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USING THE OSCILLATING DIPOLES MODEL TO STUDY THE ELECTROMAGNETIC RADIATION INDUCED BY FRACTURE OF ROCKS

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Abstract—In this paper, we make an assumption that the inertia vibrations of the electron groups in the rock fragment of the crack tips generate EMR pulses during the fracture of rocks. Based on this assumption we develop an oscillating dipoles model to analyze and simulate the EMR phenomena induced by the rock fractures. Then we use this model to simulate the EMR pulses recorded in the Rabinovitch's compression experiments on granite and chalk. Our simulations indicate a comparable accordance with Rabinovitch's experimental results. From our simulation results, we also find that the crack width associates with the maximum EMR voltage peak value.

1. INTRODUCTION

The EMR phenomenon induced by fracture of rocks has been investigated for many years by its high potential of being used as earthquake predictors [1–3]. Some representative measurements can be found, for example, in the paper by Cress et al. and Frid et al. [4, 5], who carried out a series of compression experiments on granite and chalk samples. Rabinovitch et al. proposed some numerical fitted formulations [5, 6] for their observed EMR pulses induced by rock fractures. However, the origins of the EMR phenomenon are still not well understood. Dickinson et al. [7] proposed a model of moving crack tips, in which the negative charges move with the crack tips while the positive charges accumulate at the surfaces of the cracks, thus dipole radiation occurs from these opposite charges. O'Keefe and Thiel [8]

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gave a capacitors model, in which the two sides of the crack constitute parallel plate capacitors, and the discharge of capacitors emits the EMR pulse. Rabinovitch et al. [6] presented a surface oscillating model. The model assumes that EMR is emitted by oscillating dipoles created by ions moving collectively as a surface wave on both sides of the crack. By using their numerical fitted formulations, Rabinovitch et al. got good fitting results with their experiment results. However, they did not use numerical simulation to verify their surface oscillating model. To improve Rabinovitch's work, in this paper, we present an oscillating dipoles model for the EMR induced by the rock fractures. The fracture takes place by the simultaneous rupture of atomic bonds across the fracture plane [9], so lots of positive and negative ions will be created with the cracks [10–12]. Then the impulses of fracture will drive the rock fragments on the crack tips to move. When the rock fragments were stopped suddenly, the electrons associate with them will make damped vibrations for the inertia in the rock fragments. We think that these vibrating electron groups may be the origin of the EMR of the fracture of rock. These procedures can be equivalent to lots of oscillating dipoles [13]. To verify our model, we use a line of oscillating dipoles [14, 15] to simulate the forming of an elliptical crack in the granite and chalk samples, respectively. Our simulations show a good agreement with Rabinovitch's experimental results [16, 17].

2. MODEL OF ELECTROMAGNETIC RADIATION

From Rabinovitch's compression experiments on granite [16], we suppose a two-dimensional elliptical crack is forming [18] in the rock sample. As shown in Figure 1, the crack extends along the long axis of the ellipse, and the vibrating direction of the rock grains parallels to the short axis of the ellipse. By Enomoto's rock notch experiments [11], the fracture of the rock will create positive and negative ions on the opposite surfaces of the cracks. We divide the whole crack into

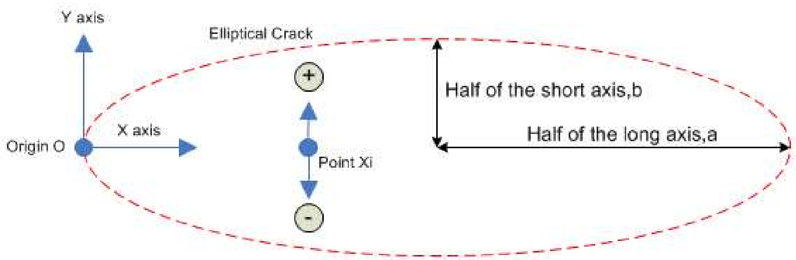

Figure 1. The elliptical crack and the oscillating directions of dipole in the crack.

many tiny charged fragments, and the vibration of each fragment is equivalent to an oscillation of a dipole. Then the whole elliptical crack is equivalent to a line of oscillating dipoles. So we can use a line of oscillating electric dipoles with equal spacing along the long axis of the ellipse to represent the forming of the crack. The centers of the oscillating dipoles all are located at the long axis of the ellipse. As shown in Figure 2, a small loop antenna is located at a distance away from the rock sample to receive the EMR pulses emitted by the crack. The Cartesian coordinates $OXYZ$ on the crack plane is shown in Figure 1 and Figure 2. The origin O is the start point of the crack. The positive X axis is along the extending direction of the crack. The positive Y axis is along the initial oscillating direction of the positive charge of dipole. The positive Z axis is normal to the plane where

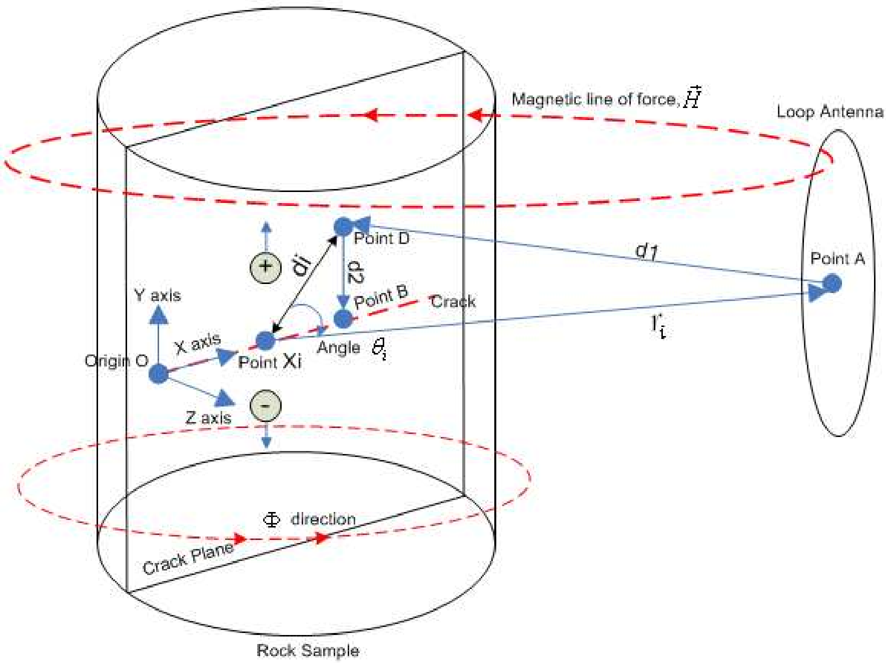

Figure 2. The EMR model of oscillating dipoles, the point Xi is the center of the dipole i in the crack. Point A is the center of the loop antenna. Point D is the projective point on the crack plane by point A. Point B is the projective point on the crack by point D. $d1$ is the distance between point A and point D. $d2$ is the distance between point D and point B. di is the distance between point D and point Xi. r_i is the distance between point Xi and point A. θ_i is an elevation angle relative to point Xi.

the small loop antenna situated. We study one of the dipoles, dipole i, which center is at point Xi, and its coordinates are $(x_i, 0, 0)$. This dipole's maximum oscillating positions are at $(x_i, \pm y_i, 0)$. Because the crack is confined by its elliptical boundary, x_i and y_i must satisfy an ellipse equation:

$$\frac{(x_i - a)^2}{a^2} + \frac{y_i^2}{b^2} = 1 \tag{1}$$

where a, b are the half of the long axis and half of the short axis of the ellipse, respectively. According to the theory of mechanical vibrations [19], the response of a spring-mass system that initially at rest for an excited impulse I is:

$$u = \frac{I}{m\omega\sqrt{1-\beta^2}} e^{-\beta\omega t} \sin(\sqrt{1-\beta^2}\omega t) \tag{2}$$

where u is the displacement of the system; m is the mass of the system; ω is the circular frequency of the vibration; β is the damped factor of the vibration; I is the pulse that exerted on the system; and t is the time. In our model, the vibrating electron groups associated with crack fragment that corresponding to the dipole i is simulated as a spring-mass system, so we have:

$$u_i = \frac{I_i}{m_i\omega_n\sqrt{1-\alpha^2}} e^{-\alpha\omega_n t} \sin(\sqrt{1-\alpha^2}\omega_n t) \tag{3}$$

where u_i is the displacement of the crack fragment; m_i is the mass of the crack fragment; I_i is the pulse which exerted on the crack fragment; ω_n is the circular frequency of the vibration; α is the damped factor of the vibration; and t is the time. Then we use the oscillation of the dipole i to represent the vibration of the corresponding electron groups in the crack fragment. So the maximum oscillating amplitude of the dipole i (y_i) can substitute the oscillation amplitude of the crack fragment, i.e., $y_i = \frac{I_i}{m_i\omega_n\sqrt{1-\alpha^2}}$. Thus the oscillating amplitude of the dipole i is:

$$u_i = y_i e^{-\alpha\omega_n t} \sin(\sqrt{1-\alpha^2}\omega_n t) \tag{4}$$

The electric dipole moment of dipole i is a function of time t [20, 21]:

$$\vec{p}_i(t) = 2Qu_i\hat{y} = 2y_iQe^{-\alpha\omega_n t} \sin(\sqrt{1-\alpha^2}\omega_n t)\hat{y} \tag{5}$$

where $\vec{p}_i$ is the electric dipole moment of the dipole i; $\hat{y}$ is the unit vector in the direction of the positive y axis; and $\pm Q$ is the charges on the dipole i. The oscillation of the dipole will generate electromagnetic waves which then propagate away from the dipole to infinity [21].

When we neglect the attenuation of the rock, the induced magnetic field by the oscillating dipole i will be:

$$\vec{B}_i = \frac{\mu_0}{4\pi}\left(\frac{1}{r_i^2}\frac{dp_i}{dt} + \frac{1}{cr_i}\frac{d^2p_i}{dt^2}\right)\sin\theta_i\hat{\phi} \tag{6}$$

where $\vec{B}_i$ is the magnetic flux density generated by dipole i; μ_0 is the permeability of the vacuum; $\frac{dp_i}{dt}$, $\frac{d^2p_i}{dt^2}$ is the one order and two order derivatives of electric dipole moment $p_i(t)$ to time t, respectively; r_i is the distance between point Xi and point A, which is the center of the loop antenna; c is the speed of light in the vacuum; θ_i is an elevation angle relative to dipole i; and $\hat{\phi}$ is a unit vector of the azimuthally direction of the dipole i (see in Figure 2).

From Figure 1 and Figure 2, we can get the geometrical relations:

$$r_i = \sqrt{d1^2 + di^2} = \sqrt{d1^2 + d2^2 + (x_i - x_b)^2} \tag{7}$$

$$\sin\theta_i \approx \frac{d1}{r_i} \tag{8}$$

where $d1$ is the distance between point A and the crack plane; $d2$ is the distance between point D and point B (see in Figure 2); x_b is the x coordinate of point B; and di is the distance between point D and point Xi. Because the loop antenna is located in the near fields created by the oscillating dipole i, the quantity r_i is far more less than the quantity c. Then Equation (6) can be simplified as:

$$\vec{B}_i \approx \frac{\mu_0}{4\pi r_i^2}\frac{dp_i}{dt}\sin\theta_i\hat{\phi} \tag{9}$$

According to Faraday's law of electromagnetic induction [21], we have:

$$E_i(t) = -\frac{d\Phi_i}{dt} = -\int_{SA}\frac{\partial\vec{B}_i}{\partial t}\cdot d\vec{A} = -\left(\frac{2\mu_0 AQy_i\omega_n^2}{4\pi r_i^2}\sin\theta_i\right)e^{-\alpha\omega_n t}$$
$$\times[(2\alpha^2-1)\sin(\sqrt{1-\alpha^2}\omega_n t) - (2\alpha\sqrt{1-\alpha^2})\cos(\sqrt{1-\alpha^2}\omega_n t)] \tag{10}$$

where $\Phi_i(t)$ is the magnetic flux, which is through the surface of the loop antenna. $E_i(t)$ is the electromotive force (EMF) in the loop antenna, and it is equal to the received EMR pulse voltage $U_i(t)$. SA is the surface of the loop antenna, and A is the area of the loop antenna. By summing all the EMR pulse voltages generated by the oscillating dipoles that associate with the crack, we get the total EMR pulse voltage induced by the crack:

$$U(t) = \sum_{i=1}^{N} U_i(t) \tag{11}$$

where $U(t)$ is the total EMR pulse voltage induced by the crack, and N is the number of the oscillating dipoles associated with the crack.

3. RESULTS AND DISCUSSION

In 2007, Rabinovitch and his co-workers [6, 22] proposed some relationships between the crack parameters and the characteristics of the EMR pulses. These relationships are shown as following:

$$L_{crack} = T v_{crack} \tag{12}$$

$$W_{crack} = \frac{\pi v_{crack}}{\omega_n} \tag{13}$$

where T is the time measured from the beginning of the EMR pulse to the maximum of the EMR pulse envelope; L_{crack} is the crack length; W_{crack} is the crack width; v_{crack} is the crack velocity; and ω_n is the circular frequency of the EMR pulse. Rabinovitch et al. [5, 23] proposed that the value of the v_{crack} equal to the Rayleigh wave velocity in the same media. In our model, from Figure 1, we can find that L_{crack} and W_{crack} can be expressed by the parameters of the ellipse:

$$L_{crack} = 2a, \quad W_{crack} = 2b \tag{14}$$

where a, b are the half length of long axis and short axis of the elliptical crack, respectively.

If we measure the values of T and ω_n of the EMR pulse from the experiment, by using the Rayleigh wave velocity in the rock for the v_{crack}, we can get the crack parameters a and b from Equations (12)–(14). The charges on each of the dipoles, Q, is based on the Enomoto's rock notch experiments [11] and adjusted by the fitting result with the experiment results of Rabinovitch et al. We use the least square method to fit the experiment results for the value of the damped factor of the vibration, α. Based on the Rabinovitch's compression experiments on the granites and chalks [5, 6], the rock samples are all standard cylinders of 100 mm in length and 53 mm in diameter. The loop antenna was placed 20 mm away from the center of the samples with its normal pointing perpendicular to the cylinder axis. The crack velocity v_{crack} was set equal to the Rayleigh

Table 1. Calculated values from Rabinovitch's compression experiments for granite and chalk. $\omega_{n,mea}$ is the measured circular frequency of the EMR pulse, and a_{cal}, b_{cal} are the calculated half length of long axis and short axis of the elliptical crack, respectively.

Type	T (s)	$\omega_{n,mea}$ (rad/s)	L_{crack} (mm)	W_{crack} (mm)	a_{cal} (mm)	b_{cal} (mm)
Granite	8.0×10^{-6}	6.98×10^{5}	9.42	5.30	4.710	2.650
Chalk	4.4×10^{-7}	6.76×10^{7}	1.05	0.11	0.525	0.055

wave velocity. Then we can program to verify our model by using Equations (10), (11). Our simulation results are shown in Figure 3 and Figure 4, respectively. The simulation parameters are listed in Table 1 and Table 2, respectively. From Figure 3 and Figure 4, we can find agreements between our simulations and Rabinovitch's experimental results [16, 17]. Meanwhile, Table 1 and Table 2 also show some differences between the theoretical values by using Rabinovitch's

Table 2. Simulation parameters for Rabinovitch's compression experiments of granite and chalk. $\omega_{n,cal}$ is the simulated circular frequency of the EMR pulse; x_b is the x coordinate of point B, S is the spacing between the dipoles; a_{sim}, b_{sim} are the simulated half length of long axis and short axis of the elliptical crack, respectively. Num is the number of cracks, used in the simulations.

Type	Q (C)	$\omega_{n,cal}$ (rad/s)	α	v_{crack} (m/s)	x_b (mm)	$d1$ (mm)	$d2$ (mm)	S (μm)	a_{sim} (mm)	b_{sim} (mm)	Num
Granite	6.2×10^{-11}	6.62×10^{5}	0.06	1178	2	20	2	10	5.0	3.00	1
Chalk	6×10^{-13}	6.65×10^{7}	0.02	2377	2	20	2	1	0.6	0.06	1

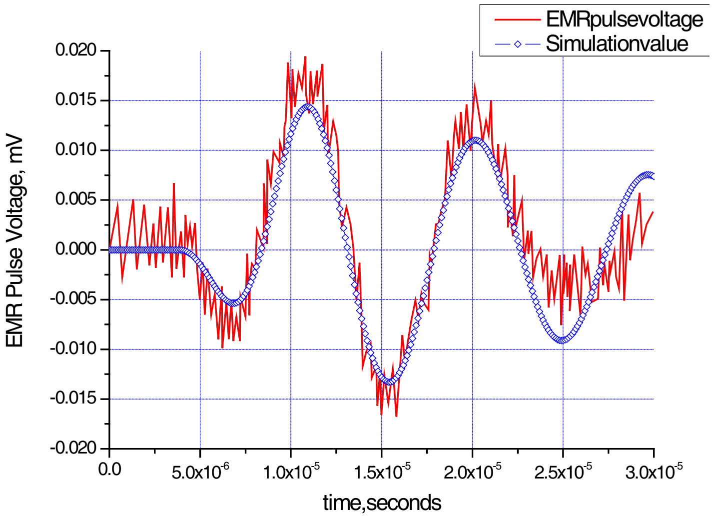

Figure 3. The simulation result for Rabinovitch's compression granite experiment (Rabinovitch et al., 1998 [16]), the dashed line and the solid line are for experimental result and simulated result, respectively.

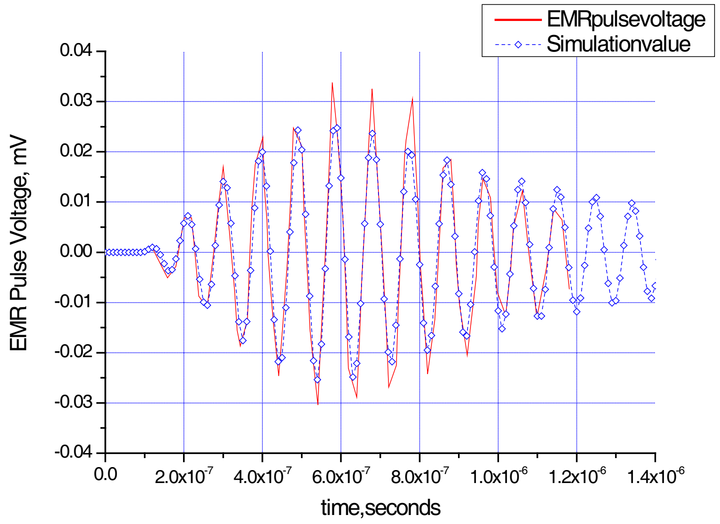

Figure 4. The simulation result for Rabinovitch's compression chalk experiment (Rabinovitch et al., 2000 [17]), the dashed line and the solid line are for experimental result and simulated result, respectively.

Equations (12), (13) and our simulation parameters. These differences may be caused by the errors of theory model with the real physical phenomenon. Because the real EMR pulses are induced by lots of different type of cracks created at random times though in our simulations these processes were simplified to the forming of a large elliptical cracks. Except these differences, from the equation $y_i = \frac{I_i}{m_i \omega_n \sqrt{1-\alpha^2}}$, we can find that the value of y_i is inversely proportional to the EMR circular frequency of the dipole i, ω_n. Relating with the ellipse equation $\frac{(x_i-a)^2}{a^2} + \frac{y_i^2}{b^2} = 1$ for the crack, we can find that the half of the crack width of the point Xi is inversely proportional to the EMR circular frequency of the dipole i. That is to say, the crack width is inversely proportional to the EMR frequency, and this result is accordant with Rabinovitch's Equation (13). For a further understanding of the relationships between the characteristics of the EMR pulses and the crack size parameters, by using our model, we study the conditions under a constant crack length and different crack widths. The result is shown in Figure 5. From Figure 5, we can find that the crack width associates with the maximum EMR voltage peak value. The higher maximum EMR voltage peak value corresponds with bigger crack width.

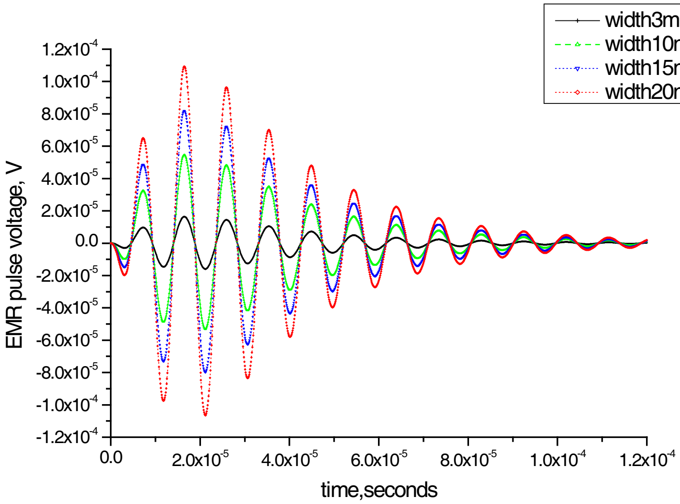

Figure 5. The simulation EMR voltage results for a constant crack length and different crack widths.

4. CONCLUSION

In this paper, we make an assumption that the inertia vibrations of the electron groups in the rock fragment of the crack tips generate EMR pulses during the fracture of rocks. Based on this assumption, we develop an oscillating dipoles model to analyze and simulate the EMR phenomena induced by the rock fractures. Then we used this model to simulate the EMR pulses recorded in the Rabinovitch's compression experiments on granite and chalk. Our simulations indicate a comparable accordance with Rabinovitch's experimental results [16, 17]. From our simulation results, we also find that the crack width associates with the maximum EMR voltage peak value, and the crack width is inversely proportional to the EMR frequency.

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