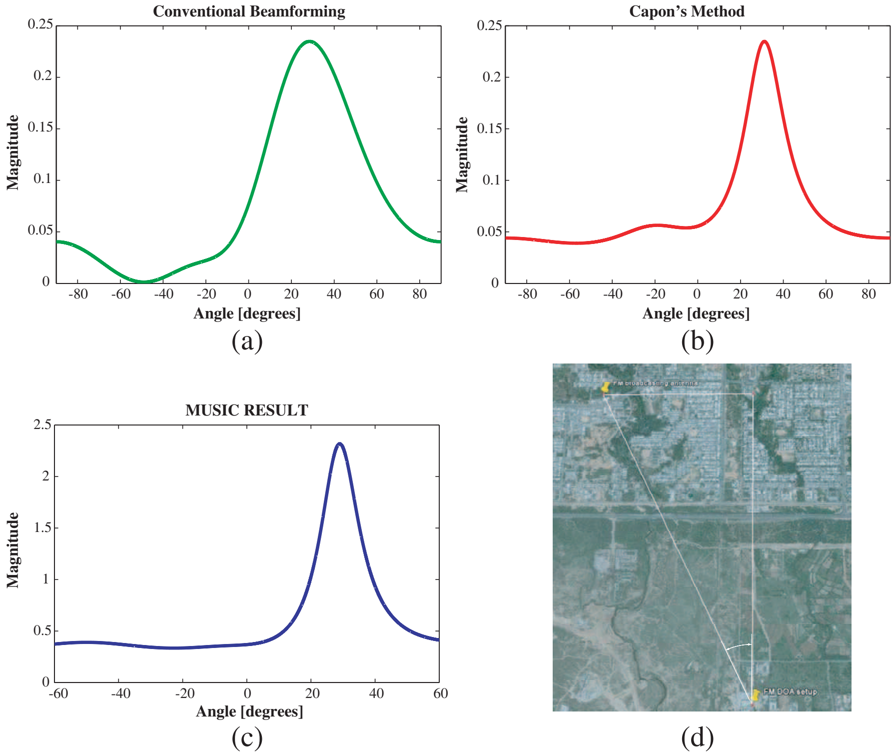

Figure 6. Simulation results. (a) Conventional beamforming; (b) Capon; (c) MUSIC; (d) Verification from Google Earth.

The maximum peak at 28.74° shows DOA for a transmitting FM point source. The increased width of the spectrum is obvious due to the poor noise resolving capability of this algorithm [7].

4.1.2. Capon's Algorithm

Capon's minimum variance method attempts to overcome the poor resolution and noise resolving problems associated with classical beamforming [7]. The technique uses some of the degrees of freedom to form a beam in the desired direction while simultaneously using the remaining degrees of freedom to form nulls in the direction of interfering signals. Capon weighting vector can be given as [7],

$$\mathbf{w_{CAP}}=\frac{\mathbf{R}'^{-1}\mathbf{a}(\theta)}{\sqrt{\mathbf{a}^H(\theta)\mathbf{a}(\theta)}} \tag{7}$$

The output power of the array $P_{\mathbf{CAP}}(\theta)$, as a function of DOA, is given by Capon's spatial spectrum.

$$P_{\mathbf{CAP}}(\theta) = \frac{1}{\mathbf{a}^H(\theta)\mathbf{R}'^{-1}\mathbf{a}(\theta)} \tag{8}$$

By computing and plotting the spectrum over the whole range of θ, the DOA can be estimated by locating the peaks in the spectrum [7]. Fig. 6(b) shows the power spectrum obtained experimentally. The maximum peak power is at 30.10° which is quite near the previous result of conventional beamforming. However, the width of spectrum is decreased due to increased capability of Capon's algorithm to counter noise sources as shown in Fig. 6(b). However, Capon's method fails if signals that are correlated with the signal of interest are present. The correlated components may be combined destructively in the process of minimizing the output power. Capon's method requires the computation of a matrix inverse, which can be computationally expensive for large antenna arrays [7].

4.2. Subspace Methods

Among various DOA (Direction of arrival) techniques subspace methods have become very popular over the decades due to their low computational complexity and good accuracy. Subspace-based methods rely on observations concerning the eigen decomposition of the covariance matrix into a signal subspace and a noise subspace [7]. Two of these methods (MUSIC and ESPRIT) were applied to determine DOA.

4.2.1. MUSIC

MUSIC is a high resolution multiple signal classification technique based on exploiting the eigen structure of the input covariance matrix. Power spectrum of MUSIC algorithm for ULA is given in Equation (9).

$$P_{\mathbf{MU}}(\theta) = \frac{1}{\mathbf{a}^H(\theta)\mathbf{E}'_{\mathbf{n}}\mathbf{E}'^H_{\mathbf{n}}\mathbf{a}(\theta)} \tag{9}$$

Eigen vectors $\mathbf{E}'_{\mathbf{n}}$ are easily obtained by either an eigen decomposition of the sample covariance matrix or a Singular Value Decomposition (SVD) of the data matrix. Fig. 6(c) shows power spectrum for MUSIC algorithm. The maximum of spectrum is indicated by the well defined peak occurring at 28.85° degrees. The width of spectrum is quite small as compared to the spectral based techniques.

The high resolution of MUSIC appears to make it the preferred method over beamforming and Capon. However, the MUSIC algorithm has a few drawbacks that prevent its application to practical systems. In case of correlated signals (multipath scenario) the covariance matrix is in not full rank, and the separation between signal and noise subspace becomes difficult. For correlations less than 0.8, MUSIC typically gives reasonable DOA estimates. If the array structure is regular, it may be possible to de-correlate the signal, thus allowing the use of MUSIC and similar subspace based estimators at the cost of increased computation complexity. Also forward-backward averaging has been applied successfully to estimate DOA for two correlated signals [7].

4.2.2. ESPRIT

ESPRIT [9] is one of the parametric methods for DOA estimation, which have low computational complexity as compared to other non-parametric algorithms [10]. ESPRIT has the capability of easily separating the "signal roots" from "noise roots". If the array contains two identical subarrays that are displaced by a known displacement vector, ESPRIT can be used for DOA estimation. Subarrays do not need to be calibrated, although they need to be identical. The geometry and response of the arrays do not have to be known, only the measurements from these arrays and the displacement between the identical arrays are required [9]. The ith DOA estimate is then obtained as,

$$\theta_i = \arcsin\left[\frac{\angle\lambda_i}{2\pi\Delta}\right] \tag{10}$$

where λ_i is the ith eigenvalue of the matrix, and Δ is known translation between two displaced identical arrays [7].

The ESPRIT results in vector of four AOAs. The maximum of those was chosen as 28.01° which is the most accurate DOA. However, ESPRIT method includes higher estimation errors, and it is not usable in the case of coherent signals [5, 7, 11] and [12].

Table 2. Experimental Results.

Algorithm	**Actual DOA**	**Estimated DOA**	% **Error**
Beamforming	28°	28.74°	2.64°
Capon's	28°	30.10°	7.50°
MUSIC	28°	28.85°	3.03°
ESPRIT	28°	28.01°	0.03°

The experimental results obtained are in close agreement with each other. The results of all these algorithms are summarized in Table 2. DOAs from different algorithms were verified using Google Earth and shown in Fig. 6(d).

5. CONCLUSION

In this paper, spectral and subspace DOA algorithms have been implemented for FM point source estimation. A complete hardware was developed from antenna to RF front end which can be used to estimate DOA for any FM point source. Spectral estimation techniques yield poor resolution results. However, when only single source is to be identified resolution threshold is not of prime concern. For more than one source, subspace estimation techniques can be used efficiently. Yet the choice of algorithm depends upon the number of sources to be identified and their properties. However, ESPRIT has proven to be the most accurate method to be used as DOA algorithm. Estimated DOAs were in close agreement to each other and were verified with Google Earth Map result. The system developed is cheaper, robust and software configurable and can find its application in monitoring of illegal FM radio operators.

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