ANALYSIS OF TERAHERTZ-INDUCED OPTICAL PHASE MODULATION IN A NONLINEAR DIELECTRIC SLAB

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Abstract—Frequency shift of the spectrum of an incident optical pulse by an intense THz pulse inducing cross-phase modulation (XPM) in a nonlinear dielectric slab is analyzed. The effect is predicted with a high degree of accuracy using the well-known transmission line matrix (TLM) technique. In this research, to model the THz-induced temporal and spatial variation of the dielectric permittivity of the nonlinear dielectric slab, the transmission lines of the TLM method are loaded with open shunt stubs. The parameters of the stubs are modified in accordance with the refractive index variation of the dielectric slab, here ZnTe, induced by the strong THz pulse. The obtained numerical results are verified with a recently reported experimental work.

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1. INTRODUCTION

Nonlinear optics plays an important role in generating, controlling, and modulating laser pulses. To this end, self-phase modulation (SPM) [1] or cross-phase modulation (XPM) in a nonlinear medium [2–8] is commonly used to modulate optical pulses. Alfano et al. are the first to demonstrate XPM [2]. They showed that propagation of intense picosecond pulses in bulk glasses leads to a spectral broadening of a copropagating weaker pulse. It has been shown that spectral broadening of a single-frequency laser pulse induced by a chaotic laser pulse in a birefringent single-mode optical fiber can generate laser pulses of variable bandwidth [9]. In addition, XPM is useful in compressing weak pulses [10]. Alternatively, ultrafast pulses copropagating in a nonlinear dispersive medium experience a significant shift of their center frequencies. This effect has been attributed to the combined effect of XPM and pulse walk-off [11]. The frequency shift of optical signals as high as some nanometers can be utilized for optical frequency switching [12].

Cross-phase modulation is commonly assigned to the Kerr effect. Yet it has been shown experimentally that a strong THz electric field induces XPM in a weak optical beam through the Pockel effect [13]. The goal of this paper is to develop a computational technique to precisely evaluate the frequency shift of an optical beam as a result of XPM in a nonlinear slab through the Pockel effect. The adopted computational technique is based on the TLM method. In this method, the ZnTe slab is first discretized by cells as small as one tenth of the optical wavelength. Each cell is then modeled by a transmission line loaded with a shunt open stub. The change of ZnTe refractive index induced by the copropagating THz pulse is modeled by varying the shunt stub parameters. In the following section, this technique will be explained in details. The obtained results along with discussions will be in Sections 3 and 4.

2. MODELING OF PROPAGATION THROUGH A NONLINEAR SLAB

A nonlinear dielectric medium is characterized by a nonlinear relation between the polarization vector and the electric field. It is expressed as

$$P_i = \epsilon_0 \sum_{j=1}^{3} \chi_{ij} E_j + 2 \sum_{j=1}^{3} \sum_{k=1}^{3} d_{ijk} E_j E_k + 4 \sum_{j=1}^{3} \sum_{k=1}^{3} \sum_{l=1}^{3} \chi_{ijkl} E_j E_k E_l + \dots,$$

(1)

where P_i is the ith component of the instantaneous polarization vector, and E_i is the ith component of the instantaneous electric field of the optical beam. The indices i, j, k, and l can be x, y, or z. Here, χ_{ij} is the linear susceptibility. d_{ijk} and χ_{ijkl} are the second- and third-order nonlinear susceptibilities, respectively [14].

As shown in Figure 1, the nonlinear dielectric slab, here ZnTe, occupies the region $0 < x < L$. The THz and optical field are plane wave propagating in the x direction and have only z component. Since the THz and optical electric field have only one component, Equation (1) can be rewritten as

$$P_z = \epsilon_0 \chi^{(1)} E_z + 2d \cdot E_z^2 + 4\chi^{(3)} E_z^3 \qquad (2)$$

where $\chi^{(1)}$, d, and $\chi^{(3)}$ are the linear, second-order, and third-order susceptibilities, and E_z is the only non-zero electric field component in the z direction. The change of the relative dielectric permittivity induced by the THz pulse in ZnTe is expressed by $\Delta\epsilon_r = \Delta\epsilon_{r1} + \Delta\epsilon_{r2}$ where $\Delta\epsilon_{r1}$ and $\Delta\epsilon_{r2}$ are the relative dielectric permittivity changes caused by the Pockel and Kerr effects, respectively [13]. The $\Delta\epsilon_{r1}$ can be obtained from the second-order term of the polarization vector, i.e.,

$$P_{zNL}(t) = 2d \cdot E_z^2 \qquad (3)$$

Since the optical pulse width is much smaller than the THz pulse width, variation of the THz pulse amplitude in the whole time duration of the optical pulse is neglected. With the use of Equation (3), the change of the relative dielectric permittivity at the optical frequency is obtained as

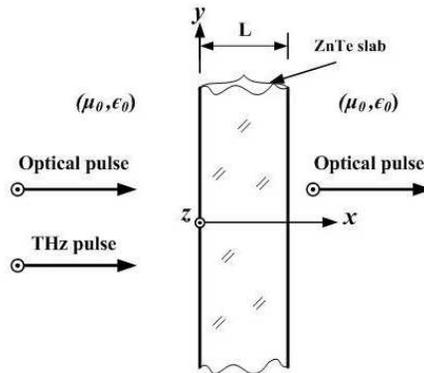$$\Delta\varepsilon_{r1} = \frac{4d}{\epsilon_0} E_T(x, t) \qquad (4)$$

Figure 1. The optical and THz pulse in the form of a plane wave applied to the ZnTe slab covering $0 < x < L$.

where ϵ_0 is the vacuum dielectric permittivity, and $E_T(x,t)$ is the calculated THz electric field as a function of time and space. By using the same process for the third-order nonlinearity polarization, the $\Delta\varepsilon_{r2}$ is achieved as $\Delta\varepsilon_{r2} = \frac{12\chi^{(3)}}{\epsilon_0}E_T^2(x,t)$. From [13], we know that the second- and third-order susceptibilities of ZnTe are $d = 4.5 \times 10^{-11}\epsilon_0$ and $\chi^{(3)} = 0.75 \times 10^{-19}\epsilon_0$, respectively. For $E_{THz} < 5 \times 10^7 \ \frac{V}{m}$, the value of $\Delta\varepsilon_{r1}$ satisfies $\Delta\varepsilon_{r1} \gg \Delta\epsilon_{r2}$. Therefore, the Pockel effect is dominant in the phase shift of the optical beam. The refractive index of ZnTe at the THz and optical frequency is $n_{THz} = 3.178$ and $n_{Opt} = 2.85$, respectively [15]. Because of the dispersion, the THz and optical signal propagate with different phase velocities. In addition, as the pulses contain various frequency components, these pulses propagate with different group velocities; thus, their temporal relative position varies across the ZnTe slab. This is known as the walk-off effect.

To calculate the wavelength shift of the optical pulse in the presence of the THz pulse, first the THz electric field in the ZnTe slab is evaluated when the optical pulse is absent. For this calculation, we expand the THz pulse in terms of N frequency components. Each of these time harmonic components of the THz pulse is then used to determine the electric field inside the slab. After superposition of these single solutions, the electric field of the THz pulse at any position inside the slab is obtained as a function of time. Figure 2 illustrates the obtained result. Then, variation of the relative permittivity is
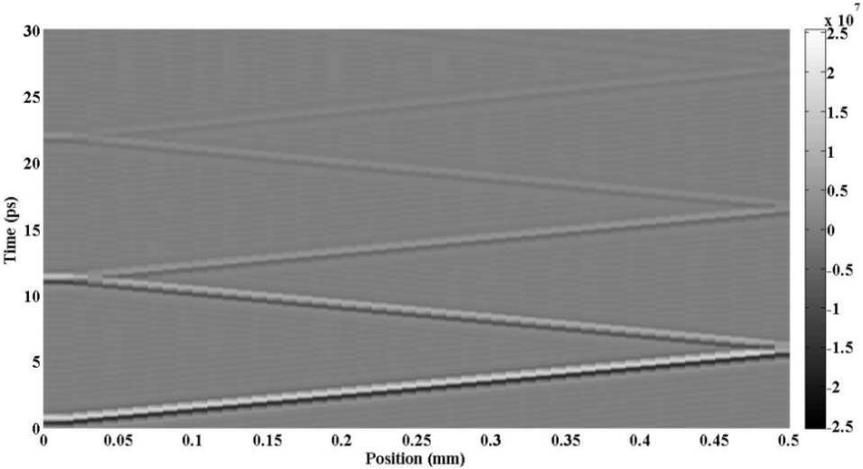

Figure 2. THz electric field inside the ZnTe slab as a function of time and space in the absence of the optical field.

calculated using the computed THz signal inside the slab. This is done using Equation (4). After the calculation of the THz-induced variation of the relative permittivity, the shift of the optical beam center frequency is determined with the help of the TLM method. The TLM method based on the Huygens principle is a powerful numerical method for solving Maxwell's equations in the time domain [16]. To do the numerical calculation, first the ZnTe slab is discretized by cells (shown in Figure 3). To minimize the numerical dispersion error, the cell size must be smaller than one-tenth of the wavelength at the maximum frequency of interest, i.e., $\lambda_{\min}$. In other words, a useful "rule of thumb" is that cell thickness $\leq \frac{\lambda_{\min}}{10}$ [18]. Therefore, we have used cells of 1/10 of the optical wavelength. One can choose the cell thickness smaller than the value selected in the manuscript, but the computation time increases significantly. In fact, there is a trade-off between the cell thickness and computation time.

In the next step, the TLM method replaces the discretized computational domain by a network of interconnected transmission lines. It is shown that there is a correspondence between the voltages and currents of the transmission line network and the electric and magnetic fields of Maxwell's equations [17]. It should be mentioned that the main contribution of this work is the modeling of time-variant
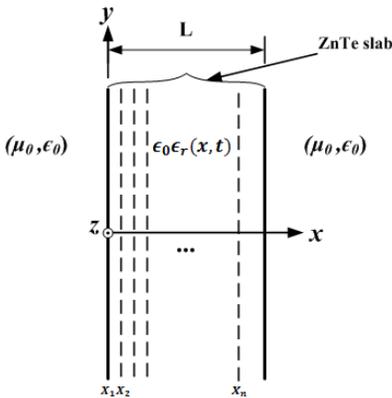

Figure 3. ZnTe slab is discretized by celles as small as a tenth of the optical wavelength. The relative permittivity of the slab varies as a function of space and time because of the THz pulse.
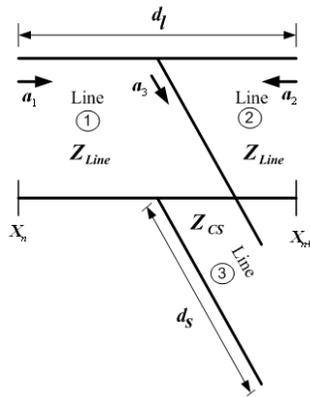
Figure 4. Transmission lines for the cell covering $X_n < x < X_{n+1}$. The change of the refractive index of ZnTe due to the THz pulse is modeled with the variation of the parameters of the shunt open stub.

dielectric permittivity. This is required in the analysis of a problem containing nonlinear materials, particularly when there are two or more time-harmonic sources. The analysis of time-variant medium is made possible by adding open shunt stubs with time varying characteristic impedance. The characteristic impedance of the shunt stubs is updated in every time step. In this work, the parameters of the stubs are changed in accordance with the refractive index variation of the ZnTe slab induced by the strong THz pulse. Figure 4 shows the three transmission lines used in a TLM cell starting at X_n and ending at X_{n+1}. Lines 1 and 2 have the same characteristic impedance Z_{Line} and a length of $\frac{d_l}{2}$. The characteristic impedance of the shunt stub, i.e., Line 3, and its length are Z_{CS} and d_s, respectively.

The change of the refractive index corresponds to a capacity per unit length of the following value $C = \varepsilon_0 \Delta \varepsilon_1$. The capacity can be modeled with a shunt open stub of the length d_s. The length of the open stub is chosen so that the time needed for a_3 going through the stub and back to the central node is equal to the time required for a_1 or a_2 to get to the central node. It means that $d_s = \frac{d_l}{4}\sqrt{\frac{\epsilon_{opt}}{\Delta \varepsilon_1}}$, where ϵ_{opt} is the dielectric permittivity at the optical frequency. Therefore, the characteristic impedance of the shunt open stub should be $Z_{Cs} = \frac{1}{4}\frac{1}{\varepsilon_0 \Delta \varepsilon_1 \upsilon_{opt}}$, where υ_{opt} is the velocity of light in the slab at the wavelength of the optical signal.

In the TLM method, both forward and backward voltage waves on each transmission line are updated in Δt time steps. To explain this procedure, we pay attention to Figure 4. Here, a_1, a_2, and a_3, i.e., the forward propagating voltages, arrive at the junction after Δt, where they are reflected according to the scattering parameters of the junction. In a general form this can be written as:

$$[a_j]^r = [S_{ji}]\,[a_i]^i \tag{5}$$

where $[a]^r$ and $[a]^i$ are the matrices of reflected and incident voltages, respectively. The elements of the scattering matrix $[S_{ji}]$ are the voltage reflection and transmission coefficients at each junction. The reflected voltage wave at time k from junction n and traveling to the left becomes incident on junction $n-1$ from the right at time $k+1$, i.e.,

$$_{k+1}a^i_{(n-1)} = {}_k a^r_n \tag{6}$$

By the same logic the incident pulse from the left at junction n and at time $k+1$ is the reflected voltage wave on junction $n-1$ from the right at time k, i.e.,

$$_{k+1}a^i_n = {}_k a^r_{n-1} \tag{7}$$

By iterative applications of this procedure, the voltage waves are updated in multiples of Δt. For a detailed description of the TLM

algorithm, the reader is referred to [18]. By calculating the total voltages of the transmission line network in the successive time intervals, the time variation of the optical beam transmitted through the ZnTe slab is determined. As will be discussed in the following section, the most prominent effect of the nonlinear interaction in the slab is the wavelength shift of the optical beam which is mainly dependent on the time delay between the optical and THz signals.

3. RESULTS AND DISCUSSIONS

In this work, it is assumed that the incident Gaussian THz pulse has a maximum amplitude of $3.5 \times 10^7 \frac{V}{m}$, a center frequency of 1 THz, and a time duration of 1 ps, while the incident Gaussian optical pulse has a center wavelength of $\lambda_{Opt} = 795$ nm and a time duration of 120 fs. The material of the slab is ZnTe, and its thickness is 0.5 mm. Figure 5 shows the spectral profiles of the optical pulse propagating with and without the THz field in the ZnTe slab. The solid-line spectrum is for the case of the optical beam propagating in the absence of the THz pulse in the crystal. The blue and red-shifted spectra correspond to the spectra of the optical beam copropagating with the THz pulse for the time delay 0.3914 and 0.8418 picoseconds, respectively .

In addition, the induced-wavelength shift of the optical signal versus the input time delay between the THz and optical signals is
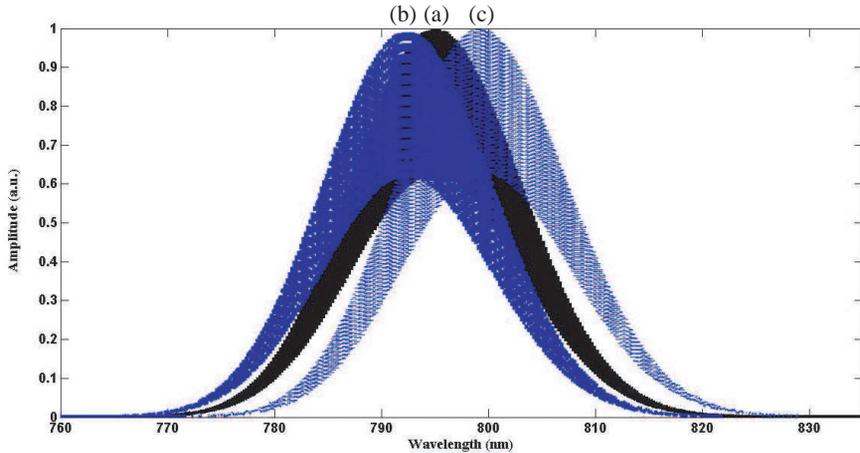

Figure 5. (a) Optical beam propagating in the ZnTe slab with no THz signal, (b) the blue shift corresponds to the case in which the input time delay is 0.3914 ps, (c) the red shift corresponds to the case in which the input time delay is 0.8418 ps.
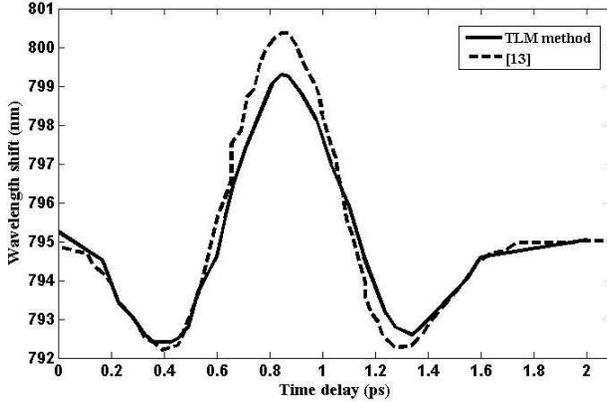
Figure 6. THz induced-wavelength shift of the optical beam versus the time delay between the THz and optical signal, the dashed-line curve corresponds to the result of [13] and the solid-line one shows the result of the TLM method.

shown in Figure 6. The obtained result using TLM method is well-matched with the recently reported experimental work [13]. It should be mentioned that the theoretical analysis presented in [13] has been based on simple assumptions. For instance, the total phase shift is computed using

$$\Delta\varphi\left(t\right) = \frac{2\pi}{\lambda_0} \int_0^L \Delta n[E_{THz}(t - \beta z)]dz \tag{8}$$

where λ_0 is the central wavelength of the probe pulse. β is a walk-off parameter, and L is the length of the ZnTe slab. This is obviously a simplified expression for $\Delta\varphi$ because it does not include several effects such as the reflection from the slab-air interface, nor the standing THz wave inside the slab, which evidently varies the refractive index in its own turn. The authors of [13] have used the simplified relation for $\Delta\varphi$ for different walk-off parameters. They noticed that the measured spectral shifts are compatible with the results of the above formula when the walk-off parameter is $\beta = 1\,\frac{\text{ps}}{\text{mm}}$. Therefore, their method is not a rigorous one to predict the frequency shift of the optical signal, because the value of β is unknown without measurement. But we have applied the TLM technique after some modifications to solve the time-variant problem of transmission through the ZnTe slab with a high degree of accuracy such that our results are compatible with the measurement results without additional assumptions. Note that our method can also be applied to more complex configurations for example a configuration of several slabs, whereas the simple $\Delta\varphi$ formula of [13]

is unable to deal with these and many other configurations.

4. CONCLUSION

In conclusion, the frequency shift of the optical beam as a result of the XPM in a ZnTe slab has been predicted accurately with the help of the TLM method. The simulated result shows good agreement with those of the reported experimental work. This agreement shows that the proposed simulation procedure is accurate. In other words, one may first calculate the THz electric field inside the slab as a function of time and space. From the THz field distribution in the slab, the temporal and spatial variation of the permittivity is computed. Then the effect of ZnTe refractive index variation on the optical pulse is evaluated.

Being a time-domain method of analysis, our proposed method is applicable to a large number of cases including but not limited to single cycle terahertz and optical pulses with different center frequencies, time durations, amplitudes, and waveforms. As long as the second-order susceptibility of the nonlinear material and its refractive index at THz and optical frequency are known, the method of this work can be utilized to analyze the nonlinear behavior of the problem. However, to verify our method, we had to apply it to a specific problem for which experimental data were available. The experimental work of [13] was a suitable candidate for this purpose. Moreover, this work forms the basis of many THz detection systems.

ACKNOWLEDGMENT

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