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CHARACTERISTICS OF ELECTROMAGNETIC WAVE PROPAGATION THROUGH A MAGNETISED PLASMA SLAB WITH LINEARLY VARYING ELECTRON DENSITY

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Abstract—Characteristics of electromagnetic wave propagation through a magnetized plasma slab with linear electron density profile is analysed. In the numerical analysis, cold, weakly ionized, collisional and steady state plasma layer is divided into sufficiently thin, adjacent subslabs, in each of which plasma parameters are constant. Reflection and transmission coefficients are calculated for discretised plasma by considering electron density profile with positive and negative slopes. Wideband absorbtion characteristic is obtained with high collision frequency and high electron density combination in linearly decreasing profile as well as wideband transmission characteristic is obtained for low collision frequency and low electron density combination in linearly increasing profile of finite length. The general results show that in steady state, plasma layer behaves as a frequency selective medium satisfying the major requirements of current shielding applications as the function of plasma parameters and the strength of external magnetic field excitation. Proposed plasma layer can be used in current shielding and stealth applications as a matching layer between the surface and the incident electromagnetic wave.

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1. INTRODUCTION

The subject of interaction of electromagnetic waves and magnetized plasmas has taken considerable interest in literature [1–18]. It has found that inhomogenous plasma behaves as frequency selective medium and can be used as a broadband radar absorbing layer in general shielding and military stealth applications, radio communications and radio astronomy [3]. Electron density profile considerably affects frequency selective characteristics of the plasma in such applications. In literature, different electron number density functions are considered such as exponential, parabolic, steppened, hyperbolic, tangent, sinusoidal and exponential with time variation [8, 14–16, 18]. In the analysis of plasma-wave interaction in literature, scattering matrix method [6], direct formulation [8] have been used. In order to simplify the solution, total plasma layer is firstly divided into very thin homogenous subslabs in each of which plasma properties are constant. Then reflection, absorbtion and transmission coefficients of the plasma in multilayered form are recursively obtained [6] without taking multiple reflection effect into account. Runge Kutta Method has also been used by Haifeng et al. [9] to analyse interaction of microwave with nonuniform and unmagnetized plasma. They concluded that the results of numerical solution coincide with the analytical solution.

Special plasma responses under certain circumstances are investigated in literature by Gradov and Stenflo [19, 20]. Nonlinear transparency of a dense magnetised plasma is discussed and it is shown that according to the parametric excitation of the surface waves, significant part of the radiation can penetrate through the plasma. In a following study, a mechanism to amplify the transmitted and reflected electromagnetic wave power from an immobile plasma is discussed and it is shown that total radiation energy can be larger than the incident energy during short periods due to release of the stored wave energy in the plasma [19]. In recent studies various new plasma applications and analysis methods have been discussed in literature [21–30].

In the present study, normally incident electromagnetic wave propagation through a magnetised plasma with linearly varying electron density profile having positive or negative slope is firstly studied as a new frequency selective medium. Various multilayered structures have been analysed in literature but plasma layer having linearly varying electron density distribution with positive and negative slopes has not been considered yet. Such a layer behaves as a tunable matching layer which is very important for maximum absorbtion of incident electromagnetic wave and interface medium in some critical shielding and stealth applications where other materials designed in multilayered form can not provide its advantages such as tunability with external magnetic field, effective collison frequency, plasma frequency and electron-cyclotron frequency.

In the analysis, plasma layer is taken as cold, weakly ionized, steady state and collisional. Background magnetic field is assumed to be uniform and parallel to the propagation direction. In the study, linearly varying electron density with different slope values are considered resulting with different reflection, absorbtion and transmission characteristics. In the analysis, inhomogenous plasma is divided into sufficiently thin, adjacent subslabs, in each of which plasma parameters are constant. Then starting with Maxwell's equations, reflected, absorbed and transmitted power expressions are derived. In the related studies in literature, multiple reflections effect while calculating the reflection and transmission coefficients is not considered. In this study, the effect of multiple reflections on the plasma response is calculated and resulting contribution is compared with the results of no multiple reflections case. The effects of collision frequency, maximum value and variation of the electron number density function and external magnetic field strength on the plasma response are presented by determining appropriate conditions for good reflection and absorbtion performances for the purpose of designing a wideband, frequency selective medium tunable via external magnetic field excitation. It is shown that due to amount of matching provided between the propagating wave and plasma medium determined by positive and negative slopes of the linear electron density, plasma layer can be used as good absorber or good reflector in practical shielding and stealth applications as an alternative to the previous multilayered designs.

2. FORMULATION

For a magnetized, source free plasma medium, plasma permittivity is in tensor form. This tensor form permittivity can be approximated by a scaler permittivity. In the case of normally incident electromagnetic wave scaler plasma permittivity is given by the Appleton's formula [1]

$$\tilde{\varepsilon}_r = 1 - \frac{(w_p/\omega)^2}{1 \pm j\frac{v_{en}}{\omega} - \frac{w_{ce}}{\omega}} \tag{1}$$

where ω is the angular frequency of the incident electromagnetic wave, ν_{en} is the effective collison frequency, w_p is the plasma frequency, w_{ce} is the electron-cyclotron frequency. The presence of the $\pm$ sign in (1) is due to two separate solutions for the refractive index. In the case of propagation parallel to the magnetic field, the '+' sign represents a left-hand circularly polarized mode, and the '−' sign represents a right-hand circularly polarized mode. In this study, magnetic field is taken parallel to the propagation direction and thus '−' sign is used in the permittivity expression in (1).

In (1), plasma frequency w_p and electron-cyclotron frequency w_{ce} are given as [1]

$$w_p^2 = e^2 \frac{N}{m\varepsilon_0} \tag{2}$$

$$w_{ce} = \frac{eB}{m} \tag{3}$$

where e is the charge of an electron, N is the electron number density, m is the mass of an electron, ε_0 is the permittivity of free space and B is the external magnetic field strength.

In this study, normally incident electromagnetic wave with progagation constant γ is taken in the form $E_i = \hat{a}_x E_0 \exp(j\omega t - \gamma z)$. In the formulation of the reflection coefficient, plasma layer is divided into equal width, sufficiently thin, adjacent subslabs through which electromagnetic wave propagation will be modelled, as shown in Fig. 1.

In each sublayer, plasma parameters are taken as constant. Electron density N along the plasma layer for a linearly varying profile with positive or negative slopes is taken along the plasma as

$$N = \begin{cases} N_m z/L \\ N_m (L - z)/L \end{cases} \tag{4}$$

where N_m is the maximum electron number density value and L is the thickness of the plasma.

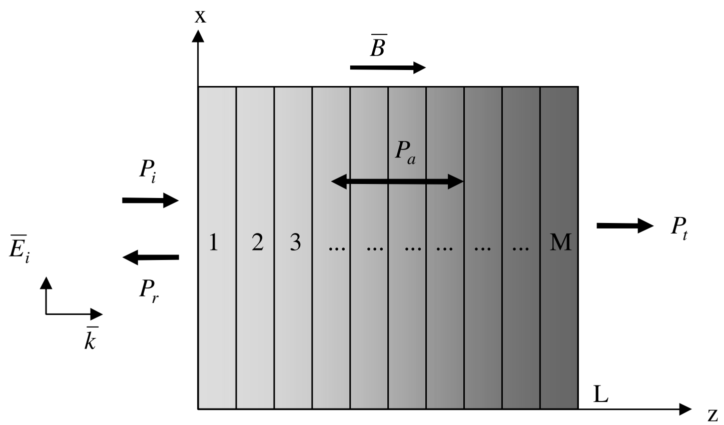

Figure 1. Electromagnetic wave propagation through a plasma layer with equi-width subslabs.

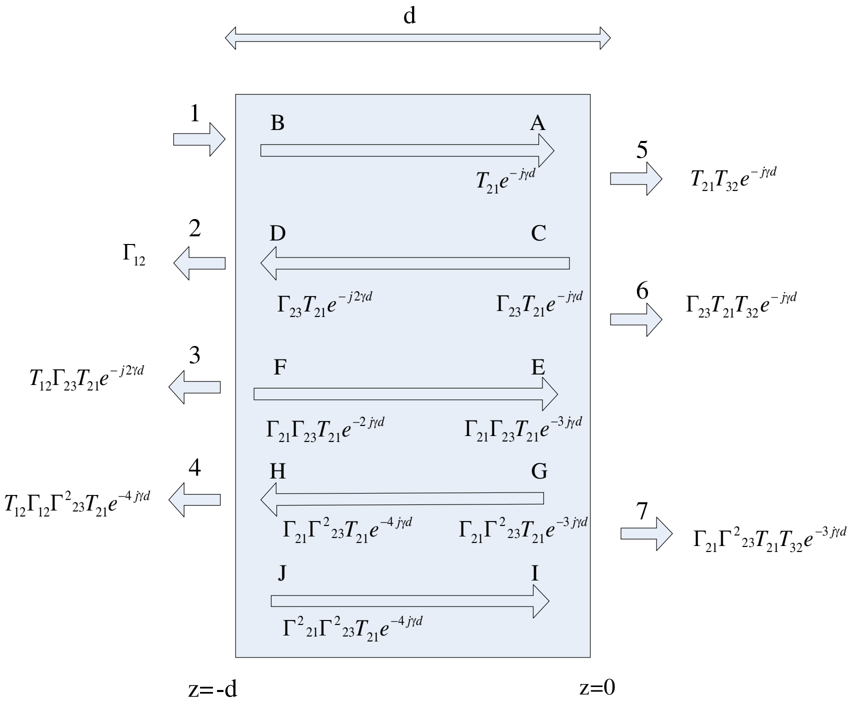

Figure 2. Representation of multiple reflection in a subslab of the plasma layer.

By taking multiple reflections into consideration as shown in Fig. 2, reflection coefficient $\Gamma(j, z)$ at the j'th interface and total reflection coefficient at $z = -d$ interface $(j = 1, z = -d)$ for normal incidence case can be obtained as [31]

$$\Gamma_{in}(j = 1, z = -d) = \Gamma_{12} + \frac{T_{12}T_{21}\Gamma_{23}e^{-2\gamma d}}{1 - \Gamma_{21}\Gamma_{23}e^{-2\gamma d}} \tag{5}$$

From Fig. 2, the relation between the reflection and transmission coefficients can be given as

$$\Gamma_{21} = -\Gamma_{12} \tag{6}$$

$$T_{12} = 1 + \Gamma_{21} = 1 - \Gamma_{12} \tag{7}$$

$$T_{21} = 1 + \Gamma_{12} \tag{8}$$

When multiple reflections are ignored due to highly lossy plasma, by taking $|\Gamma_{12}| \ll 1$ and $|\Gamma_{23}| \ll 1$, reflection coefficient on the $(j + 1)$th interface is obtained as [31]

$$\Gamma_{in}(j+1) = \frac{\sqrt{\tilde{\varepsilon}_r(j)} - \sqrt{\tilde{\varepsilon}_r(j+1)}}{\sqrt{\tilde{\varepsilon}_r(j)} + \sqrt{\tilde{\varepsilon}_r(j+1)}} \tag{9}$$

The reflected power is then calculated by using attenuation constants $\alpha(i)$ of each subslabs and reflection coefficients $\Gamma(j)$ evaluated at their interfaces as

$$P_r = P_i \left\{ \Gamma_{in}(2)|^2 + \sum_{j=2}^{M} \left(|\Gamma_{in}(j)|^2 \prod_{i=1}^{j-1} (\exp[-4\alpha(j)d](1-|\Gamma_{in}(j)|^2)) \right) \right\} \tag{10}$$

where d is the thickness of the subslabs, M is the total number of subslabs and α is the attenuation constant of the medium which is the real part of the propagation constant $\gamma = \alpha + j\beta$.

Then, total transmitted power and absorbed power in the plasma slab can be calculated from

$$P_t = P_i \prod_{i=1}^{M} \left\{ \exp\left[-2\alpha(j)d\right] \left(1 - |\Gamma_{in}(j)|^2\right) \right\} \tag{11}$$

and

$$P_a = P_i - P_r - P_t \tag{12}$$

3. RESULTS AND DISCUSSION

In this section, firstly the plasma slab is assumed to have linearly increasing electron density profile ended with the maximum electron number density value, N_m. Total plasma length is taken as $L = 24\,\text{cm}$. In order to investigate the effect of multiple reflections in plasma-EM wave interaction problem, normalized reflected power variation with and without multiple reflections are shown in Fig. 3. It is seen that very small difference occurs between two cases. This is the result of high value of imaginary part of the complex plasma permittivity of highly lossy plasma and thus high rate of EM wave absorbtion during propagation. For this reason, in the following parts results are presented by ignoring the multiple reflections.

Another important point in Fig. 3 is that plasma layer behaves nearly as a nonreflecting medium along a wide frequency range which is due to provided good matching between the EM wave and the plasma by means of the slowly increasing electron density value along the direction of propagation.

In Fig. 4, the normalized absorbed power variation for different collision frequency values is shown when $N_m = 1 \times 10^{18}\,\text{m}^{-3}$ and $B = 0.25\,\text{T}$. As the collision frequency increases, absorbtion band gets wider. For the lower values of collision frequency, absorbtion band gets narrow and normalized absorbed power decreases sharply.

The plasma slab having both high electron density and high collision frequency values can absorb all of the wave power along wide frequency range even though such a case is not so easy to achieve [8].

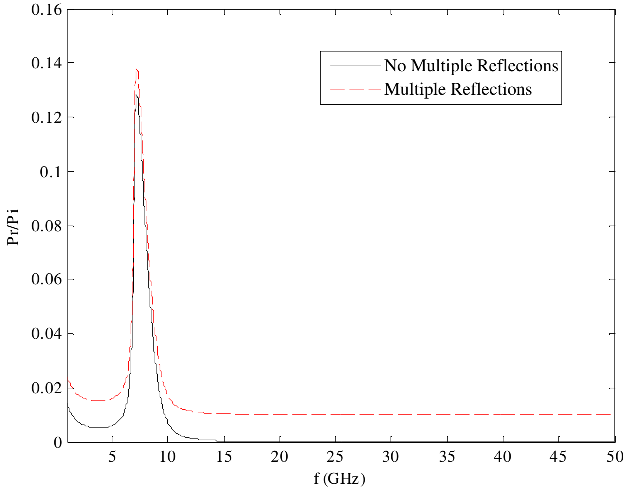

Figure 3. Normalized reflected power for $N_m = 1 \times 10^{18}\,\mathrm{m}^{-3}$, $\nu_{en} = 1\,\mathrm{GHz}$, $B = 0.25\,\mathrm{T}$.

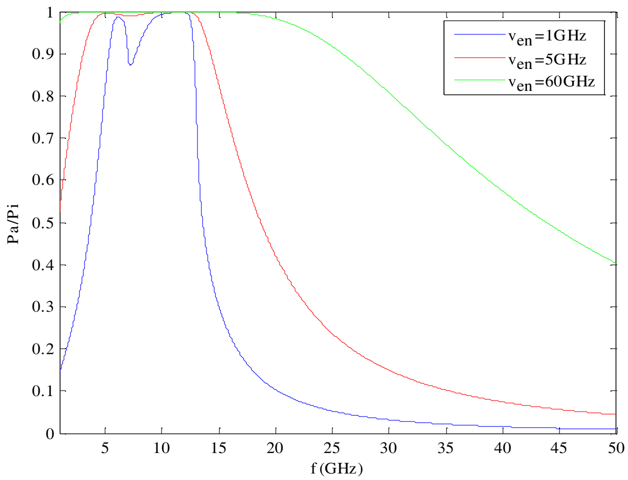

Figure 4. Normalized absorbed power. $N_m = 1 \times 10^{18}\,\mathrm{m}^{-3}$, $B = 0.25\,\mathrm{T}$.

As shown in Fig. 5, for the lower values of N_m, nearly all of the wave power is transmitted especially in higher frequencies and the overall transmission characteristic is the dual of absorbtion characteristic due to very small reflection as shown in Fig. 3. This means that nearly all of the incident power is absorbed or transmitted according to the frequency of the electromagnetic wave with ignorable amount of reflection.

In Fig. 5 indicates that in order to decrease transmitted power and thus improve the absorbtion, it is required to increase electron number density along the plasma layer while collision frequency has low value. In this case very narrow absorbtion band is expected as the dual of the transmission chracteristic again with very small reflection.

In the second part of the results, plasma slab is assumed to have linearly decreasing electron density profile with maximum value N_m at the beginning of the plasma layer. The normalized reflected power variation is shown in Fig. 6 for $N_m = 1\times10^{18}\,\mathrm{m}^{-3}$ and $B = 0.25\,\mathrm{T}$ case. It is clear that reflected approaches to zero as the collision frequency increases especially at high operational frequencies.

The plasma slab is more likely to be a reflector as the collision frequency decreases. It can be generally concluded that reflected power amount is higher with respect to the previous case, i.e., linearly increasing electron density profile case as shown in Fig. 3 due to the high input impedance exerted on the electromagnetic wave by the plasma as the result of high electron density value N_m. For high collision frequency values, plasma behaves as a good absorber according to the value of N_m and ν_{en} as shown in Fig. 7. Double

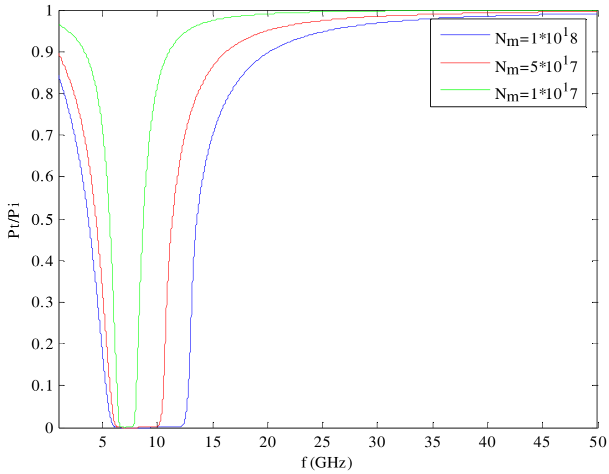

Figure 5. Normalized transmitted power. $\nu_{en} = 1\,\mathrm{GHz}$, $B = 0.25\,\mathrm{T}$.

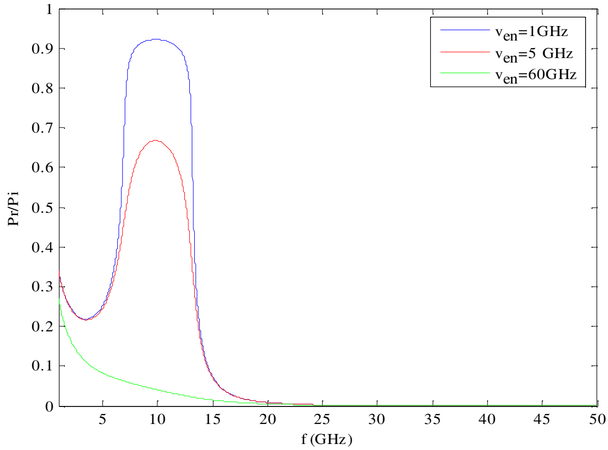

Figure 6. Normalized reflected power. $N_m = 1 \times 10^{18}\,\mathrm{m}^{-3}$, $B = 0.25\,\mathrm{T}$.

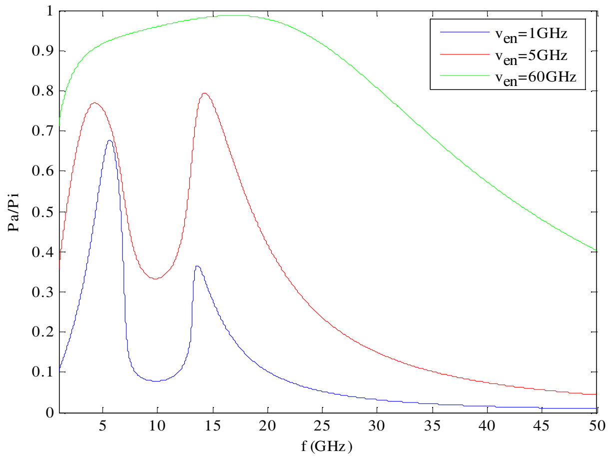

Figure 7. Normalized absorbed power. $N_m = 1 \times 10^{18}\,\mathrm{m}^{-3}$, $B = 0.25\,\mathrm{T}$.

absorbtion bands are observed for $\upsilon_{en} = 1\,\mathrm{GHz}$ and $\nu_{en} = 5\,\mathrm{GHz}$ and plasma slab behaves as a bandreject filter. The reflection characteristic of the plasma slab becomes stronger when the collison frequency is lower around 10 GHz. Normalized absorbed power increases as the collison frequency increases accompanied with single and wider absorbtion band.

Normalized transmitted power can be determined from (11) which is not the dual of the absorbed power as shown in the previous part due to increased amount of reflected power.

In the last part of the study, linearly increasing and decreasing profiles are compared in terms of reflection and absorbtion characteristics in Fig. 8 and Fig. 9 for $\nu_{en} = 1\,\text{GHz}$, $N_m = 5 \times 10^{18}\,\text{m}^{-3}$ and $B = 0.1\,\text{T}$.

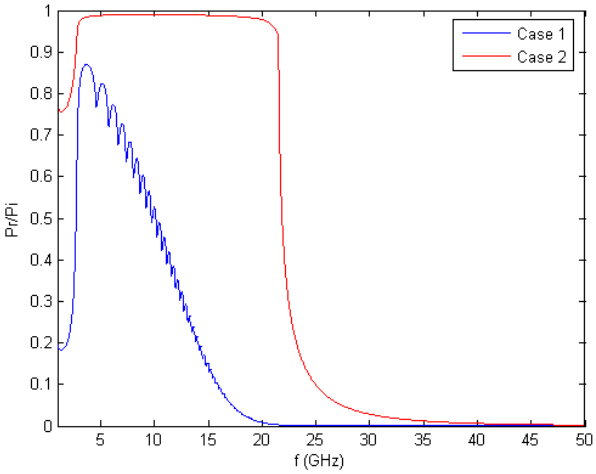

Figure 8. Normalized reflected power comparison for linearly increasing (case 1) and decreasing profiles (case 2). $\nu_{en} = 1\,\text{GHz}$, $N_m = 5 \times 10^{18}\,\text{m}^{-3}$, $B = 0.1\,\text{T}$.

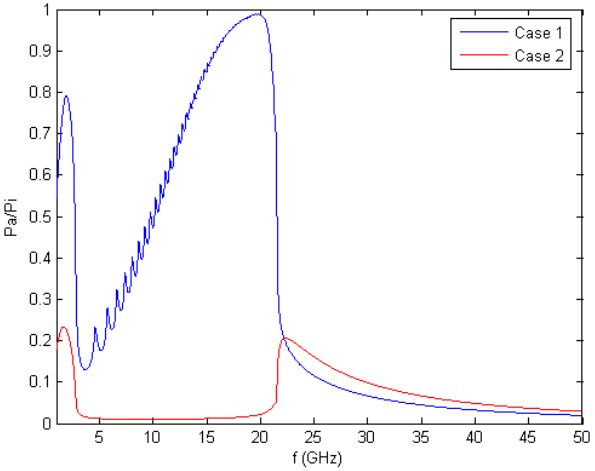

Figure 9. Normalized absorbed power comparison for linearly increasing (case 1) and decreasing profiles (case 2). $\nu_{en} = 1\,\text{GHz}$, $N_m = 5 \times 10^{18}\,\text{m}^{-3}$, $B = 0.1\,\text{T}$.

It is shown that linearly decreasing profile shows better reflector characteristics while linearly increasing profile exhibiting better absorbtion performance due to amount of power matching provided at the input of the plasma layer. It must be noted that according to selected values for magnetic field, maximum electron density and collision frequency center frequency (i.e., the resonant frequency) and the bandwidth of reflection and absorbtion characteristics considerably change. Under such conditions proposed plasma layer provides wideband frequency selectivity which is desirable in critical shielding applications.

4. CONCLUSION

In this study, interaction of electromagnetic wave and finite length plasma slab having linearly varying electron density profile under dc magnetic field excitation is firstly analysed. The effects of linear profile with positive and negative slopes on reflection, absorbtion and transmission characteristics are determined. It is shown that plasma response considerably changes due to adjustment of the slope of the electron density. According to the amount of matching provided at the interface, wideband absorbtion or transmission characteristics can be obtained. In the analysis, effect of multiple reflections are included and shown to be ignorable due to high loss provided by the plasma itself. It is shown that by tuning the plasma parameters such as collision frequency, maximum value and slope of the electron number density and external magnetic field strength, proposed plasma medium can be used as broadband absorber in shielding and stealth applications.

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