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PARALLEL MOM SOLUTION OF JMCFIE FOR SCATTERING BY 3-D ELECTRICALLY LARGE DIELECTRIC OBJECTS

Z.-W. Cui, Y.-P. Han, Q. Xu, and M.-L. Li

School of Science
Xidian University
No. 2 Taibai Road, Xi'an, Shaanxi, China

Abstract—In this paper, we apply the parallel method of moments (MOM) to solve the Electric and Magnetic Current Combined Field Integral Equation (JMCFIE) for scattering by large, three-dimensional (3-D), arbitrarily shaped, homogeneous dielectric objects. We first derive the JMCFIE formulation which produces well-conditioned matrix equation when the MOM with Galerkin's type testing and Rao-Wilton-Glisson (RWG) functions is applied. We then develop a parallel conjugate gradient (CG) method on personal computer (PC) clusters using message passing interface (MPI) for solving the matrix equation obtained with JMCFIE. The matrix is decomposed by the row and stored in distributed memory of the node. Several numerical results are presented to demonstrate the accuracy and capability of the proposed method.

1. INTRODUCTION

The electromagnetic scattering of arbitrarily shaped 3-D homogeneous dielectric objects has been extensively studied because of its importance in the area of wireless communication and radar. Analytical solutions are available for only very limited geometries such as a sphere and a spheroid. For dielectric objects having an arbitrary shape, one has to resort to some approximate numerical techniques. The surface integral equation (SIE) approach is often preferred because it limits the discretization of the unknown quantity to the surface of the object. In order to formulate the surface integral equation, several ways have been proposed, including the PMCHW formulation [1], the TENENH formulation [2] and the JMCFIE formulation [3–5] and

Corresponding author: Z.-W. Cui (zhiweicui@163.com).

so on. In this paper, we will describe the JMCFIE formulation in detail, which is a combination of two CFIEs [6], denoted by JCFIE (CFIE for the electric surface current J) and MCFIE (CFIE for the magnetic surface current M). This formulation leads to a better conditioned matrix equation than the TENENH formulation and PMCHW formulation [7] and hence, gives more rapidly converging iterative solutions when the matrix equation is solved with the CG method [8]. However, the matrix equation obtained with JMCFIE is solved by CG method requiring $O\left(N^2\right)$ memory space and $O\left(N^2\right)$ operations for the matrix-vector multiplications in each iteration, where N is the number of unknowns. This significantly limits the size of the object to be handled. One solution to this problem is to employ fast numerical algorithms to reduce the memory requirement and computational complexity of the matrix equation solution. The Fast Multipole Method (FMM) [9] can reduce the memory requirement and computational complexity to $O\left(N^{1.5}\right)$. The memory requirement and computational complexity can be further reduced to $O\left(N1\log N\right)$ using the multilevel fast multipole algorithm (MLFMA) [10, 11]. Contrary to these fast numerical algorithms, the parallel MOM based on MPI of PC clusters is investigated to solve electromagnetic scattering from 3-D electrically large dielectric objects in this paper. Compared with the fast multipole algorithm the parallel MOM [12–15] based on PC clusters has its merits such as easier programming, smaller investment, more flexible architecture and so on.

This paper is organized as follows. The MOM solution of JMCFIE for scattering by 3-D homogeneous dielectric objects is described in detail. According to the properties of MPI, the CG method for solving the matrix equation obtained with JMCFIE is parallelized. The numerical results for the radar cross section (RCS) of a sphere and a Cone-Column are presented to demonstrate the accuracy and capability of the proposed method.

2. JMCFIE FORMULATION AND ITS DISCRETIZATION

Consider the problem of electromagnetic scattering by an arbitrarily shaped and homogeneous body D characterized by a permittivity ε_2 and a permeability μ_2 which is surrounded by an infinite and homogeneous medium having permittivity ε_1 and permeability μ_1. Introducing equivalent electric and magnetic currents $\mathbf{J}_S$ and $\mathbf{M}_S$ on the surface of D which are related to the surface fields by $\mathbf{J}_S = \hat{n}_1 \times \mathbf{H}$ and $\mathbf{M}_S = \mathbf{E} \times \hat{n}_1$, respectively, as shown in Fig. 1. Where $\hat{n}_1$ denotes the outer unit normal of D. Applying the equivalence principle to the

exterior fields [2], we obtain

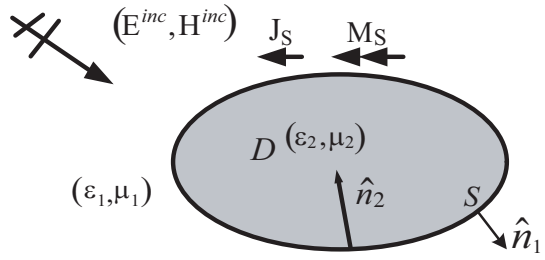

Figure 1. Surface equivalence theorem.

The electric field integral equation (EFIE) and magnetic field integral equation (MFIE) outside D

$$\text{EFIE}\,(1) \quad \left|Z_1\mathbf{L}_1\left(\mathbf{J}_S\right) - \mathbf{K}_1\left(\mathbf{M}_S\right) = -\mathbf{E}^{inc}\right|_{\tan} \tag{1}$$

$$\text{MFIE}\,(1) \quad \left|\mathbf{L}_1\left(\mathbf{M}_S\right) + Z_1\mathbf{K}_1\left(\mathbf{J}_S\right) = -Z_1\mathbf{H}^{inc}\right|_{\tan} \tag{2}$$

where $Z_1 = \sqrt{\mu_1/\varepsilon_1}$, $\left(\mathbf{E}^{inc}, \mathbf{H}^{inc}\right)$ denote the incident fields, and the operators $\mathbf{L}_1$ and $\mathbf{K}_1$ are defined as

$$\mathbf{L}_1(\mathbf{X}) = -\mathbf{j}k_1 \int_s \left[\mathbf{X}\left(\mathbf{r}'\right) + \frac{1}{k_1^2}\nabla\left(\nabla'\cdot\mathbf{X}\left(\mathbf{r}'\right)\right)\right] G_1\left(\mathbf{r},\mathbf{r}'\right) dS' \tag{3}$$

$$\mathbf{K}_1(\mathbf{X}) = -0.5\mathbf{X}\left(\mathbf{r}\right)\times\hat{n}_1 + P\cdot V\cdot\int_s \nabla G_1\left(\mathbf{r},\mathbf{r}'\right)\times\mathbf{X}\left(r'\right) dS' \tag{4}$$

where S denotes the surface D, $P\cdot V\cdot$ is used to denote the principal value, $G_1\left(\mathbf{r},\mathbf{r}'\right) = e^{-jk_1R}/(4\pi R)$, with $R = |\mathbf{r}-\mathbf{r}'|$, $k_1 = \omega\sqrt{\varepsilon_1\mu_1}$.

By applying the equivalence principle to the inner fields, the corresponding EFIE and MFIE inside D can be written in the following equations

$$\text{EFIE}\,(2) \quad \left|Z_2\mathbf{L}_2\left(\mathbf{J}_s\right) - \mathbf{K}_2\left(\mathbf{M}_s\right) = 0\right|_{\tan} \tag{5}$$

$$\text{MFIE}\,(2) \quad \left|\mathbf{L}_2\left(\mathbf{M}_s\right) + Z_2\mathbf{K}_2\left(\mathbf{J}_s\right) = 0\right|_{\tan} \tag{6}$$

where $Z_2 = \sqrt{\mu_2/\varepsilon_2}$ and the operators $\mathbf{L}_2$ and $\mathbf{K}_2$ are defined similarly to $\mathbf{L}_1$ and $\mathbf{K}_1$, provided that all the subscripts are changed from "1" to "2".

The idea of JMCFIE is to consider two CFIEs, called here as an electric current CFIE (JCFIE), and a magnetic current CFIE (MCFIE). These equations can be written as

$$\text{JCFIE}\,(l) \quad \alpha\text{EFIE}\,(l) + \beta Z_1\hat{n}_l\times\text{MFIE}\,(l) \tag{7}$$

$$\text{MCFIE}\,(l) \quad \alpha Z_1\text{MFIE}\,(l) - \beta\hat{n}_l\times\text{EFIE}\,(l) \tag{8}$$

where α and β are the coupling coefficients of the equations, $0 < \alpha < 1$ and $\beta = 1 - \alpha$, $l = 1$ stands for outside D and $l = 2$ stands for inside D. The JMCFIE formulation is obtained by combining these equations in a similar fashion as the EFIEs and MFIEs are combined in the PMCHW formulation, as follows

$$\text{JCFIE}\,(1) + \text{JCFIE}\,(2) \tag{9}$$

$$\text{MCFIE}\,(1) + \text{MCFIE}\,(2) \tag{10}$$

For numerical solutions, JMCFIE is discretized with the RWG functions using a Galerkin scheme. Discretization of JMCFIE leads to $2N \times 2N$ dense matrix equation in the form of

$$\begin{bmatrix} [Z_{11}]_{N\times N} & [Z_{12}]_{N\times N} \\ [Z_{21}]_{N\times N} & [Z_{22}]_{N\times N} \end{bmatrix} \begin{Bmatrix} J \\ M \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b_2 \end{Bmatrix} \tag{11}$$

where N denotes the total number of edges on S, J and M are vectors of N elements involving the coefficients of the electric and magnetic currents, respectively. It should be pointed out that all the coefficients of the magnetic currents are divided by Z_1 in order to improve the conditioning of the discretized matrix equation The matrix and excitation vector elements are given by [16]

$$[Z_{11}] = \alpha\left(Z_1 P_1^{TE} + Z_2 P_2^{TE}\right) + \beta\left(Z_1 Q_1^{NE} + Z_1 Q_2^{NE}\right) \tag{12}$$

$$[Z_{12}] = \alpha\left(-Z_1 Q_1^{TE} - Z_1 Q_2^{TE}\right) + \beta\left(\frac{Z_1^2}{Z_1} P_1^{NE} + \frac{Z_1^2}{Z_2} P_2^{NE}\right) \tag{13}$$

$$[Z_{21}] = \alpha\left(Z_1 Q_1^{TE} + Z_1 Q_2^{TE}\right) + \beta\left(-Z_1 P_1^{NE} - Z_2 P_2^{NE}\right) \tag{14}$$

$$[Z_{22}] = \alpha\left(\frac{Z_1^2}{Z_1} P_1^{TE} + \frac{Z_1^2}{Z_2} P_2^{TE}\right) + \beta\left(Z_1 Q_1^{NE} + Z_1 Q_2^{NE}\right) \tag{15}$$

$$b_{1i} = \alpha\left[-\iint_S \mathbf{g}_i \cdot \mathbf{E}^{inc} dS\right] - \beta\left[-Z_1 \iint_S \hat{n}_1 \times \mathbf{g}_i \cdot \mathbf{H}^{inc} dS\right] \tag{16}$$

$$b_{2i} = \alpha\left[-Z_1 \iint_S \mathbf{g}_i \cdot \mathbf{H}^{inc} dS\right] + \beta\left[-\iint_S \hat{n}_1 \times \mathbf{g}_i \cdot \mathbf{E}^{inc} dS\right] \tag{17}$$

where

$$P_{lij}^{TE} = \iint_S \mathbf{g}_i \cdot \mathbf{L}_l\left(g_j\right) dS \tag{18}$$

$$Q_{lij}^{TE} = \iint_S \mathbf{g}_i \cdot \mathbf{K}_l\left(g_j\right) dS \tag{19}$$

$$P_{lij}^{NE} = -\iint_S \hat{n}_l \times \mathbf{g}_i \cdot \mathbf{L}_l\left(g_j\right) dS \tag{20}$$

$$Q_{lij}^{NE} = -\iint_S \hat{n}_l \times \mathbf{g}_i \cdot K_l\left(g_j\right) dS \tag{21}$$

where $\mathbf{g}_i$ and $\mathbf{g}_j$ denotes the RWG [17] vector basis functions.

3. CONJUGATE GRADIENT METHOD AND ITS PARALLELIZATION

In this paper, we apply the CG method to solve the matrix equation obtained with JMCFIE. In order to handle large problems, we parallelize the CG method on PC Clusters using MPI [18]. The main Clusters configuration is as follows: 1) System composed of: 8 PCs; 2) For each PC CPU: Intel Pentium4, 2.4 GHz; Memory: 1 GB; 3) Switch: TP-Link TL-R402M 1000M; 4) Operating system: Windows XP; 5) Programming environment: Visual Fortran 6.5.

We first present a CG algorithm [19] for solving Equation (11), as follows:

Given an initial guess $J(1)$ and $M(1)$, we have

$$\begin{aligned} r_J(1) &= b_1 - (Z_{11} \cdot J(1) + Z_{12} \cdot M(1)) \\ r_M(1) &= b_2 - (Z_{21} \cdot J(1) + Z_{22} \cdot M(1)) \\ p_J(1) &= Z_{11}^H \cdot r_J(1) + Z_{21}^H \cdot r_M(1) \\ p_M(1) &= Z_{12}^H \cdot r_J(1) + Z_{22}^H \cdot r_M(1) \end{aligned}$$

Iterate for $i = 1, 2, \ldots$ until convergence

$$\begin{aligned} \alpha(i) &= [\langle r_J(i), r_J(i)\rangle + \langle r_M(i), r_M(i)\rangle]/ \\ &\quad [\langle p_J(i), p_J(i)\rangle + \langle p_M(i), p_M(i)\rangle] \\ J(i+1) &= J(i) + \alpha(i) \cdot p_J(i) \\ M(i+1) &= M(i) + \alpha(i) \cdot p_M(i) \\ r_J(i+1) &= r_J(i) - \alpha(i) \cdot (Z_{11} \cdot p_J(i) + Z_{12} \cdot p_M(i)) \\ r_M(i+1) &= r_M(i) - \alpha(i) \cdot (Z_{21} \cdot p_J(i) + Z_{22} \cdot p_M(i)) \\ \beta(i) &= [\langle r_J(i+1), r_J(i+1)\rangle + \langle r_M(i+1), r_M(i+1)\rangle]/ \\ &\quad [\langle p_J(i+1), p_J(i+1)\rangle + \langle p_M(i+1), p_M(i+1)\rangle] \\ p_J(i+1) &= Z_{11}^H \cdot r_J(i+1) + Z_{21}^H \cdot r_M(i+1) + \beta(i) \cdot p_J(i) \\ p_M(i+1) &= Z_{12}^H \cdot r_J(i+1) + Z_{22}^H \cdot r_M(i+1) + \beta(i) \cdot p_M(i) \end{aligned}$$

Terminate when

$$\sqrt{\frac{\langle r_J(i+1), r_J(i+1)\rangle + \langle r_M(i+1), r_M(i+1)\rangle}{\langle b_1, b_1\rangle + \langle b_2, b_2\rangle}} < \varepsilon$$

where, the r_J and r_M are residual vectors, the p_J and p_M are search vectors, $\langle , \rangle$ stand for the inner space product for two vectors $\mathbf{x} = [x_1, x_2, \ldots, x_n]$, $\mathbf{y} = [y_1, y_2, \ldots, y_n]$, $\langle \mathbf{x}, \mathbf{y}\rangle = \sum_{i=1}^{n} x_i y_i^*$, superscript "$H$" denotes transpose conjugate, superscript "$*$" denotes conjugate $\varepsilon = 0.001$ is the residual.

In the implementation of the CG method presented above, one iteration contains eight complex matrix vector products. So the distribution of matrix and solving are the keys of the parallel computation. In this paper, the impedance matrix is decomposed by the row and stored in distributed memory of the nodes. Here, we take the 4×4 matrix shown in Fig. 2(a) and Fig. 2(b) as an example to demonstrate the procedure of the parallelized method to calculate the product of the matrix and the vector [12]. The first subscript of the matrix's and vector's elements denotes the number of the processes where the elements are stored. In Fig. 2(a), the subscript "(0-3)" correspond to the process j.

The product of the matrix and the vector:

Process 0: $\sum_{i=1}^{4} a_{0i} \times x_{oi} = b_0$

Process j: $\sum_{i=1}^{4} a_{ji} \times x_{ji} = b_j \, (j = 1, 2, 3)$, then send b_j to process 0, the final product $y = [b_0, b_1, b_2, b_3]$

The product of the transpose conjugate matrix and the vector:

Process 0: $b_{0i} = a_{0i}^* x_{01} \, (i = 1, 2, 3, 4)$, enables to obtain the vector $[b_{01}, b_{02}, b_{03}, b_{04}]$

Process j: $b_{ji} = a_{ji}^* x_{j(j+1)} \, (i = 1, 2, 3, 4)$, enables to obtain the vector $[b_{j1}, b_{j2}, b_{j3}, b_{j4}] \quad (j = 1, 2, 3)$, then send the vector to process 0, the final product $y = [y_1, y_2, y_3, y_4]$, where $y_m = \sum_{k=0}^{3} b_{km} \, (m = 1, 2, 3, 4)$.

$$\begin{bmatrix} a_{01} & a_{02} & a_{03} & a_{04} \\ a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} \begin{bmatrix} x_{(0\text{-}3)1} \\ x_{(0\text{-}3)2} \\ x_{(0\text{-}3)3} \\ x_{(0\text{-}3)4} \end{bmatrix} \qquad \begin{bmatrix} a_{01}^* & a_{11}^* & a_{21}^* & a_{31}^* \\ a_{02}^* & a_{12}^* & a_{22}^* & a_{32}^* \\ a_{03}^* & a_{13}^* & a_{23}^* & a_{33}^* \\ a_{04}^* & a_{14}^* & a_{24}^* & a_{34}^* \end{bmatrix} \begin{bmatrix} x_{01} \\ x_{12} \\ x_{23} \\ x_{34} \end{bmatrix}$$

(a) (b)

Figure 2. (a) The product of the matrix and the vector. (b) The product of the transpose conjugate matrix and the vector.

4. NUMERICAL RESULTS AND DISCUSSIONS

The parallel MOM (PMOM) described above is implemented for the solution of the JMCFIE formulation. In all considered cases the coupling coefficient is $\alpha = 0.5$. Firstly, we consider the problem of plane-wave scattering by a dielectric sphere. The sphere has a

diameter of $2\lambda_0$ and a relative permittivity $\varepsilon_r = 4.0$, where λ_0 is the free space wavelength. The surface is discretized with about $\lambda_0/15$ mesh size. The number of triangles and unknowns are 3788 and 11364, respectively. Fig. 3 shows the bistatic radar cross section (RCS) of the sphere, obtained using the JMCFIE formulation and the exact Mie series. The two results are in good agreement. Fig. 4 displays the residual norm versus the number of iterations from which we see clearly that JMCFIE formulation gives more rapidly converging iterative solutions than PMCHW and TENENH.

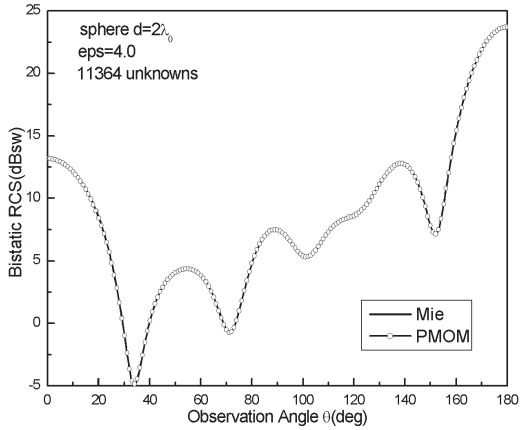

Figure 3. Bistatic RCS of a dielectric sphere in the E plane.

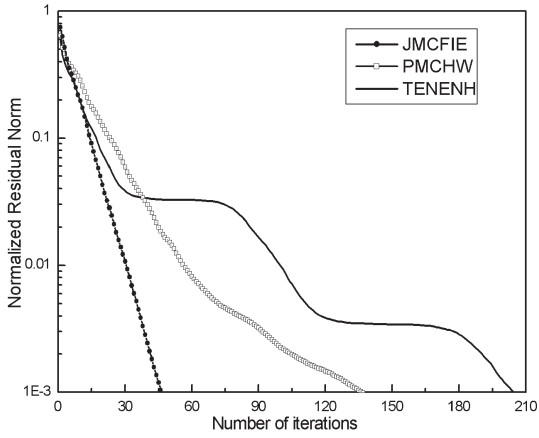

Figure 4. The normalized residual norm versus the number of iterations.

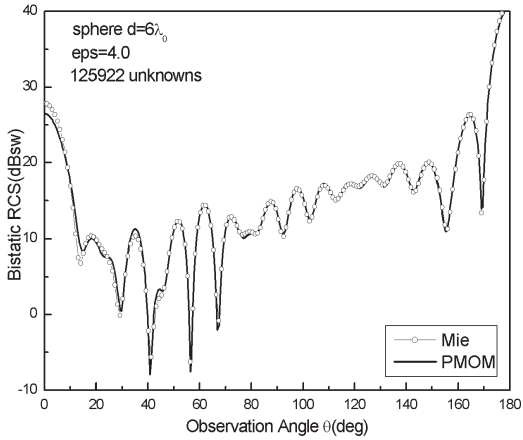

Figure 5. Bistatic RCS of a large dielectric sphere.

Table 1. The comparison of computing time for a same surface realization on different numbers of the processors.

number of the processors	filling times/s	solving times/s	total times/s	speedup
1 processors	2173	675	2848	1.0
2 processors	1152	363	1515	1.87
4 processors	576	183	759	3.75
8 processors	294	89	383	7.43

The comparison of computing time for the above problem on different numbers of the processes are presented in Table 1. From the table, it is easily found that the acceleration ratio is almost in direct proportion to the numbers of processes involved on the computing parallel system. This indicates that the proposed method can solve the problem of scattering by 3-D electrically large dielectric objects.

Then, we use the parallel MOM to calculate the RCS of a relatively large dielectric sphere, which has a diameter of $6\lambda_0$ and a relative permittivity $\varepsilon_r = 4.0$. Discretization of the surface with about $\lambda_0/12$ mesh size leads to a matrix equation with 125922 unknowns The numerical result is complemented with 8 processes and the computation time is 8426 s. Fig. 5 depicts the bistatic RCS as a function of the observation angle in the E plane. We observe that the computational result is in agreement with the analytical result obtained by a Mie-series solution.

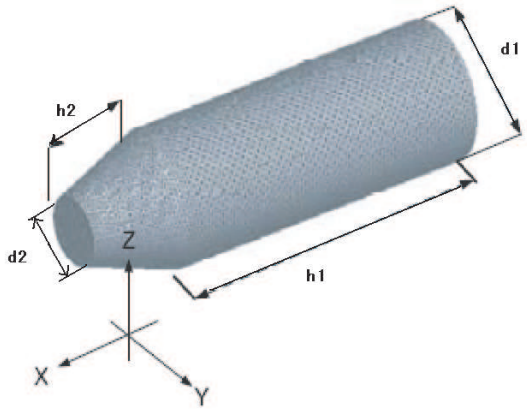

Figure 6. Geometry for Cone-Column object.

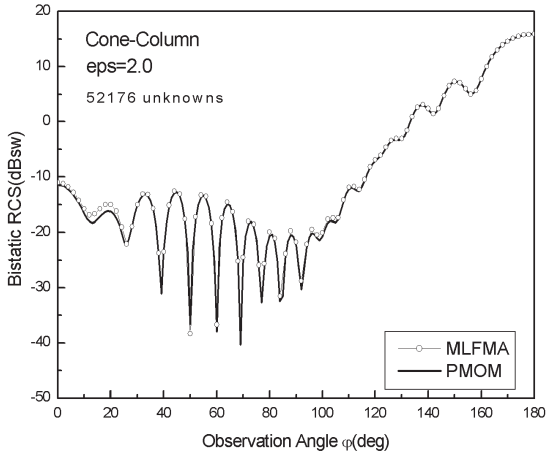

Figure 7. Bistatic RCS of Cone-Column dielectric object.

Finally, we consider the electromagnetic scattering from a Cone-Column dielectric object as shown in Fig. 6, where $d_1 = 2\lambda_0$, $d_2 = 1\lambda_0$, $h_1 = 5\lambda_0$, $h_2 = 2\lambda_0$, the relative permittivity $\varepsilon_r = 2.0$, the total of unknowns is 52176. The object is illuminated by a plane wave propagating in the $-x$ direction with the electric field polarized in the y direction. Fig. 7 shows the bistatic RCS in xoy plane based on PMOM. The calculated result of MLFMA is also given for comparison. The two results agree well to each other. Where, the parallel MOM program is

performed on PC clusters with 8 processes, the computation times is 3942 s. The MLFMA code is performed on an IBM server xSeries 366, Xeon MP 3.66 GHz, 16 GB memory, the computation times is 4136 s.

5. CONCLUSION

In this paper, we derive the JMCFIE formulation for scattering by 3-D homogeneous dielectric objects using the RWG functions as both the expansion and testing functions. The numerical results suggest that the JMCFIE formulation leads to a better conditioned matrix equation than the traditional CFIE and PMCHW formulations. In order to calculate electrically large dielectric objects, we build a PC cluster and develop a parallel MOM program based on MPI. The numerical results indicated that the proposed method is a powerful and efficient scheme for solving scattering from 3-D electrically large dielectric objects. It should be pointed out that the parallel MOM is a sufficiently straightforward way to solve relatively large scattering problems. Our future work is to parallelize the multilevel fast multipole algorithm in order to handle very large problems.

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