

## PROPERTIES OF ELECTROMAGNETIC FIELDS AND EFFECTIVE PERMITTIVITY EXCITED BY DRIFTING PLASMA WAVES IN SEMICONDUCTOR-INSULATOR INTERFACE STRUCTURE AND EQUIVALENT TRANSMISSION LINE TECHNIQUE FOR MULTI-LAYERED STRUCTURE

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**Abstract**—Strong interests are recently emerging for development of solid-state devices operating in the so-called “terahertz gap” region for possible application in radio astronomy, industry and defense. To fill the THz gap by using conventional electron approach or transit time devices seems to be very difficult due to the limitation that comes from the carrier transit time where extremely small feature sizes are required. One way to overcome this limitation is to employ the traveling wave type approach in semiconductors like classical traveling wave tubes (TWTs) where no transit time limitation is imposed. In this paper, the analysis method to analyze the properties of drifting plasma waves in semiconductor-insulator structure based on the transverse magnetic (TM) mode analysis is presented. Two waves components (quasi-lamellar wave and quasi-solenoidal wave), electromagnetic fields ( $E_y$ ,  $E_z$  and  $H_x$ ) and  $\omega$ - and  $k$ -dependent effective permittivity are derived where these parameters are the main parameters to explain the interaction between propagating electromagnetic waves and drifting carrier plasma waves in semiconductor. A method to determine the surface impedances in semiconductor-insulator multi-layered structure using equivalent transmission line representation method is also presented since multi-layered structure is also a promising structure for fabricating such a so-called plasma wave device.

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## 1. INTRODUCTION

The idea to replace the electron beam in a traveling wave tube with drifting carriers in a semiconductor has motivated tremendous theoretical work to show and evaluate the possibilities of interactions between drifting carriers and propagating slow electromagnetic waves. The resulted convective instability by the interactions would lead to the possibility of constructing a new type electromagnetic wave solid-state amplifier.

In 1964, Solymar and Ash [1] published a one-dimensional analysis of an  $n$ -type semiconductor traveling wave amplifier predicting high gain per centimeter. They assumed a single species of charge carrier with infinite recombination lifetime obtaining a characteristic equation for the interaction that is reducible to the well known traveling wave tube case. This one-dimensional analysis may be valid for the coupling that takes place directly in the semiconductor bulk but in the case of using external circuit, the coupling is realized only through a surface of semiconductor which sandwiching a thin insulating layer, contacts with the slow-wave circuit. Thus, the coupling through semiconductor surface is essentially of two or three dimensions and hence, two- or three- dimensional analysis would be required for the understanding of amplification by this process.

In 1966, Sumi [2,3] published an analysis of semiconductor traveling wave amplification by drifting carriers in a semiconductor in which he predicted 100 dB/mm gain for an InSb device operated at 4 GHz at liquid nitrogen temperature. The analysis consisted of evaluating the transverse admittance at the surface of a collision-dominant semiconductor and equating it to the transverse admittance at the surface of a developed helix (slow-wave structure). In this analysis, all the electromagnetic fields in the semiconductor are included for the estimation of propagation constants and the amplification is attained beyond the threshold that the electronic gain exceeds all the semiconductor loss.

In 1968, Zotter [4] corrected algebraic errors in Sumi's paper and numerically evaluated the available gain for different semiconductor materials, predicting an even higher gain per millimeter. In 1969, Steele and Vural [5] have extended Sumi's analysis to consider the interaction with a generalized admittance wall including the effects of surface charge and currents. In 1970, Ettenberg [6] published an analysis by following essentially the same method of Solymar and Ash [1] which applicable for two carrier species, e.g., electrons and holes, and derived a maximum resistivity for a given material for which the single dominant carrier approximation remains valid. In their

analysis, carrier-lattice collisions, diffusion and carrier recombination were taken into account.

Although those theories were very different but they agreed on one point, the gain may be very high (several hundred dB/mm). Motivated by the possibilities of amplification with such high gain, in particular demonstrated theoretically by Solyman and Sumi, some innovative experimental study at 77 K down to 2.4 K was performed [7, 8]. In 1991, Thompson et al. also claimed a gain of 13 dB/mm at 8 GHz with an applied transverse *dc* field of 1.5 kV/cm using *n*-type GaAs with interdigitated fingers and *dc* segmented fan antenna [9]. In their experiments, they observed the change of reflection coefficients between the biased and unbiased states of the device which they assumed to be caused by the traveling wave interaction without any theoretical explanation. At best only marginal internal electronic gain was observed and it was not clear that the gain mechanism corresponded to the predicted mode of operation.

These innovative experimental results demonstrated by various group since 1960s till 1990s experiments did not show any net gain and only an interaction much weaker than predicted by theory was observed. Many effects may contribute to divergence between theory and experiment. One of the reasons is mainly due to the strongly collision-dominant (CD) nature of semiconductor plasma. Further accurate theoretical approach and proper device design supported by the remarkable progress in semiconductor materials, fabrication techniques and measurement technologies should open new hope towards the realization of solid-state THz device utilizing plasma wave interaction.

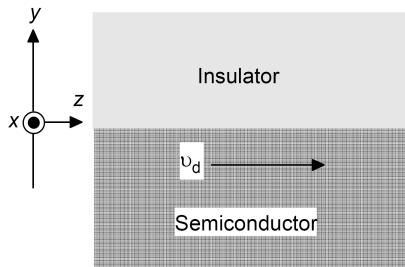
Recently, we have reported theoretically the phenomena of negative conductance in the frequency range of several GHz up to THz region at temperature of 300 K using III-V high-electron-mobility-transistor (HEMT) semiconductor with interdigital structure [10]. A generalized three-dimensional (3D) transverse magnetic (TM) mode analysis to analyze the characteristics of two-dimensional electron gas (2DEG) drifting carrier plasma at III-V hetero-interface was presented. The detail of the theoretical approach for that structure was presented in Reference [11]. Indeed, we have aggressively presented some experimental results which absolutely can be explained well with our theoretical approach [12–15].

In this paper, we report an extended and systematic approach in detail to perform the three-dimensional analysis of the interactions between carrier plasma waves and electromagnetic waves at semiconductor-insulator interface structure for the readers to understand the concept of drifting plasma and its interaction in such struc-

ture. It includes the determination of electromagnetic fields in semiconductor drifting plasma using the combination of well known Maxwell's equations and carrier kinetic equation based on semiconductor fluid model, the determination of boundary condition at semiconductor-insulator interface, and the derivation of the effective permittivity of a semi-infinite semiconductor drifting plasma which are described in Section 2, Section 3 and Section 4, respectively. These parameters are the main parameters to explain the interaction between propagating electromagnetic waves and drifting carrier plasma waves in semiconductor. In Section 5, the analysis technique on the multi-layered structure using transmission line representations is also presented since multi-layered structure is also a promising structure for fabricating such a so-called plasma wave device. Finally, the conclusion is summarized in Section 6.

## 2. ELECTROMAGNETIC FIELDS IN SEMICONDUCTOR DRIFTING PLASMA

To derive the electromagnetic fields in semiconductor drifting plasma, the following assumptions are applied. (a) Only one sort of carriers exists in the semiconductor layer, (b) the semiconductor layer is isotropic and (c) mobility is not changed with electric field. The assumptions are also made where the change of electromagnetic field components, electron density, electron drift velocity are very small and electrons drift in the  $z$  direction with a factor of  $\exp[j(\omega t - kz)]$ . Here,  $k$  is the propagation constant in  $z$  direction as illustrated in Fig. 1. Basically, we generalized the transverse magnetic (TM) mode analysis by Sumi [2, 3] in such a way that the inertia effect of the electron in the nearly collision free (NCF) situation is included. Since the collision frequency,  $\nu$ , in the semiconductor plasma falls typically in the THz or sub-THz region at room temperature, and even in a lower frequency range at lower temperature, the NCF case is a realistic possibility.



**Figure 1.** Semiconductor-insulator interface and its coordinate.

The electromagnetic fields are obtained by the three groups of equations mentioned as follows.

(1) The electron kinetic equation.

$$\frac{d\vec{v}}{dt} = \frac{\partial\vec{v}}{\partial t} + \vec{v} \cdot \nabla\vec{v} = -\frac{q}{m^*} \left( \vec{E} + \vec{v} \times \vec{B} \right) - \frac{v_{th}^2}{n} \nabla n - \nu\vec{v} \quad (1)$$

The Eq. (1) is obtained by applying the charge-current conservation principle derived from zeroth momentum term of Boltzman transport equation into the first momentum term of Boltzman transport equation. The left-hand side of Eq. (1) represents an acceleration term caused by external force applied to electrons. The first term, second term and third term of the right-hand side of Eq. (1) represents acceleration term caused by Lorentz force, diffusion term and the collision term, respectively. The acceleration term caused by Lorentz force was not considered in the Sumi's analysis [2, 3]. The acceleration term caused by Lorentz force shows the inertia effect experienced by electrons when there is an introduction of external electromagnetic fields. The collision term shows the effect due to the collisions among the electrons or the collisions between the electrons and ionized impurities. The diffusion term show the diffusion effect due to the movement of electrons caused by electron temperature ambience.

(2) The charge and current equations.

$$\rho = -qn \quad (2)$$

$$\vec{j} = -qn\vec{v} \quad (3)$$

(3) The Maxwell's equations.

$$\nabla \times \vec{E} = -\frac{\partial\vec{B}}{\partial t} \quad \text{and} \quad \nabla \times \vec{H} = -\mu_o \frac{\partial\vec{H}}{\partial t} \quad (4)$$

$$\nabla \times \vec{H} = \vec{j} + \frac{\partial\vec{D}}{\partial t} \quad \text{and} \quad \nabla \times \vec{H} = \vec{j} + \varepsilon \frac{\partial\vec{E}}{\partial t} \quad (5)$$

$$\nabla \cdot \vec{D} = \rho \quad \text{and} \quad \nabla \cdot \vec{E} = -\frac{q}{\varepsilon}n \quad (6)$$

$$\nabla \cdot \vec{B} = 0 \quad \text{and} \quad \nabla \cdot \vec{H} = 0 \quad (7)$$

$$\vec{D} = \varepsilon\vec{E} \quad (8)$$

$$\vec{B} = \mu_o\vec{H} \quad (9)$$

From the small signal theory,  $\vec{v}$ ,  $n$ ,  $\vec{E}$  and  $\vec{H}$  can be represented by summation of *dc* component and *ac* component. Symbol '0' represents

the *dc* component while symbol ‘1’ represents *ac* component. Here,  $v_d$  is a vector quantity.

$$\vec{v} = \vec{v}_0 + \vec{v}_1 = v_d + \vec{v}_1 \quad (\vec{v}_0 = v_d) \quad (10)$$

$$n = n_0 + n_1 \quad (11)$$

$$\vec{E} = \vec{E}_0 + \vec{E}_1; \quad \vec{H} = \vec{H}_1 \quad (\vec{H}_0 = 0) \quad (12)$$

Again, as mentioned previously, the assumptions are made where the change of electromagnetic field components, electron density, electron drift velocity are very small and electrons drift in the  $z$  direction with a factor of  $\exp[j(\omega t - kz)]$ . Here,  $\vec{v}_0$  is replaced by  $v_d$  and  $k$  is the propagation constant in  $z$  direction. Then, the Eqs. (1)–(7) are converted as follows.

$$j \left( \omega - kv_d - \frac{j}{\tau} \right) \vec{v}_1 = -\frac{q}{m^*} \left( \vec{E}_1 + \mu_0 v_d \times \vec{H}_1 \right) - \frac{v_{th}^2}{n_0} \nabla n_1 \quad (13)$$

$$\rho_1 = -qn_1 \quad (14)$$

$$\vec{j}_1 = -q(n_0 \vec{v}_1 + n_1 v_d) \quad (15)$$

$$\nabla \times \vec{E}_1 = -j\omega\mu_0 \vec{H}_1 \quad (16)$$

$$\nabla \times \vec{H}_1 = \vec{j}_1 + j\omega\varepsilon \vec{E}_1 \quad (17)$$

$$\nabla \cdot \vec{E}_1 = -\frac{q}{\varepsilon} n_1 \quad (18)$$

$$\nabla \cdot \vec{H}_1 = 0 \quad (19)$$

If the frequency of the input electromagnetic field is small enough compared to collision frequency,  $\nu$  where  $|\omega - kv_d| \ll \frac{1}{\tau}$  is assumed, then, the acceleration term  $j(\omega - kv_d)\vec{v}_1$  of Eq. (13) can be omitted.

The following Eq. (20) is built from Eqs. (13) and (18).

$$\vec{v}_1 = -\mu \vec{E}_1 + \frac{\varepsilon D}{qn_0} \nabla \nabla \cdot \vec{E}_1 - \mu \mu_0 v_d \times \vec{H}_1 \quad (20)$$

Then, the following Eq. (21) which shows the relation of Eqs. (13)–(19) and (20) is obtained.

$$\begin{aligned} \nabla \times \nabla \times \vec{E}_1 &= \omega^2 \varepsilon \mu_0 \vec{E}_1 - j\omega q n_0 \mu_0 \mu \vec{E}_1 + j\omega \varepsilon \mu_0 D \nabla \nabla \cdot \vec{E}_1 \\ &\quad - j\omega \varepsilon \mu_0 v_d \left( \nabla \cdot \vec{E}_1 \right) + q \mu n_0 \mu_0 v_d \times \nabla \times \vec{E}_1 \end{aligned} \quad (21)$$

By introducing the  $\nabla \times \nabla \times \vec{E}_1 = \nabla \nabla \cdot \vec{E}_1 - \nabla^2 \vec{E}_1$ ,  $c = (\varepsilon \mu_0)^{-\frac{1}{2}}$  and

$\omega_c = \frac{q\mu n_0}{\varepsilon}$  into Eq. (21), then we can obtain Eq. (22).

$$\begin{aligned} & \left[ \nabla^2 + \frac{\omega^2}{c^2} \left( 1 - j \frac{\omega_c}{\omega} \right) \right] \vec{E}_1 \\ &= \left( 1 - j \frac{\omega D}{c^2} \right) \nabla \nabla \cdot \vec{E}_1 + j \frac{\omega}{c^2} v_d \left( \nabla \cdot \vec{E}_1 \right) - \frac{\omega_c}{c^2} v_d \times \nabla \times \vec{E}_1 \end{aligned} \quad (22)$$

The above Eq. (22) is the fundamental equation for determining the electromagnetic fields in the semiconductor drifting plasma. Here, the effect of magnetic field is also considered.

In the further analysis, only the *ac* component of electromagnetic fields are going to be dealt with, then, the above fundamental equation can be rewritten as follows by omitting the symbol '1'.

$$\begin{aligned} & \left[ \nabla^2 + \frac{\omega^2}{c^2} \left( 1 - j \frac{\omega_c}{\omega} \right) \right] \vec{E} \\ &= \left( 1 - j \frac{\omega D}{c^2} \right) \nabla \nabla \cdot \vec{E} + j \frac{\omega}{c^2} v_d \left( \nabla \cdot \vec{E} \right) - \frac{\omega_c}{c^2} v_d \times \nabla \times \vec{E} \end{aligned} \quad (23)$$

In the following step, we analyze the transverse magnetic (TM) waves propagating along the interface between the insulator layer and semi-infinite semiconductor layer as schematically shown in Fig. 1. Here, the interface between those layers is set at  $y = 0$ . This work is going to deal with the coupling between the electromagnetic waves and drifting carrier waves which normally the phase velocity of the electromagnetic waves need to be slowed down by a so-called slow wave structure so that it can be just slightly higher than the drift velocity of drifting carrier waves. One of the slow wave structures that have been studied by our group is known as an interdigital-gate slow wave structure [10, 13–15]. The electromagnetic waves which exist in the slow-wave structure can be classified into transverse magnetic (TM) waves and transverse electric (TE) waves. However, the electric field direction of TE waves is in  $x$  direction where it is vertical to its traveling direction and electron drifting direction. As a result, the coupling of TE waves with the drifting electrons is assumed not to occur. Hence, only the TM waves will contribute to the interactions with the drifting electrons. In other word, in  $x$  direction, only the magnetic field,  $H_x$  will have an effect on the drifting of electrons while the electric field,  $E_x$  will not have an effect on the drifting of electrons.

In addition, the field component in the  $x$  direction is also ignored with the reason that the width of semiconductor in the  $x$  direction is small enough compared to the wavelength of microwaves. Thus, we can assume that

$$E_x = 0 \quad \text{or} \quad \frac{\partial}{\partial x} = 0 \quad (24)$$

Hence, in this analysis, the components of the electromagnetic fields that will be determined are  $H_x$ ,  $E_y$  and  $E_z$ .

The following equations, Eq. (25) and Eq. (26) are obtained from the extension of Eq. (23).

$$\begin{aligned} & \left( \frac{\partial^2}{\partial y^2} + \frac{\omega^2}{c^2} \left( \left( 1 - \frac{kv_d}{\omega} \right) - j \left( \frac{\omega_c}{\omega} + \frac{Dk^2}{\omega} \right) \right) \right) E_z \\ &= -jk \left( 1 - \frac{\omega^2}{k^2 c^2} \frac{kv_d}{\omega} - j \frac{\omega D}{c^2} \right) \frac{\partial E_y}{\partial y} \end{aligned} \quad (25)$$

$$\begin{aligned} & \left( j \frac{\omega D}{c^2} \frac{\partial^2}{\partial y^2} - k^2 + \frac{\omega^2}{c^2} \left( 1 - j \frac{\omega_c}{\omega} \right) + j \frac{\omega_c}{c^2} kv_d \right) E_y \\ &= jk \left( 1 - j \frac{\omega D}{c^2} - j \frac{\omega_c v_d}{kc^2} \right) \frac{\partial E_z}{\partial y} \end{aligned} \quad (26)$$

From Eqs. (25) and (26), the following differential equation is derived to relate  $E_y$  and  $E_z$ .

$$\left( \frac{\partial^2}{\partial y^2} - \Gamma_l^2 \right) \left( \frac{\partial^2}{\partial y^2} - \Gamma_s^2 \right) \begin{pmatrix} E_y \\ E_z \end{pmatrix} = -\xi^2 \frac{\partial^2}{\partial y^2} \begin{pmatrix} E_y \\ E_z \end{pmatrix} \quad (27)$$

Here,

$$\Gamma_s = \sqrt{k^2 - \frac{\omega^2}{c^2} \left( 1 - j \frac{\omega_c}{\omega} \right) - j \frac{\omega_c}{c^2} kv_d} \quad (28)$$

$$\Gamma_l = \sqrt{\frac{1}{D} (\omega_c + Dk^2 + j(\omega - kv_d))} \quad (29)$$

$$\xi^2 = \sqrt{\frac{\omega_c v_d^2}{D c^2}} \quad (30)$$

The above Eqs. (28) and (29) can also be expressed in the forms as the following.

$$\begin{aligned} \Gamma_s &= \sqrt{k^2 - \frac{\omega^2}{c^2} \left( 1 - \frac{\omega_p^2}{\omega(\omega - kv_d - j\nu)} \right) - \frac{\omega_p^2 kv_d}{c^2 (\omega - kv_d - j\nu)}} \\ &= \sqrt{k^2 - \frac{\omega^2}{c^2} \left( 1 - j \frac{\omega_c^*}{\omega} \right) - j \frac{\omega_c^* kv_d}{c^2}} \end{aligned} \quad (31)$$

$$\begin{aligned} \Gamma_l &= \sqrt{k^2 + \frac{\omega_c}{D} - \frac{(\omega - kv_d - j\nu)(\omega - kv_d)}{\nu D}} \\ &= \sqrt{k^2 + j \frac{1}{\lambda_D^2} \frac{(\omega - kv_d - j\omega_c^*)}{\omega_c^*}} \end{aligned} \quad (32)$$



Here,

$$\lambda_D \equiv \frac{1}{\omega_p} \sqrt{D\nu} = \sqrt{\frac{\epsilon k_B T_e}{nq^2}} \text{ (debye length)}$$

$$\omega_c \equiv \frac{\omega_p^2}{\nu} \text{ (dielectric relaxation frequency) } (\omega_p: \text{ plasma frequency})$$

$$\omega_c^* \equiv -j \frac{\omega_p^2}{\omega - kv_d - j\nu}$$

Assuming  $\Gamma$  as a propagation constant in the  $y$  direction, the following Eq. (33) can be formed from Eq. (27).

$$(\Gamma^2 - \Gamma_l^2) (\Gamma^2 - \Gamma_s^2) = -\xi^2 \Gamma^2 \tag{33}$$

From Eqs. (27) and (33), it can be seen that there is a coupling between  $\Gamma_l$  wave and  $\Gamma_s$  wave. Here  $\Gamma_l$  wave and  $\Gamma_s$  are known as the decay constants of quasi-lamellar wave ( $l$ -wave) and quasi-solenoidal wave ( $s$ -wave), respectively. The properties of these waves are going to be mentioned later. However, in a general semiconductor material, drift velocity of carriers are very small compared to the light speed in semiconductor and thus, a condition of  $(v_d/c)^2 \ll 1$  is usually valid which means that the term of  $\xi^2 \ll 1$  can be considered. As a result, the right-hand side of Eq. (27) can be ignored. In other word, the coupling between  $\Gamma_l$  wave and  $\Gamma_s$  wave will become very small. If the coupling is considered, then, the propagation constant of the TM waves can be expressed as

$$\Gamma_{\pm}^2 = \frac{(\Gamma_s^2 + \Gamma_l^2 - \xi^2) \pm \sqrt{(\Gamma_l^2 - \Gamma_s^2)^2 - 2\xi^2 (\Gamma_l^2 + \Gamma_s^2) + \xi^4}}{2} \tag{34}$$

Next, the following boundary conditions of semi-infinite semiconductor are made.

$$E_y(y = -\infty) = 0 \tag{35}$$

$$E_z(y = -\infty) = 0 \tag{36}$$

From Eq. (27), it is shown that  $E_y$  and  $E_z$  component are constructed by two factors,  $\Gamma_+$  and  $\Gamma_-$ . Thus, those components can be expressed in term of  $\Gamma_+$  and  $\Gamma_-$  as follows.

$$E_y = A_{yl} e^{\Gamma_+ y} + A_{ys} e^{\Gamma_- y} \tag{37}$$

$$E_z = A_{zl} e^{\Gamma_+ y} + A_{zs} e^{\Gamma_- y} \tag{38}$$

$A_{yl}$ ,  $A_{ys}$ ,  $A_{zl}$ ,  $A_{zs}$  are the coefficients determined by the boundary condition at semiconductor-insulator interface.

Next, Eqs. (37) and (38) are introduced into Eqs. (25) and (26). Here, the assumption of

$$\frac{\omega D}{c^2} \ll 1 \tag{39}$$

is made due to the diffusion constant in GaAs is only a few tens  $\text{cm}^2/\text{s}$  at the considered frequency. Also, it will be shown later that to make the interactions, the propagation velocity of microwave and electron drift velocity should be nearly equal,

$$\frac{\omega}{k} \cong v_d \quad (40)$$

where the conditions of

$$\left(\frac{v_d}{c}\right)^2 \ll 1, \quad \frac{\omega}{kv_d} \left(\frac{v_d}{c}\right)^2 \ll 1 \quad \text{or} \quad \frac{\omega_c v_d}{kc^2} \ll 1 \quad (41)$$

can be considered. However, the term  $\frac{\omega_c v_d}{kc^2}$  cannot be ignored in the following Eq. (42).

$$A_{yl} = j \frac{\Gamma_+}{k} \frac{1 - j \frac{\omega D}{c^2}}{1 - \frac{\omega}{kv_d} \left(\frac{v_d}{c}\right)^2 - j \frac{\omega D}{c^2}} A_{zl} \cong j \frac{\Gamma_+}{k} \frac{A_{zl}}{1 - \frac{\omega}{kv_d} \left(\frac{v_d}{c}\right)^2} \quad (42)$$

$$A_{ys} = j \frac{k}{\Gamma_-} \frac{1 - j \frac{\omega D}{c^2} - j \frac{\omega_c v_d}{kc^2}}{1 - j \frac{\omega D}{c^2}} A_{zs} \cong j \frac{k}{\Gamma_-} A_{zs} \quad (43)$$

For Eq. (43), the conditions of  $\frac{\omega D}{c^2} \ll 1$  and  $\frac{\omega_c v_d}{kc^2} \ll 1$  can be applied. From Eq. (16),  $H_x$  is obtained as

$$H_x = j \frac{1}{\mu_o \omega} \left( -jk E_y + \frac{\partial E_z}{\partial y} \right). \quad (44)$$

Eqs. (42) and (43) are introduced into Eq. (37) which produces the following Eq. (45).

$$E_y = j \frac{\Gamma_+}{k} \frac{A_{zl}}{1 - \frac{\omega}{kv_d} \left(\frac{v_d}{c}\right)^2} e^{\Gamma_+ y} + j \frac{k}{\Gamma_-} A_{zs} e^{\Gamma_- y} \quad (45)$$

$H_x$  can be expressed again by considering Eqs. (38), (44) and (45).

$$H_x = j \frac{\varepsilon v_d \Gamma_+}{k} A_{zl} e^{\Gamma_+ y} + \frac{j}{\mu_o \omega \Gamma_-} (k^2 - \Gamma_-^2) A_{zs} e^{\Gamma_- y} \quad (46)$$

Assuming that the coupling between  $l$ -wave and  $s$ -wave is very weak where  $\xi^2 = 0$  then the following assumptions are valid.

$$\Gamma_+ \cong \Gamma_l \quad (47)$$

$$\Gamma_- \cong \Gamma_s \quad (48)$$

Replacing  $A_{zl}$  with  $A_l$  and  $A_{zs}$  with  $A_s$  then finally the electromagnetic field components are obtained as follows.

$$E_y = j \frac{\Gamma_l}{k} \cdot \frac{A_l}{1 - j \frac{\omega}{kv_d} \left(\frac{v_d}{c}\right)^2} e^{\Gamma_l y} + j \frac{k}{\Gamma_s} A_s e^{\Gamma_s y} = j \frac{\Gamma_l}{k} \cdot A_l e^{\Gamma_l y} + j \frac{k}{\Gamma_s} A_s e^{\Gamma_s y} \quad (49)$$

$$E_z = A_l e^{\Gamma_l y} + A_s e^{\Gamma_s y} \quad (50)$$

$$H_x = \frac{j \omega \varepsilon}{k} \left( \frac{\Gamma_l}{k} \cdot \frac{kv_d}{\omega} A_l e^{\Gamma_l y} + \frac{k}{\Gamma_s} \left( 1 - j \frac{\omega_c}{\omega} \right) A_s e^{\Gamma_s y} \right) \quad (51)$$

Here, the first term and the second term of the right-hand side of Eqs. (49), (50) and (51) represents the quasi-lamellar component and the quasi-solenoidal component, respectively. The quasi-lamellar component satisfies the condition of

$$\nabla \times \vec{E} \cong 0 \quad \text{or} \quad \nabla \times \vec{E}_l \cong 0 \quad (52)$$

while the quasi-solenoidal component satisfies the condition of

$$\nabla \cdot \vec{E} \cong 0 \quad \text{or} \quad \nabla \cdot \vec{E}_s \cong 0 \quad (53)$$

which can be confirmed using Eqs. (53) and (50). Here, again  $\Gamma_s$  and  $\Gamma_l$  are referred as the decay constant of solenoidal wave (*s-wave*) and lamellar wave (*l-wave*), respectively.

The electric fields in semiconductor  $\vec{E}$  are formed by  $\vec{E}_l$  and  $\vec{E}_s$ .

$$\vec{E} = \vec{E}_l + \vec{E}_s \quad (54)$$

The divergence of electric fields is given as

$$\nabla \cdot \vec{E} = -\frac{qn}{\varepsilon} \quad (55)$$

Then, Eq. (55) is converted to the following equation.

$$\nabla \cdot (\vec{E}_l + \vec{E}_s) = -\frac{qn}{\varepsilon} \quad \nabla \cdot \vec{E}_l \cong -\frac{qn}{\varepsilon} \quad (56)$$

Eq. (56) shows that the behavior of electrons in the semiconductor near to the interface of semiconductor-insulator is mainly influenced by  $\vec{E}_l$ .

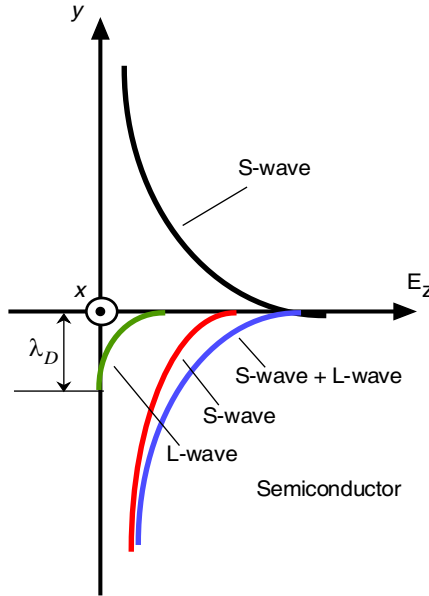
The rotation of electric fields is given as

$$\nabla \times \vec{E} = -j\omega\mu_o\vec{H}_1 \quad (57)$$

Then, Eq. (57) is converted to the following equation.

$$\nabla \times (\vec{E}_l + \vec{E}_s) = -j\omega\mu_o\vec{H}_1 \quad \nabla \times \vec{E}_s \cong -j\omega\mu_o\vec{H}_1 \quad (58)$$

The above Eq. (58) shows that the magnetic field in the semiconductor is mainly influenced by  $\vec{E}_s$ . In addition, it is shown from Eq. (46) that the behavior of electrons is not influenced by  $\vec{E}_s$ . In other word, the lamellar component represents the longitudinal component which can influence the electron distribution near to the interface of semiconductor-insulator structure while the solenoidal component represents the transverse component which can influence the *x* direction magnetic field. These *l-wave* and *s-wave* are schematically shown in Fig. 2.



**Figure 2.** *S*-wave and *l*-wave in semiconductor-insulator structure.

The term “quasi” means that  $\nabla \times \vec{E}_l$  and  $\nabla \cdot \vec{E}_s$  are not perfectly have a value of zero as shown in Eqs. (52) and (53). This also gives a meaning that  $\vec{E}_l$  and  $\vec{E}_s$  is not strictly independent between each other. The electromagnetic fields of TM waves in semiconductor,  $E_y$ ,  $E_z$ , and  $H_x$  are given by Eqs. (49), (50) and (51), respectively.

### 3. BOUNDARY CONDITION AT SEMICONDUCTOR-INSULATOR INTERFACE

In reality, due to various causes, the surface states will exist at the semiconductor-insulator interface. It is generally believed that the response time of the surface states are very slow, lying in the kHz to MHz region. In this analysis, the surface recombination of carriers at the semiconductor-insulator interface is ignored with the reason that the frequency range dealt in this work is high enough compared to the frequency range of surface recombination. Generally, in normal condition, the surface charge,  $\rho_{sur}$  and surface current,  $J_{sur}$  exist due to the existence of carriers in semiconductor.  $J_{sur}$  is considered only in the  $z$  direction. Thus, the boundary conditions are determined as

below which relates both  $\rho_{sur}$  and  $J_{sur}$ .

$$\varepsilon_1 E_{y1} - \varepsilon_2 E_{y2} = \rho_{sur} \tag{59}$$

$$H_{x1} - H_{x2} = -J_{sur} \tag{60}$$

Here, the subscript “1” represents the dielectric layer and subscript “2” represents the semiconductor layer. The following Eq. (61) is also obtained from the condition of charge-current conservation principles.

$$j\omega\rho_{sur} = jk J_{sur} + \vec{j}_{y2} \tag{61}$$

$\vec{j}_{y2}$  is the conductive current of  $y$  direction component in semiconductor.

The boundary conditions used in the previous analysis works done by other researchers are summarized according to the condition of (A) without consideration of diffusion and (B) with consideration of diffusion.

(A) Without consideration of diffusion (zero temperature proximity  $T_e = 0$  K)

In this situation,  $l$ -wave is terminated and only  $s$ -wave exists in semiconductor. Kino, G. S. [16] considered the existence of both  $\rho_{sur}$  and  $J_{sur}$  at the interface which is related by the equation  $J_{sur} = \rho_{sur}v_d$ . This treatment is equivalent to the Hahn’s boundary condition used in the electron beam theory.

(B) With consideration of diffusion

(i) Sumi [2, 3] applied the condition of  $\rho_{sur} = 0, J_{sur} = 0$  in his analysis.

(ii) Blotekjaer [17] applied the condition of  $\varepsilon_1 E_{y1} = \varepsilon_2 E_{y2}$  and  $\vec{j}_{y2} = 0$  in his analysis. It can be seen that Eqs. (49), (50) and (51) is equivalent to the item (i) and (ii) if the diffusion is being considered.

(iii) Mizushima et al. [18] considered that  $J_{sur} = 0$  when signal frequency is nearly equal to dielectric relaxation frequency and  $\rho_{sur}$  is represented by scalloped charges referring to Hahn’s boundary condition.

(iv) Steele et al. [19] applied the condition of  $\rho_{sur} \neq 0, J_{sur} \neq 0$  in his analysis. According to his analysis, to achieve the condition of  $\rho_{sur} = 0, J_{sur} = 0$ ,  $dc$  magnetic field with infinitive value has to be applied in  $x$  direction.

If there is no occurrence of diffusion where the carriers do not perform thermal motion, the charges will only appear at the surface. This also means that only  $s$ -wave exists in semiconductor bulk. Nevertheless, if the carriers perform thermal motion, the charges will be distributed in the semiconductor bulk near to the surface. To assume that the diffusion current is not existing at the surface, this

condition,  $\rho_{sur} = 0$ ,  $J_{sur} = 0$  has to be set. The penetration of space-charges will result in the existence of  $l$ -wave. In the collision-dominant condition, ( $\omega_C^* = \omega_C$ ) and ( $\omega \ll \omega_C$ ), the penetration distance is almost equal the Debye length,  $\lambda_D$ , where this statement can be understood from Eq. (32).

With respect to the above considerations, we proceeded the analysis based on the boundary condition where  $\rho_{sur} = 0$ ,  $J_{sur} = 0$ , meaning that the diffusion is considered.

Then, the boundary conditions are re-determined as follows.

$$\varepsilon_1 E_{y1} = \varepsilon_2 E_{y2} \quad (62)$$

$$H_{x1} = H_{x2} \quad (63)$$

$$E_{z1} = E_{z2} \quad (64)$$

Using these boundary conditions, the ratio of  $A_l/A_s$  for the electric field in the  $z$  direction is obtained as follows.

$$\frac{A_l}{A_s} = j \frac{k^2}{\Gamma_s \Gamma_l} \frac{\omega_c}{\omega - kv_d} \quad (65)$$

By considering Eq. (65), the electromagnetic field components,  $E_y$ ,  $E_z$ , and  $H_x$  are rewritten as

$$E_y = j \frac{k}{\Gamma_s} A_s \left( e^{\Gamma_s y} - j \frac{\omega_c}{(\omega - kv_d)} e^{\Gamma_l y} \right) \quad (66)$$

$$E_z = A_s \left( e^{\Gamma_s y} - j \frac{k^2}{\Gamma_s \Gamma_l} \frac{\omega_c}{(\omega - kv_d)} e^{\Gamma_l y} \right) \quad (67)$$

$$H_x = \frac{j\varepsilon_2 \omega}{\Gamma_s} A_s \left( \left( 1 - j \frac{\omega_c}{\omega} \right) e^{\Gamma_s y} - j \frac{\omega_c}{(\omega - kv_d)} \frac{kv_d}{\omega} e^{\Gamma_l y} \right) \quad (68)$$

The following Eq. (69) is obtained from Eqs. (51) and (65).

$$\frac{H_{xl}}{H_{xs}} = \eta = -j \frac{kv_d \omega_c}{(\omega - j\omega_c)(\omega - kv_d)} \quad (69)$$

The above Eq. (69) shows that the  $s$ -wave component and  $l$ -wave component of electromagnetic fields have to be excited in order to be satisfied.

#### 4. EFFECTIVE PERMITTIVITY OF A SEMI-INFINITE SEMICONDUCTOR DRIFTING PLASMA

In this section, in order to derive the  $\omega$ - and  $k$ -dependent effective permittivity of semi-infinite semiconductor drifting plasma,  $E_y$ ,  $E_z$  and

$H_x$  components are classified into lamellar component and solenoidal component. The following group of equations was used in the analysis.

$$Y_s \equiv \frac{j\omega\varepsilon^*}{\Gamma_s}; \quad Y_l \equiv j\frac{\varepsilon v_d \Gamma_l}{k}; \quad \varepsilon^* = \varepsilon \left(1 - j\frac{\omega_c^*}{\omega}\right) \quad (70)$$

Here,  $Y_l$  and  $Y_s$  are the admittances of  $l$ -wave and  $s$ -wave, respectively.  $\varepsilon^*$  is the effective permittivity.

First, the expression for Eq. (51) of  $H_x$  is rewritten as follows, assuming  $\omega_c \cong \omega_c^*$ .

$$\begin{aligned} H_x &= \frac{j\omega\varepsilon}{k} \left( \frac{\Gamma_l}{k} \cdot \frac{kv_d}{\omega} A_l e^{\Gamma_l y} + \frac{k}{\Gamma_s} \left(1 - j\frac{\omega_c}{\omega}\right) A_s e^{\Gamma_s y} \right) = \frac{j\omega\varepsilon \Gamma_l v_d}{k\omega} A_l e^{\Gamma_l y} \\ &\quad + \frac{j\omega\varepsilon}{\Gamma_s} \left(1 - j\frac{\omega_c}{\omega}\right) A_s e^{\Gamma_s y} = \frac{j\varepsilon v_d \Gamma_l}{k} A_l e^{\Gamma_l y} + \frac{j\omega\varepsilon^*}{\Gamma_s} A_s e^{\Gamma_s y} \quad (71) \end{aligned}$$

$S$ -wave component of  $E_y$ ,  $E_z$  and  $H_x$  can be expressed as the following by referring to Eqs. (49), (50) and (71), respectively.

$$\begin{aligned} E_{sy} &= A_s^+ \frac{jk}{\Gamma_s} e^{\Gamma_s y} - A_s^- \frac{jk}{\Gamma_s} e^{-\Gamma_s y} = \frac{jk}{\Gamma_s} (A_s^+ e^{\Gamma_s y} - A_s^- e^{-\Gamma_s y}) \\ &= Y_s \frac{k}{\omega\varepsilon^*} (A_s^+ e^{\Gamma_s y} - A_s^- e^{-\Gamma_s y}) \quad (72) \end{aligned}$$

$$E_{sz} = A_s^+ e^{\Gamma_s y} + A_s^- e^{-\Gamma_s y} \quad (73)$$

$$\begin{aligned} H_{sx} &= -A_s^+ \frac{j\omega\varepsilon^*}{\Gamma_s} e^{\Gamma_s y} + A_s^- \frac{j\omega\varepsilon^*}{\Gamma_s} e^{-\Gamma_s y} = -\frac{j\omega\varepsilon^*}{\Gamma_s} (A_s^+ e^{\Gamma_s y} - A_s^- e^{-\Gamma_s y}) \\ &= -Y_s (A_s^+ e^{\Gamma_s y} - A_s^- e^{-\Gamma_s y}) \quad (74) \end{aligned}$$

$L$ -wave component of  $E_y$ ,  $E_z$  and  $H_x$  can also be expressed as the following by referring also to Eqs. (49), (50) and (71), respectively.

$$\begin{aligned} E_{ly} &= A_l^+ j\frac{\Gamma_l}{k} e^{\Gamma_l y} - A_l^- j\frac{\Gamma_l}{k} e^{-\Gamma_l y} = j\frac{\Gamma_l}{k} (A_l^+ e^{\Gamma_l y} - A_l^- e^{-\Gamma_l y}) \\ &= j\frac{Y_l}{\varepsilon v_d} (A_l^+ e^{\Gamma_l y} - A_l^- e^{-\Gamma_l y}) \quad (75) \end{aligned}$$

$$E_{lz} = A_l^+ e^{\Gamma_l y} + A_l^- e^{-\Gamma_l y} \quad (76)$$

$$\begin{aligned} H_{lx} &= -A_l^+ j\frac{\varepsilon v_d}{k} \Gamma_l e^{\Gamma_l y} + A_l^- j\frac{\varepsilon v_d}{k} \Gamma_l e^{-\Gamma_l y} \\ &= -j\frac{\varepsilon v_d}{k} \Gamma_l (A_l^+ e^{\Gamma_l y} - A_l^- e^{-\Gamma_l y}) = -Y_l (A_l^+ e^{\Gamma_l y} - A_l^- e^{-\Gamma_l y}) \quad (77) \end{aligned}$$

The calculation of  $H_x/E_y$  is performed as follows.

$$\frac{H_x}{E_y} = \frac{H_{sx} + H_{lx}}{E_{sy} + E_{ly}} = \frac{Y_s (A_s^+ - A_s^-) + Y_l (A_l^+ - A_l^-)}{\frac{kY_s}{\omega\varepsilon^*} (A_s^+ - A_s^-) + \frac{Y_l}{\varepsilon v_d} (A_l^+ - A_l^-)} = \frac{\omega\varepsilon}{k}$$

$$\begin{aligned}
 & Y_s (A_s^+ - A_s^-) + Y_l (A_l^+ - A_l^-) \\
 &= \frac{\omega \varepsilon}{k} \left[ \frac{k Y_s}{\omega \varepsilon^*} (A_s^+ - A_s^-) + \frac{Y_l}{\varepsilon v_d} (A_l^+ - A_l^-) \right] \\
 & Y_s \left( 1 - \frac{\varepsilon}{\varepsilon^*} \right) (A_s^+ - A_s^-) = Y_l \left( \frac{\omega}{k v_d} - 1 \right) (A_l^+ - A_l^-) \\
 & \frac{(A_l^+ - A_l^-)}{(A_s^+ - A_s^-)} = \frac{Y_s (1 - \frac{\varepsilon}{\varepsilon^*})}{Y_l \left( \frac{\omega}{k v_d} - 1 \right)} = \frac{\omega \varepsilon^*}{\Gamma_s} \frac{k}{\varepsilon v_d \Gamma_l} \frac{(1 - \frac{\varepsilon}{\varepsilon^*})}{\left( \frac{\omega}{k v_d} - 1 \right)} \\
 & \frac{(A_l^+ - A_l^-)}{(A_s^+ - A_s^-)} = \frac{\omega k^2 (\varepsilon^* - \varepsilon)}{\Gamma_s \Gamma_l \varepsilon (\omega - k v_d)} = -j \frac{k^2}{\Gamma_s \Gamma_l} \frac{\omega_c^*}{(\omega - k v_d)} \equiv K
 \end{aligned} \tag{78}$$

Then, the admittance at interface is determined as follows:

$$\begin{aligned}
 Y|_{y=0} &= -\frac{H_x}{E_z} = -\frac{[-Y_s (A_s^+ - A_s^-) - Y_l (A_l^+ - A_l^-)]}{(A_s^+ + A_s^-) + (A_l^+ + A_l^-)} \\
 &= \frac{Y_s (A_s^+ - A_s^-) + Y_l (A_l^+ - A_l^-)}{(A_s^+ + A_s^-) + (A_l^+ + A_l^-)}
 \end{aligned} \tag{79}$$

For the case of semi-infinite,  $A_s^- = A_l^- = 0$ . The following calculation is obtained by introducing Eq. (78) into (79).

$$\begin{aligned}
 Y|_{y=0} &= Y_s \frac{1 + \frac{Y_l}{Y_s} \frac{A_l^+}{A_s^+}}{1 + \frac{A_l^+}{A_s^+}} = Y_s \frac{1 - j \frac{\Gamma_s \varepsilon v_d \Gamma_l k^2 \omega_c^*}{\omega \varepsilon^* k \Gamma_s \Gamma_l (\omega - k v_d)}}{1 - j \frac{k^2}{\Gamma_s \Gamma_l} \frac{\omega_c^*}{(\omega - k v_d)}} = Y_s \frac{1 - j \frac{k v_d \omega_c^*}{(\omega - j \omega_c^*)(\omega - k v_d)}}{1 - j \frac{k^2}{\Gamma_s \Gamma_l} \frac{\omega_c^*}{(\omega - k v_d)}}
 \end{aligned} \tag{80}$$

Assuming that  $\frac{k v_d}{(\omega - j \omega_c^*)} \ll \frac{\omega_c^*}{(\omega - k v_d)}$ , then the admittance is obtained as follows.

$$Y|_{y=0} = \frac{j \omega \varepsilon}{\Gamma_s} \cdot \frac{1 - j \frac{\omega_c^*}{(\omega - k v_d)}}{1 - j \frac{k^2}{\Gamma_s \Gamma_l} \frac{\omega_c^*}{(\omega - k v_d)}} = \frac{j \omega \varepsilon_{eff}}{\Gamma_s} \tag{81}$$

Eq. (81) can also be obtained directly from Eqs. (67) and (68). Finally, the effective permittivity is drawn out from Eq. (81).

$$\varepsilon_{eff} = \varepsilon \frac{1 - j \frac{\omega_c^*}{(\omega - k v_d)}}{1 - j \frac{k^2}{\Gamma_s \Gamma_l} \frac{\omega_c^*}{(\omega - k v_d)}} \tag{82}$$

Here,

$$\omega_c^* = -j \frac{\omega_p^2}{\omega - k v_d - j \nu}; \quad \omega_p = \sqrt{\frac{q^2 n_0}{m^* \varepsilon}} \text{ (Plasma frequency)}$$



Again, the  $\omega$ - and  $k$ -dependent effective permittivity can also be expressed in the following form.

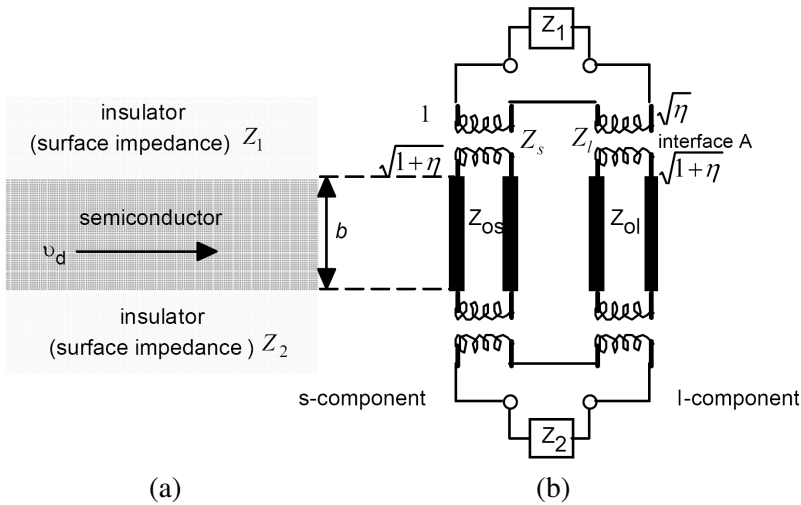
$$\varepsilon_{eff} = \varepsilon \frac{1 - \frac{\omega_p^2}{(\omega - kv_d)(\omega - kv_d - j\nu)}}{1 - \frac{k^2}{\Gamma_s \Gamma_l} \frac{\omega_p^2}{(\omega - kv_d)(\omega - kv_d - j\nu)}} \quad (83)$$

This effective permittivity is used to describe the dielectric response of the semiconductor plasma to the TM surface wave excitation. It is noted here that the derived effective permittivity presented in Reference [10] is only applicable to the two-dimensional electron gas (2DEG) structure but the derived effective permittivity as expressed in Eq. (83) is applicable to the semiconductor-insulator bulk structure. The details on the derivation of the effective permittivity for 2DEG structure can be found in Reference [20]. We have shown that the transverse decay constant of  $s$ -wave,  $\Gamma_s$  and longitudinal decay constant of  $l$ -wave,  $\Gamma_l$  for both structures are different which result in the different expression of effective permittivity. It can also be understood that the thermal velocity is related to those decay constants.

## 5. TRANSMISSION LINE REPRESENTATIONS FOR MULTI-LAYERED STRUCTURES

In Sections 2, 3 and 4, the properties of electromagnetic fields and effective permittivity excited by drifting plasma waves in semiconductor-insulator single structure are derived. These parameters are basically the main parameters to be used in further analysis to predict or indicate the interaction between propagating electromagnetic waves and drifting carrier plasma waves in semiconductor. The examples of analysis procedures can be found in References [10] and [12], where we presented the formulation to calculate the admittance of interdigital gate slow-wave structure on bulk semiconductor structure and semiconductor with 2DEG structure, respectively, in order to understand the condition of interaction. In this section, the another analysis technique on the multi-layered structure using transmission line representations is presented since multi-layered structure is also an interesting structure for fabricating such a so-called plasma wave device. Those derived basic parameters can be directly applied in this transmission line representation to calculate the surface impedance and hence, the conductance characteristics.

This section describes an analysis on the multi-layered structure of insulator-semiconductor-insulator (I-S-I) structure by using transmission line representations. The analyzed structure and its equivalent



**Figure 3.** (a) Schematic insulator-semiconductor-insulator structure and (b) its transverse equivalent circuit.

circuit is shown in Fig. 3. In this structure, the semiconductor layer with a thickness of  $b$  is sandwiched by two insulator layers having surface impedance of  $Z_1$  and  $Z_2$ . It is assumed here that the  $s$ -wave component and  $l$ -wave component are excited independently. Using equivalent transmission line representation, the characteristic impedances of  $s$ -wave,  $Z_{os}$  and  $l$ -wave,  $Z_{ol}$  in semiconductor is given as

$$Z_{os} = \frac{\Gamma_s}{j\varepsilon(\omega - j\omega_c)} \tag{84}$$

$$Z_{ol} = \frac{k}{j\varepsilon v_d \Gamma_l} \tag{85}$$

The problem that may occur during the determination of surface impedance of semiconductor layer is the contribution level of  $s$ -wave and  $l$ -wave. It was mentioned in the previous section that  $s$ -wave and  $l$ -wave have to be excited in order to satisfy Eq. (69). Due to this condition, the surface impedance,  $Z_s$  and  $Z_l$  determined from  $s$ -wave and  $l$ -wave will be contributed by a ratio of  $1/(1 + \eta)$  and  $\eta/(1 + \eta)$ , respectively as shown in Fig. 3.

Assuming that the surface impedance at the interface  $A$  as shown in Fig. 3 is  $Z$  and the surface impedance of dielectric at the back side is  $Z_2$ , then  $Z$  is given as

$$Z = \left( \frac{Z_2 + Z_{sh}}{Z_2 + Z_{op}} \right) Z_{op} \tag{86}$$

Here  $Z_{sh}$  represents the surface impedance at interface  $A$  when  $Z_2$  is made short-circuited ( $Z_2 = 0$ ) while  $Z_{op}$  represents the surface impedance at interface  $A$  when  $Z_2$  is made open-circuited ( $Z_2 = \infty$ ).

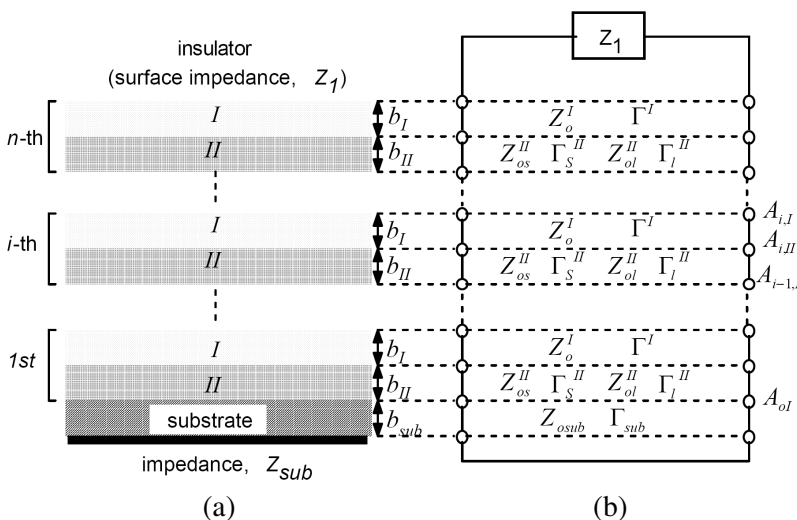
For simplicity,  $s$ -wave and  $l$ -wave are assumed to be short-circuited, then  $Z_{sh}$  and  $Z_{op}$  can be expressed as follows.

$$Z_{sh} = \frac{1}{1 + \eta} Z_{os} \tanh \Gamma_s b + \frac{\eta}{1 + \eta} Z_{ol} \tanh \Gamma_l b \tag{87}$$

$$Z_{op} = \frac{1}{1 + \eta} Z_{os} \coth \Gamma_s b + \frac{\eta}{1 + \eta} Z_{ol} \coth \Gamma_l b \tag{88}$$

Figure 4 shows the structure where the insulator layer and the semiconductor layer are structured to form a multi-layered structure. Again, the surface impedance  $Z$  at the interface  $A$  is derived. Here, ‘ $I$ ’ represents the insulator layer while ‘ $II$ ’ represents the semiconductor layer.  $Z_{os}^{II}, Z_{ol}^{II}, \Gamma_s^{II}$ , and  $\Gamma_l^{II}$  are the characteristic impedance and the propagation constant of  $s$ -wave and  $l$ -wave in semiconductor layer, respectively. In the other hand,  $Z_o^I$  and  $\Gamma_I$  are the characteristic impedance and the propagation constant in insulator layer, respectively.

The surface impedance  $Z_{i,I}, Z_{i,II}$  and  $Z_{i-1,I}$  at the  $A_{i,I}, A_{i,II}$  and  $A_{i-1,I}$  interface, respectively, are given as follows. Here,  $i$  is the number of structure where a pair of semiconductor layer and insulator



**Figure 4.** (a) Schematic multi-layered structure and (b) its transverse equivalent circuit.

layer represents one structure.

$$Z_{i,I} = Z_o^I \frac{Z_{i,II} + Z_o^I \tanh \Gamma_s^{II} b_{II}}{Z_o^I + Z_{i,II} \tanh \Gamma_s^{II} b_{II}} \quad (89)$$

$$Z_{i,II} = \left( \frac{Z_{i-1,I} + Z_{sh}}{Z_{i-1,I} + Z_{op}} \right) Z_{op} \quad (90)$$

$$Z_{sh} = \frac{1}{1 + \eta} Z_{os}^{II} \tanh \Gamma_s^{II} b_{II} + \frac{\eta}{1 + \eta} Z_{ol}^{II} \tanh \Gamma_l^{II} b_{II} \quad (91)$$

$$Z_{op} = \frac{1}{1 + \eta} Z_{os}^{II} \coth \Gamma_s^{II} b_{II} + \frac{\eta}{1 + \eta} Z_{ol}^{II} \coth \Gamma_l^{II} b_{II} \quad (92)$$

The surface impedance of substrate at  $A_{0,I}$  interface is given by

$$Z_{sub} = Z_{0sub} \tanh \Gamma_{sub} b_{sub} \quad (93)$$

Here,  $\Gamma_{sub} = \sqrt{k^2 - \omega^2/c_{sub}^2}$ . Hence, the surface impedance at interface  $A$  can be determined from Eqs. (89)–(93). From the obtained surface impedance, the conductance characteristics can be determined. The phenomena of negative conductance exist when the coupling interaction between the propagating electromagnetic waves and drifting plasma waves is achieved. It has been presented that the negative conductance characteristics occur when the drift velocity of carriers is slightly exceeds the phase velocity of electromagnetic waves [10].

## 6. CONCLUSION

An improved and reliable method to analyze the properties of semiconductor plasma in a semiconductor-insulator structure based on the transverse magnetic (TM) mode analysis was presented. Two waves components (quasi-lamellar wave and quasi-solenoidal wave), electromagnetic fields ( $E_y$ ,  $E_z$  and  $H_x$ ) and  $\omega$ - and  $k$ -dependent effective permittivity were derived. A method to determine the surface impedances in semiconductor-insulator complex structure using equivalent transmission line representation method was also presented since a multi-layered structure is also a promising structure for fabricating a plasma wave device.

## ACKNOWLEDGMENT

This work was partly supported by Ministry of Science, Technology and Innovation (MOSTI), Malaysia and Ministry of Higher Education (MOHE), Malaysia through Science Fund Vote 03-01-06-SF0283

and Fundamental Research Grant Scheme Vote 78205 & 78417, respectively.

## APPENDIX A.

The list of symbols and their definitions are summarized as follows.

- $\vec{v}$ : electron mean drift velocity,
- $q$ : electronic charge,
- $m^*$ : effective mass of electron,
- $\vec{E}$ : electric field in semiconductor,
- $\vec{H}$ : magnetic field in semiconductor,
- $\mu_o$ : permeability of free-space,
- $n$ : electron density,
- $k_B$ : Boltzman constant,
- $T_e$ : electron temperature,
- $\tau$ : relaxation time of electrons,
- $\epsilon$ : dielectric permittivity of semiconductor.
- $\vec{D}$ : electric flux density,
- $\vec{B}$ : magnetic flux density,
- $\rho$ : charge density,
- $\vec{j}$ : conductive current density,
- $\mu$ : mobility of semiconductor  $\mu = \frac{q\tau}{m^*}$ ,
- $D$ : diffusion constant  $D = \frac{kT}{m^*\nu}$  or  $D = v_{th}^2 \tau$ ,
- $c$ : light velocity in semiconductor  $c = \frac{1}{\sqrt{\epsilon\mu_o}}$ ,
- $\omega_c$ : dielectric relaxation frequency  $\omega_c = \frac{q\mu n_o}{\epsilon}$ ,
- $v_{th}$ : mean thermal velocity  $v_{th} = \sqrt{\frac{k_B T_e}{m^*}}$ ,
- $\nu$ : collision frequency  $\nu = \frac{1}{\tau}$ ,
- $\omega$ : angular frequency,
- $\omega_p$ : plasma frequency.

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