ELEMENT POSITION PERTURBATION FOR A NAR-ROW SPOT BEAM WITH APPLICATIONS TO SATEL-LITE COMMUNICATION ANTENNAS

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Abstract—Design of array antennas for satellite applications is always a trade-off between physical constrains and pattern requirements. In this paper, the focus is on the design of a large array antenna for earth coverage applications using spot beams. The array antenna has a diameter of 1 m and consists of circular polarized horn antennas positioned in a non-uniform grid. By using a binary coded genetic algorithm (BCGA) the desired element positions and their excitations are optimized to fulfill the pattern requirements. In addition thinning has been used to study the possibility of maintaining good antenna performance when reducing the number of elements. The proposed antenna design has robust side lobe level, beam width and gain; all remain virtually unchanged under a change of operating frequency $\pm 7\%$ and under lobe steering over earth $\pm 8.8^{\circ}$.

1. INTRODUCTION

Antennas for satellite communication are of interest for worldwide coverage, and some examples of recent applications can be found in [1-4], see also [5]. In the work presented here the focus is on the design of a large array antenna for use on satellites. One main issue is the overall weight of the antenna, roughly proportional to the number of antenna elements, which is traded against antenna performance.

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Consequently, it is very important to design arrays with as few elements as possible while still trying to maintain the radiation pattern design goals. Different optimization realizations of the antenna are discussed to examine the trade-off possibilities for the design.

We consider the design of a one meter in diameter array antenna intended for a geostationary orbit. The array should be used to cover the surface of the earth with spot beams. From the altitude 36000 km the earth covers a cone angle of 17.6°. The downlink band is 17.7–20.2 GHz and each channel has 250 MHz band width. The goal is to create a number of narrow spot beams in a hexagonal pattern as shown in Figure 1, where each spot beam has a 1° separation and a half-power beam width of 1.16°. The beams should be steerable over the $\pm 8.8°$ region as to cover the visible earth with preferably small distortions. The resulting radiation pattern design is created by using digital beam forming and a four or a seven frequency scheme reuse by scanning multiple single spot beams; see Figure 1 for the four frequency reuse case.

In the current work, three goal parameters are considered for the antenna pattern: The 3 dB beam width (BW), the side-lobe level (SLL), and the directivity. It is well known that these parameters depend strongly on the array aperture, tapering, the number of elements, and the element grid and spacing, see e.g., [6] for a stochastic treatment. The goals cannot be optimized independently and to establish theoretical values of the inter-relation between the above parameters a continuous source aperture is considered. To use an idealized aperture with a *continuous* source will give an overestimate of the obtainable values for a *discrete* sampling as represented by the antenna elements. For a circular array antenna with a diameter of 1 m. assuming a radial tapering of the idealized current across the aperture. we find [5, 7] a 3 dB beam width of 1.15° , a side-lobe level of -24.6 dB and a directivity of 44.7 dB for the center frequency (18.95 GHz). It is desired to approach these quantities over the entire down-link spectrum with as few elements as possible. Thus, the goal of this work is to investigate the possibility of creating a high gain single spot beam array pattern with as low side-lobe level as possible, in order to have a high isolation between the same frequency cells.

From a manufacturing point of view a regular grid is preferable. However, due to expected large element spacing in this particular design, grating lobes would dominate the array pattern. To suppress such grating lobes within the $\pm 8.8^{\circ}$ earth coverage area one can use a non-uniform grid, see e.g., [8]. An additional benefit of using different inter-element spacing in the grid layout is that it helps improving the robustness for frequency change and beam scanning [1]. The proposed



Figure 1. Four frequency reuse pattern for earth coverage.

antenna elements to be used in the array are circular polarized horns with a diameter of 15 mm. They are chosen since they are robust high gain antennas known to have a rather low element to element coupling for inter element distances of λ or greater see e.g., [9].

To succeed with the pattern design a good synthesis procedure is needed. There are a large number of antenna pattern synthesis methods like Dolph-Chebyshev, Taylor methods [7] or alternating projections [10]. Unfortunately, all of them optimize the excitation for a fixed and given element position layout. Here, a critical factor in the design is the element positions which must be included in the optimization procedure. Thus we need to use a non-standard synthesis method; in the present case we use an evolution algorithm.

The present paper is concerned with antenna design and antenna properties such as that the robustness of scanning and beam steering and frequency sweep, rather than design of evolution algorithms. Evolution based algorithms for electromagnetic problems have become well-known, and applied to a wide range of areas, see e.g., [11]. Such algorithms applied to pattern synthesis have appeared e.g., as pattern property perseverance starting from a linear equidistantly spaced array through thinning for the linear and square array antennas see e.g., [12, 13]. In the present paper, the main optimization improvement is due to inter-element distance perturbation on a 2D circular disk jointly with thinning. Pattern perturbation has also been studied in see e.g., [14, 15] for arrays with elements on a line.

The layout of this paper is as follows; we start by looking at the problem formulation (Section 2), then the genetic algorithm is considered with focus of the modifications needed to solve this particular problem (Section 3). In Section 4 some results are shown and the paper is concluded in Section 5.

2. PROBLEM FORMULATION

The upcoming optimization is to maximize a cost function weighting the directivity, the beam width and the SLL's against element positions and excitations. These quantities are all interrelated through the electric far-field formula given by

$$\bar{E}_{Total}(\bar{R}) = \frac{e^{-ikR}}{R} \bar{\varepsilon}_{Element}(u, v) F(w, u, v) ,$$

$$w = (w_1, w_2, \dots w_N) , \ u = \sin\theta\cos\phi, \ v = \sin\theta\sin\phi, \ (1)$$

where F is the array factor, w, is the excitation complex vector, and (u, v) are the directional cosines. The element pattern, $\bar{\varepsilon}_{Element}$, is the isolated element pattern of the horn antenna[†]. An idealized horn antenna is simulated in HFSS at 18.95 GHz. To use this pattern in the optimization, an idealization of its response is given by the φ -symmetric antenna pattern

$$|\bar{\varepsilon}(\theta)| = 3.1e^{0.33\theta^3 - 1.6\theta^2 + 0.18\theta} \qquad \theta \in [0,\pi].$$

Now, consider an array of elements where each element has the position (d_{xn}, d_{yn}) . The array factor is then given by:

$$F(w, u, v) = \sum_{n} w_n e^{ik(d_{xn}u + d_{yn}v)}.$$
(3)

To reduce the optimization problem in size we use a mirror symmetric arrangement of elements. Hence, N + 1 elements are placed in the first quadrant of the 1 meter disc and subsequently mirrored along the x- and the y-axes into the other three quadrants, see Figure 2. The symmetry introduces the following simplification in the array factor.

$$F(w, u, v) = 2\sum_{n=0}^{N} w_n \left[\cos \left(k (d_{xn}u + d_{yn}v) \right) + \cos \left(k (d_{xn}u - d_{yn}v) \right) \right].$$
(4)

The mirror symmetric algorithm introduces generically a non-uniform element density across the mirror axes. In order to avoid such a density problem we introduce elements that initially are placed exactly on the mirror axes. These elements are then allowed to move along the respective mirror axes only. Elements that are not positioned on the mirror axes are allowed to move in both the x- and the y- directions. Figure 2 shows an example of the upper right corner of an optimized array geometry.

 $^{^\}dagger$ Note, mutual coupling is not included in the analysis. It is assumed that the coupling is low enough when having large inter-element spacing as is the case here.



Figure 2. Array geometry, the 126 elements in the first quadrant is mirrored along the axes to generate the 497 element case (with center element). The color corresponds to the normalized excitation amplitude, and the corresponding radiation pattern is given in Figure 3.

3. OPTIMIZATION BY A GENETIC ALGORITHM

As a synthesis method we use a binary coded genetic algorithm (BCGA), to optimize the desired element positions and their excitations to get the spot beam pattern. An additional feature with this algorithm is the possibility of including element thinning [12, 13, 16] of the array, of interest to additionally reduce the number of elements while maintaining the desired pattern properties [10]. The thinning algorithm, however, increases the search space considerably. In order to perform thinning, we introduce a binary variable b_n which equals one if the element is active and zero if the element is switched off. Thus modifying the array factor to:

$$F(w, u, v) = 2\sum_{n=0}^{N} b_n w_n [\cos\left(k(d_{xn}u + d_{yn}v)\right) + \cos\left(k(d_{xn}u - d_{yn}v)\right)].$$
(5)

Before starting the optimization an initial grid layout is defined. In this case, a regular grid is used, i.e., elements are placed $a\lambda$ apart on a square of side $\xi^{\ddagger}/2 - \Delta$ where $a \in [1.5, 2.5]$ and Δ is the absolute

^{\ddagger} ξ is the radius of the circular array.

value of the maximum distance each element can be perturbed. The elements are initially placed on a square with side 1 m; the elements which lie outside the disc with radius $\xi = 0.5$ m are removed.

To rank individual realizations in the optimization population we use a cost function that aims to obtaining a narrow spot beam with minimum side lobe levels. The cost function, which then should be maximized, is defined as follows

$$C(w, b, d_x, d_y) = 20 \log \left(\frac{\max(\left| \bar{E}_{Total} \right|)}{\max(\left| \bar{E}_R \right|)} \right).$$
(6)

Here R is an annulus around the center spot beam of inner radius 1°, which is the region of the angular variables where we want to minimize the SLL's. The unknown variables are the excitation vector w, the thinning vector b, and the position vectors d_x , d_y . The variables are binary coded to be suitable for a genetic algorithm and then included into a chromosome, S. The chromosome consist of two parts $S = (S_1, S_2)$ where S_1 contains the vectors $w = w_R + iw_I$, d_x , d_y for which each element (w_R, w_I, dx, dy) in the vectors are encoded using 6 bits. The second part S_2 contains the *b*-vector which is digital in character.

The BCGA algorithm is rather standard; see [10, 17-21]. However, for completeness we shortly discuss its main elements. Using the cost-function (6) we rank the randomly generated members of the population and keep the 50% fittest. Elitism [17] is then used to ensure that the population has a monotonic increase in the performance of the BCGA. The next generation is generated by selecting *parents* via a tournament selection procedure. Crossovers between the selected parents yield off springs resulting in a new population of parents and off springs. Finally, we apply mutation with a mutation rate of 0.1. The numbers of initial chromosomes were taken to be 200.

For S_1 , we apply a standard single bit crossover with a rate 0.8 and in the mutation step one bit is inverted. However, for S_2 a modified procedure is needed to keep the total number of elements fixed in the array. The crossover and the mutations are modified according to the following two algorithms respectively. At a certain bit number, selected randomly, the S_2 crossover algorithm for e.g., the father chromosome begins with an ordered pair partitioning of his chromosome starting from the chosen bit number. A similarly partition is done for the mother chromosome. Now, each corresponding partitioned pair in the mother and father chromosome is compared with each other by comparing the summed bit state. If they have equal summed bit states they are exchanged. e.g., a father pair of (1, 0) can be exchanged with the ordered corresponding mother pair if it is either (0, 1) or

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(1, 0). Similarly a father pair (1, 1) can only be exchanged with a mother (1, 1) state (null operation). The mutation algorithm for S_2 is as follows: given a (new) randomly chosen bit position we apply a randomly selected permutation among the remaining bits in the chromosome using Matlab's permute command.

4. RESULTS

Applying the above described algorithm, with and without thinning, several designs have been investigated. The main difference between the considered cases is the number of elements in the circular array and the interval in which each element is allowed to be perturbed. Then, the robustness of the set-up has been verified by scanning the beam and changing the frequency as well a study of a tapered excitation. Due to space limitations one particular set-up will be shown here, for a complete set of results the interested reader is referred to [22].

In the first example discussed in this paper, the starting grid is made up of elements placed 2.5λ apart (i.e., a = 2.5) and each element is allowed to be moved horizontally and vertically in the interval $[-\Delta, \Delta]$. In Figure 3, the x- and the y-projection of the normalized power pattern is shown for the case when $\Delta = \lambda$ with no thinning allowed in this case. The elements are placed according to the rules outlined above, and the total number of elements is 497 (the layout is seen in Figure 2).

Applying the modified BCGA algorithm, the directivity of the resulting pattern obtained was 35.4 dB with a maximum SLL 16.9 dB below the main beam peak, the convergence of the algorithm is shown in Figure 4(a). However, the maximum peak SLL over the surface of earth[§] is -18 dB and the half-power beam width is 0.96° . This is not exactly equal to the theoretical results assuming a continuous source, as expected.

To get a view of the array layout in terms of element spacing, Figure 4(b) shows a cumulative distribution function (CDF) of the minimum spacing^{||} between the elements. Looking at the graph, the average separation between the elements is 1.8λ (2.8 cm) and 8% of the elements have a spacing less than λ . Hence, the majority of the elements (92%) are separated with considerably larger distances (more than λ). This indicates that the effect of the mutual coupling can

 $[\]S$ Note, in the following text the value mentioned as "SLL" is the maximum peak value of the side lobe level throughout the whole space (not only over the surface of earth) unless stated otherwise.

 $[\]parallel$ Minimum spacing is the minimum distance between the element of interest and its neighboring elements.

be neglected without affecting the results much, as indicated earlier. Furthermore, if we would have optimized for a uniform geometry we expect to see very high side lobe levels and poor directivity. This example depicts the advantage of optimizing for element positions.

Several examples adding more elements have been studied [22], but the effect is (surprisingly) limited. Adding more elements will result in



Figure 3. Normalized power pattern. The entire 3D pattern is here projected into an x-view and a y-view, thus the above pattern includes peaks at all positions on the sphere. (a) x-view, (b) y-view.



Figure 4. (a) Convergence of the cost function (6) as a function of the number of iterations of generation updates. (b) The cumulative distribution function of the minimum element separation, the 497 elements case.

an increased gain, but the side lobe level remains around $-18 \,\mathrm{dB}$. An interesting question then is to see the effect of thinning. By allowing the optimizer to turn off certain elements, we can test if the optimizer finds a better solution with more degrees of freedom. In the example shown here, we start again with the set-up as above (i.e., the elements are place 2.5 λ apart) but $\Delta = 1.25\lambda$ (i.e., the total number of elements is still 497). By allowing thinning, 7% of the elements were turned off in the optimization procedure giving a directivity of the resulting pattern of $35 \,\mathrm{dB}$ with max peak SLL of $-16 \,\mathrm{dB}$ ($-18.4 \,\mathrm{dB}$ over the surface of earth), the beam width is 0.94° . Hence, the set-up considered seems to be quite stable where it is possible to adjust the number of elements without losing much in performance. The robustness is important in these applications, and to investigate this further a robustness analysis was made with respect to frequency and beam scanning for the case without thinning (Figures 3, 4). We have scanned the beam to two different scan angles; they are $\theta = 4$, $\phi = 60$ and $\theta = 8.8$, $\phi = 0$. The scanned beam can be seen in Figure 5. As seen in the pictures, scanning over the half-cone of interest for earth coverage, the results are stable enough.

To check robustness with respect to frequency two extreme frequencies of the given band were picked; 17.7 GHz and 20.2 GHz, respectively. In this test, the assumption was made that the element pattern does not change much over this frequency range. The results from this test are summarized in Table 1. Thus the realized pattern seems to be rather robust over the frequency and angle interval.



Figure 5. Beam scanning, the *xy*-view of the normalized power pattern. Scanned for (a) $\theta = 8.8^{\circ}$, $\phi = 0^{\circ}$. (b) $\theta = 4^{\circ}$, $\phi = 60^{\circ}$. The power in decibel is shown in the color bar/scale.

	Frequency = 17.7 GHz	Frequency $20.2\mathrm{GHz}$
Max peak SLL	$-17.3\mathrm{dB}$	$-16.97\mathrm{dB}$
3 dB BW	1.05°	0.89°
Directivity	$35.4\mathrm{dB}$	$34.8\mathrm{dB}$

 Table 1. Robustness test of the array.

To determine the stability of the array pattern with respect to its excitation, we return to the radial tapered continuous source, and let the excitations in the physical array be the sampled values of the continuous source amplitude. The resulting beam width is 1.2° , SLL of -20 dB over earth and max peak SLL of -14.8 dB, and the directivity is 34.7 dB. We note here a slight improvement of the SLL which is traded against the beam-width and the directivity. This test indicates that also the element positions are rather robust in changes of the excitation.

The above resulting values are noticeably lower than that of the continuous source, and to examine if the Genetic Algorithm will approach these values given a larger number of initial elements we consider the following simulation: We place elements 1.5λ apart and allow the elements to move in the interval $[-\lambda, \lambda]$. Total number of elements placed in such a way is 1377. A 46% thinning is performed resulting in 750 switched-on elements. The resulting pattern from the aperture has a directivity of 36.78 dB with max peak SLL of -16 dB over the surface of earth and BW is 0.94°. Notice that the SLL did not improve, we got a slight improvement in directivity, for an essentially equally wide beam. This trend seems rather generic for the chosen cost function, while testing for several initial element inter-distances and number of elements [22].

Finally, as the reader may have noticed, it may happen in some examples (Figure 2) that there is a physical overlap between adjacent elements. In all examples shown here this overlap has been overseen, assuming that each element is a point source having a radiation pattern equal to the horn antenna to be used in practice. To investigate this in more detail, the physically overlapping elements have been merged into one element with an excitation equal to the sum of the two overlapping elements. This cleaning improves marginally the SLL over earth to -18.1 dB and beam width is 0.95° and directivity 35.3 dB.

5. CONCLUSION

In this paper, we show that a BCGA algorithm can optimize antenna patterns with respect to antenna position with a rather simple cost function. We note that a -18 dB max peak SLL for a 1° beam width and 36 dB directivity seems to be rather generic for this array size. Thus, choosing a realization with only 497 elements as presented here a result close to the theoretical limit is possible. We have shown that the given element positions enable a robust response to different element excitations and that the element patters persist across beam-sweeps and the down-link frequency band. Hence, the results indicate that it will be possible to realize an array antenna fulfilling the requirements needed for this satellite application.

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