Progress In Electromagnetics Research B, Vol. 21, 329–346, 2010

FOCAL REGION FIELDS OF CASSEGRAIN SYSTEM PLACED IN HOMOGENEOUS CHIRAL MEDIUM

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Abstract—In this paper, the high frequency electromagnetic field expressions for two dimensional Cassegrain system embedded in a chiral medium are presented. Due to failure of Geometrical Optics (GO) at the caustic region, Maslov's method is used to find the field expressions. Two different cases have been analyzed. Firstly, the chirality parameter ($k\beta$) is adjusted to support positive phase velocity (PPV) for both left circularly polarized (LCP) and right circularly polarized (RCP) modes traveling in the medium. Secondly, $k\beta$ is adjusted such that one mode travels with PPV, and the other mode travels with negative phase velocity (NPV). The results for both cases are presented in the paper.

1. INTRODUCTION

Chirality means handedness, and a handed object is called chiral. Therefore, chiral objects are either left or right handed. Interaction between electromagnetic wave and chiral objects results in the rotation of the wave polarization to the right or left depending on the handedness of the object. If these handed objects are uniformly distributed and randomly oriented and form a macroscopically homogeneous medium then such a composite is called a chiral medium [1]. A chiral medium supports both LCP and RCP modes. Depending upon the value of chirality parameter the medium may supports NPV propagation for both modes, or NPV for one mode and PPV for the other mode [2]. Different reflectors are placed in chiral medium due to its unique characteristics over an ordinary medium like polarization control, impedance matching and cross coupling of electric and magnetic fields. By changing the chiral media

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parameters ϵ, μ and $k\beta$ the desirable values of the wave impedance and propagation constants can be achieved by which reflections can be adjusted (decreased or increased). In this respect, the chiral medium can be controlled by variations of three parameters ϵ, μ, $k\beta$, whereas an achiral medium has only two variable parameters, ϵ, μ. Moreover, in the case of negative reflection caused by NPV, it also gives the advantage of invisibility. Due to these characteristics of chiral medium, we have embedded the Cassegrain system in chiral medium in this problem. Depending upon the values of chirality parameter two cases are considered. In the first case, chiral medium supports PPV for both the LCP and RCP modes. In the second case, chiral medium supporting PPV for one mode and NPV for the other mode is taken into account. Maslov's method is used to study the fields in the focal region, which combines the simplicity of asymptotic ray theory and the generality of the Fourier transform method. This is achieved by representing the GO fields in hybrid coordinates consisting of space coordinates and wave vector coordinates by representing the field in terms of six coordinates [4]. Analysis of focusing systems has been worked out by various authors using Maslov's method [5–9].

2. REFLECTION OF PLANE WAVE FROM PERFECT ELECTRIC CONDUCTOR (PEC) PLACED IN CHIRAL MEDIUM

Reflection of plane waves from simple PEC plane placed in chiral medium is discussed in [10]. We recapitulate it here to introduce our notations and present it in a form suitable for our present work. Consider reflection of RCP wave from PEC plane lying along xy-plane as shown in Figure 1. An RCP wave traveling with phase velocity ω/kn_2 and unit amplitude is incident on the plane making angle α with z-axis. Reflected wave is composed of two waves with opposite handedness. An LCP wave is reflected making an angle $\alpha_1 = [\sin^{-1}(n_2/n_1 \sin\alpha)]$ and with amplitude $2\cos\alpha/(\cos\alpha + \cos\alpha_1)$. The phase velocity of LCP wave is ω/kn_1. An RCP wave is reflected

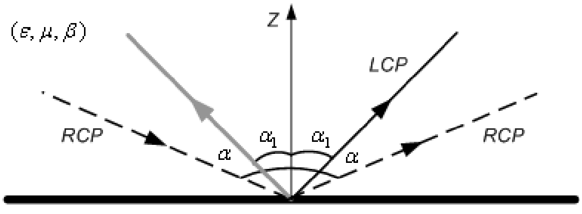


Figure 1. Reflection of RCP wave from PEC plane.

making angle α and amplitude $(\cos\alpha - \cos\alpha_1)/(\cos\alpha + \cos\alpha_1)$. If we take $k\beta < 1$, then $n_1 > n_2$ and $\alpha_1 < \alpha$, i.e., LCP wave bends towards normal, because it is traveling slower than RCP wave. For $k\beta > 1$, α_1 is negative, and the wave is reflected in the wrong way. It may be called negative reflection (shown as gray in Figure 1). This means that for $k\beta > 1$, LCP reflected wave sees the chiral medium as NPV medium.

Similarly, when an LCP wave with unit amplitude is incident on PEC plane making angle α with z-axis, as shown in Figure 2, we get two reflected waves of opposite handedness. An RCP wave is reflected at angle $\alpha_2 = [\sin^{-1}(n_1/n_2 \sin\alpha)]$ with amplitude $2\cos\alpha/(\cos\alpha + \cos\alpha_2)$ and an LCP wave at angle α and amplitude $(\cos\alpha - \cos\alpha_2)/(\cos\alpha + \cos\alpha_2)$. If we take $k\beta < 1$, then $n_1 > n_2$ and $\alpha_2 > \alpha$. If $k\beta = 0$ then only normal reflection takes place, and if $k\beta$ increases the difference between the angle α and α_1, α_2 increases. For $k\beta > 1$, we have negative reflection for RCP reflected wave (shown as gray in Figure 2 [11]).

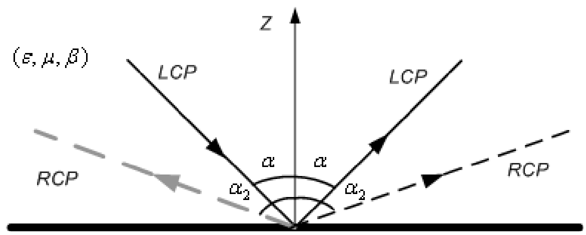


Figure 2. Reflection of LCP wave from PEC plane.

3. GEOMETRICAL OPTICS FIELDS OF TWO DIMENSIONAL CASSEGRAIN SYSTEM PLACED IN CHIRAL MEDIUM

Cassegrain system consists of two reflectors. One is parabolic main reflector, and the other is hyperbolic subreflector as shown in Figure 3. We will consider the receiving characteristics of this system. Both RCP and LCP waves are incident on main parabolic reflector, and it will cause four reflected waves designated as LL, RR, LR and RL [11]. These four waves are then incident on the secondary hyperbolic subreflector and will cause eight reflected waves designated as LLL, RRR, LLR, RRL, RLR, RLL, LRR and LRL. Only four of these waves (LLL, RRR, LLR, RRL) will converge in the focal region while the other four waves (RLR, RLL, LRR, LRL) will diverge. In this paper, we are considering only four converging rays after reflection from hyperbolic subreflector as shown in Figure 4.

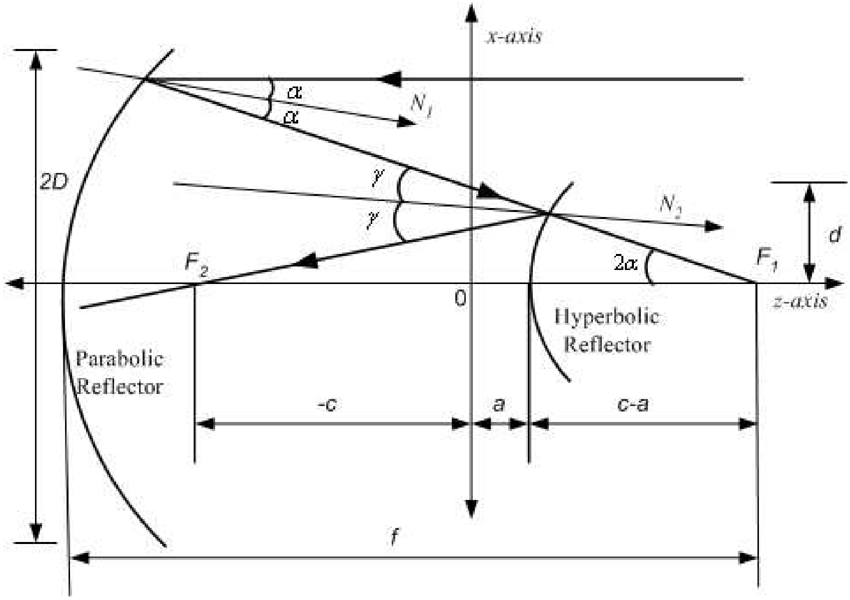


Figure 3. Cassegrain system.

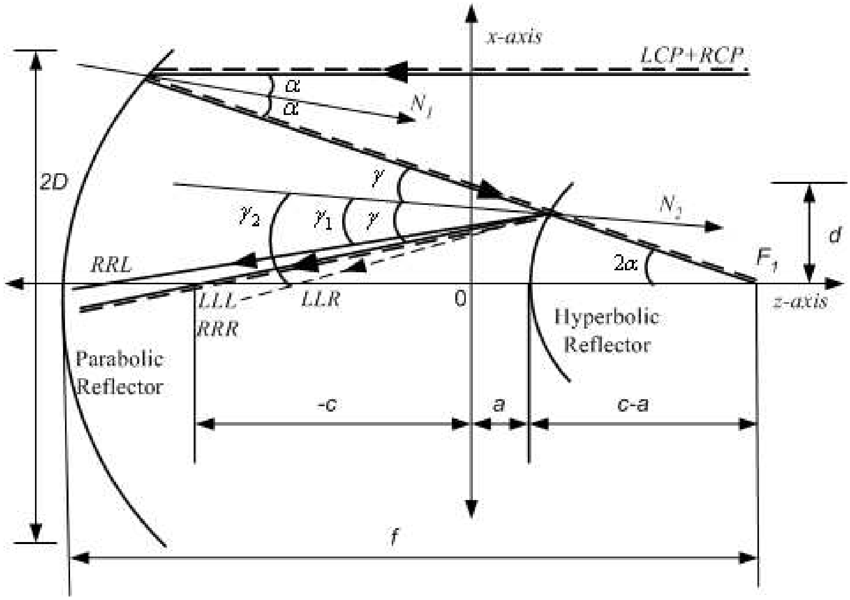


Figure 4. Cassegrain system in chiral medium.

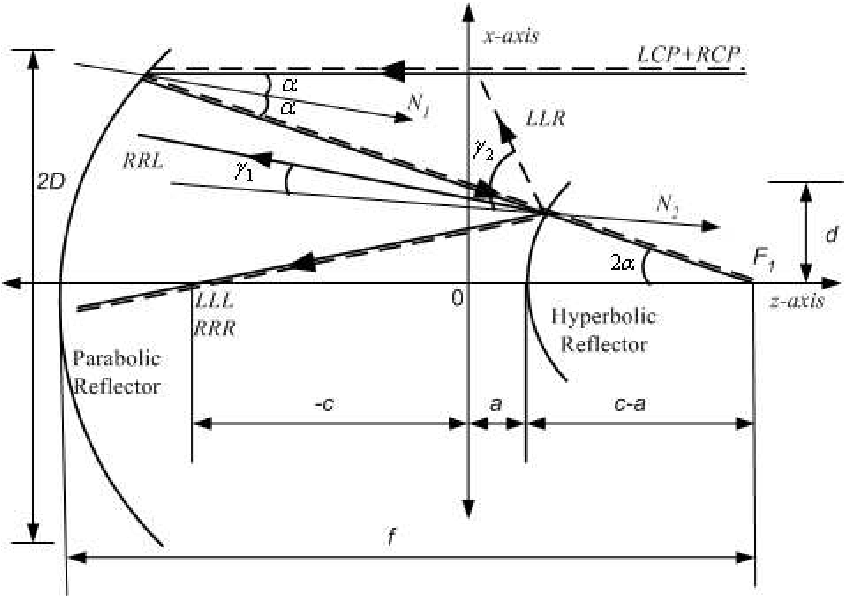


Figure 5. Cassegrain system in chiral medium, $k\beta > 1$.

For $k\beta > 1$, $n_1 = \frac{1}{1-k\beta} < 0$ and $n_2 = \frac{1}{1+k\beta} > 0$, so LCP wave travels with NPV and RCP wave with PPV. For $k\beta < -1$ RCP wave travels with NPV and LCP wave with PPV. We have depicted here the case of $k\beta > 1$ only because for $k\beta < -1$, we can get the solutions from $k\beta > 1$ by interchanging the role of LCP and RCP modes [12]. Cassegrain system for $k\beta > 1$ is shown in Figure 5. LLL and RRR rays are reflected at the same angle while RRL and LLR rays have different responses. It can be seen that only two rays (LLL and RRR) are contributing to the focus, while RRL and LLR rays are divergent. Equations for parabolic and hyperbolic reflectors of Cassegrain system are given by

$$\zeta_1 = \frac{\xi_1^2}{4f} - f + c, \quad \zeta_2 = a\left[1 + \frac{\xi_2^2}{b^2}\right]^{1/2}, \quad c^2 = a^2 + b^2 \tag{1}$$

where (ξ_1, ζ_1) and (ξ_2, ζ_2) are the Cartesian coordinates of the points on the parabolic and hyperbolic reflectors, respectively. Incident waves on main parabolic reflector having unit amplitude are given by

$$Q_L = \exp(jkn_1 z), \quad Q_R = \exp(jkn_2 z) \tag{2}$$

Consider the case of normal incidence such that these waves are incident at an angle α with surface normal $\overrightarrow{N_1}$ as shown in Figure 3 to

Figure 5. The reflected wave vectors of LL, RR, RL and LR waves are given by [11]. We will take two, RR and LL, waves that will incident on hyperbolic subreflector and converge as well after reflection. The wave vectors of these (LLL, RRR, RRL and LLR) reflected waves by the hyperbolic subreflector are given as.

$$\overrightarrow{P_{LLL}} = -n_1 \sin(2\alpha - 2\psi)\hat{i}_x - n_1 \cos(2\alpha - 2\psi)\hat{i}_z \tag{3a}$$

$$\overrightarrow{P_{RRR}} = -n_2 \sin(2\alpha - 2\psi)\hat{i}_x - n_2 \cos(2\alpha - 2\psi)\hat{i}_z \tag{3b}$$

$$\overrightarrow{P_{RRL}} = -n_1 \sin(\gamma_1 - \psi)\hat{i}_x - n_1 \cos(\gamma_1 - \psi)\hat{i}_z \tag{3c}$$

$$\overrightarrow{P_{LLR}} = -n_2 \sin(\gamma_2 - \psi)\hat{i}_x - n_2 \cos(\gamma_2 - \psi)\hat{i}_z \tag{3d}$$

Corresponding initial amplitudes for these four reflected rays are

$$A_{0LLL} = \left[\frac{\cos\alpha - \cos\alpha_2}{\cos\alpha + \cos\alpha_2}\right]\left[\frac{\cos\gamma - \cos\gamma_2}{\cos\gamma + \cos\gamma_2}\right] \tag{4a}$$

$$A_{0RRR} = \left[\frac{\cos\alpha - \cos\alpha_1}{\cos\alpha + \cos\alpha_1}\right]\left[\frac{\cos\gamma - \cos\gamma_1}{\cos\gamma + \cos\gamma_1}\right] \tag{4b}$$

$$A_{0RRL} = \left[\frac{\cos\alpha - \cos\alpha_1}{\cos\alpha + \cos\alpha_1}\right]\left[\frac{2\cos\gamma}{\cos\gamma + \cos\gamma_1}\right] \tag{4c}$$

$$A_{0LLR} = \left[\frac{\cos\alpha - \cos\alpha_2}{\cos\alpha + \cos\alpha_2}\right]\left[\frac{2\cos\gamma}{\cos\gamma + \cos\gamma_2}\right] \tag{4d}$$

And the corresponding initial phases are

$$S_{0LLL} = -n_1\zeta_1 = n_1\left[2f\frac{\cos 2\alpha}{1+\cos 2\alpha} - c\right] \tag{5a}$$

$$S_{0RRR} = -n_2\zeta_1 = n_2\left[2f\frac{\cos 2\alpha}{1+\cos 2\alpha} - c\right] \tag{5b}$$

$$S_{0RRL} = -n_2\zeta_1 = n_2\left[2f\frac{\cos 2\alpha}{1+\cos 2\alpha} - c\right] \tag{5c}$$

$$S_{0LLR} = -n_1\zeta_1 = n_1\left[2f\frac{\cos 2\alpha}{1+\cos 2\alpha} - c\right] \tag{5d}$$

where

$$\overrightarrow{N_1} = -\sin\alpha\hat{i}_x + \cos\alpha\hat{i}_z,\ \sin\alpha = \frac{\xi_1}{\sqrt{\xi_1^2 + 4f^2}},\ \cos\alpha = \frac{2f}{\sqrt{\xi_1^2 + 4f^2}} \tag{6a}$$

$$\overrightarrow{N_2} = -\sin\psi\hat{i}_x + \cos\psi\hat{i}_z,\ \sin\psi = \frac{-1}{\sqrt{R_1 R_2}}\frac{a}{b}\xi_2,\ \cos\psi = \frac{1}{\sqrt{R_1 R_2}}\frac{b}{a}\zeta_2 \tag{6b}$$

$$\gamma_1 = \sin^{-1}\left(\frac{n_2}{n_1}\sin\gamma\right),\ \gamma_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\gamma\right),\ \gamma = (2\alpha - \psi) \tag{6c}$$

In the above equations R_1 and R_2 are the distances from the point (ξ_2, ζ_2) to the focal points $z = -c$ and $z = c$, respectively with $c^2 = a^2 + b^2$. The Cartesian coordinates of the rays reflected by the hyperbolic subreflector are given by.

$$x_{LLL} = \xi_2 + P_{xLLL} t, \qquad x_{RRR} = \xi_2 + P_{xRRR} t \tag{7a}$$

$$x_{RRL} = \xi_2 + P_{xRRL} t, \qquad x_{LLR} = \xi_2 + P_{xLLR} t \tag{7b}$$

$$z_{LLL} = \zeta_2 + P_{zLLL} t, \qquad z_{RRR} = \zeta_2 + P_{zRRR} t \tag{7c}$$

$$z_{RRL} = \zeta_2 + P_{zRRL} t, \qquad z_{LLR} = \zeta_2 + P_{zLLR} t \tag{7d}$$

Where

$$t_1 = \sqrt{(\xi_2 - \xi_1)^2 + (\zeta_2 - \zeta_1)^2}, \quad t = \sqrt{(x - \xi_2)^2 + (z - \zeta_2)^2} \tag{8}$$

The Jacobian transformation of reflected rays can be found using $\left[J(t) = \frac{D(t)}{D(0}\right]$, where $D(t)$ for all four rays is given in Appendix A.

$$J_{LLL} = 1 - \frac{n_1 t}{R_1} \tag{9a}$$

$$J_{RRR} = 1 - \frac{n_2 t}{R_1} \tag{9b}$$

$$J_{RRL} = 1 - \frac{n_1 t \cos\gamma}{R_2 \cos\gamma_1} \left[\frac{n_2 R_1 \cos\gamma - a\left(\sqrt{n_1^2 - n_2^2 \sin^2\gamma} + n_2 \cos\gamma\right)}{R_1 \sqrt{n_1^2 - n_2^2 \sin^2\gamma}} \right] \tag{9c}$$

$$J_{LLR} = 1 - \frac{n_2 t \cos\gamma}{R_2 \cos\gamma_2} \left[\frac{n_1 R_1 \cos\gamma - a\left(\sqrt{n_2^2 - n_1^2 \sin^2\gamma} + n_1 \cos\gamma\right)}{R_1 \sqrt{n_2^2 - n_1^2 \sin^2\gamma}} \right] \tag{9d}$$

The GO field for each ray can now be written as

$$U(r)_{LLL} = A_{0LLL}(\xi)[J_{LLL}]^{-1/2} \exp\left[-jk\left(S_{0LLL} + n_1^2 t + n_1 t_1\right)\right] \tag{10a}$$

$$U(r)_{RRR} = A_{0RRR}(\xi)[J_{RRR}]^{-1/2} \exp\left[-jk\left(S_{0RRR} + n_2^2 t + n_2 t_1\right)\right] \tag{10b}$$

$$U(r)_{RRL} = A_{0RRL}(\xi)[J_{RRL}]^{-1/2} \exp\left[-jk\left(S_{0RRL} + n_1^2 t + n_1 t_1\right)\right] \tag{10c}$$

$$U(r)_{LLR} = A_{0LLR}(\xi)[J_{LLR}]^{-1/2} \exp\left[-jk\left(S_{0LLR} + n_2^2 t + n_2 t_1\right)\right] \tag{10d}$$

where $A_0(\xi)$ and $S_0(\xi)$ are the initial phases and amplitudes. Their expressions are given in Eqs. (4a)–(5d). The phase functions are given by

$$S_{LLL} = S_{0LLL} + n_1 t_1 + n_1^2 t - z(x, P_{zLLL}) P_{zLLL} + z P_{zLLL} \tag{11a}$$

$$S_{RRR} = S_{0RRR} + n_2 t_1 + n_2^2 t - z(x, P_{zRRR}) P_{zRRR} + z P_{zRRR} \tag{11b}$$

$$S_{RRL} = S_{0RRL} + n_1 t_1 + n_1^2 t - z(x, P_{zRRL}) P_{zRRL} + z P_{zRRL} \tag{11c}$$

$$S_{LLR} = S_{0LLR} + n_2 t_1 + n_2^2 t - z(x, P_{zLLR}) P_{zLLR} + z P_{zLLR} \tag{11d}$$

In these phase functions S_0 and t_1 are given above. While the extra terms are given by

$$\begin{aligned} S_{exLLL} &= n_1^2 t - z(x, P_{zLLL}) P_{zLLL} + z P_{zLLL} \\ &= n_1^2 t - (\zeta_2 + P_{zLLL} t) P_{zLLL} + z P_{zLLL} \\ &= (P_{xLLL})^2 t + (z - \zeta_2) P_{zLLL} = (x - \xi_2) P_{xLLL} + (z - \zeta_2) P_{zLLL} \\ &= -n_1 [x \sin(2\alpha - 2\psi) + z \cos(2\alpha - 2\psi)] \\ &\quad + n_1 [\xi_2 \sin(2\alpha - 2\psi) + \zeta_2 \cos(2\alpha - 2\psi)] \end{aligned} \tag{12a}$$

Similarly

$$\begin{aligned} S_{exRRR} &= -n_2 [x \sin(2\alpha - 2\psi) + z \cos(2\alpha - 2\psi)] \\ &\quad + n_2 [\xi_2 \sin(2\alpha - 2\psi) + \zeta_2 \cos(2\alpha - 2\psi)] \end{aligned} \tag{13a}$$

$$\begin{aligned} S_{exRRL} &= -n_1 [x \sin(\gamma_1 - \psi) + z \cos(\gamma_1 - \psi)] \\ &\quad + n_1 [\xi_2 \sin(\gamma_1 - \psi) + \zeta_2 \cos(\gamma_1 - \psi)] \end{aligned} \tag{13b}$$

$$\begin{aligned} S_{exLLR} &= -n_2 [x \sin(\gamma_2 - \psi) + z \cos(\gamma_2 - \psi)] \\ &\quad + n_2 [\xi_2 \sin(\gamma_2 - \psi) + \zeta_2 \cos(\gamma_2 - \psi)] \end{aligned} \tag{13c}$$

Since GO becomes infinite at caustic region, we found approximate field at the caustic by Maslov's method. To calculate the field at caustic region we need expression $\left[J(t)\frac{\partial P_z}{\partial z}\right]$ for all four rays, reflected form hyperbolic subreflector, which are found below (see Appendix B).

$$\left[J(t)_{LLL} \frac{\partial P_{zLLL}}{\partial z}\right] = \frac{n_1 \sin^2(2\alpha - 2\psi)}{R_1} \tag{14a}$$

$$\left[J(t)_{RRR} \frac{\partial P_{zRRR}}{\partial z}\right] = \frac{n_2 \sin^2(2\alpha - 2\psi)}{R_1} \tag{14b}$$

$$\begin{aligned} \left[J(t)_{RRL} \frac{\partial P_{zRRL}}{\partial z}\right] &= \left[\frac{n_1^2 \sin^2(\gamma_1 - \psi) \cos^2 \gamma}{\cos \gamma_1 \sqrt{n_1^2 - n_2^2 \sin^2 \gamma}}\right] \\ &\quad \times \left[\frac{n_2 b R_1 - a\sqrt{(R_1 R_2)(n_1^2 - n_2^2 \sin^2 \gamma)} - n_2 ab}{b R_1 R_2}\right] \end{aligned} \tag{14c}$$

$$\begin{aligned} \left[J(t)_{LLR} \frac{\partial P_{zLLR}}{\partial z}\right] &= \left[\frac{n_2^2 \sin^2(\gamma_2 - \psi) \cos^2 \gamma}{\cos \gamma_2 \sqrt{n_2^2 - n_1^2 \sin^2 \gamma}}\right] \\ &\quad \times \left[\frac{n_1 b R_1 - a\sqrt{(R_1 R_2)(n_2^2 - n_1^2 \sin^2 \gamma)} - n_1 ab}{b R_1 R_2}\right] \end{aligned} \tag{14d}$$

After substituting all the required parameters and simplifying them we will get the following final expressions at caustic region.

$$U(r)_{LLL} = \sqrt{\frac{k}{2j\pi}}\left[\int_{A_1}^{A_2} + \int_{-A_1}^{-A_2}\right] A_{0LLL}(\xi)\sqrt{R_1} \times \exp[-jk\{S_{0LLL} + n_1 t_1 + S_{exLLL}\}]d(2\alpha) \tag{15a}$$

$$U(r)_{RRR} = \sqrt{\frac{k}{2j\pi}}\left[\int_{A_1}^{A_2} + \int_{-A_1}^{-A_2}\right] A_{0RRR}(\xi)\sqrt{R_1} \times \exp[-jk\{S_{0RRR} + n_2 t_1 + S_{exRRR}\}]d(2\alpha) \tag{15b}$$

$$U(r)_{RRL} = \sqrt{\frac{k}{2j\pi}}\left[\int_{A_1}^{A_2} + \int_{-A_1}^{-A_2}\right] A_{0RRL}(\xi)\frac{1}{\sqrt{n_1^2 - n_2^2\sin^2\gamma}} \times \left[\frac{R_1 R_2 b n_2 \cos\gamma_1}{abn_2 + a\sqrt{(R_1R_2)(n_1^2 - n_2^2\sin^2\gamma)} - bn_2R_1}\right]^{-1/2} \times \exp[-jk\{S_{0RRL} + n_1 t_1 + S_{exRRL}\}]d(2\alpha) \tag{15c}$$

$$U(r)_{LLR} = \sqrt{\frac{k}{2j\pi}}\left[\int_{A_1}^{A_2} + \int_{-A_1}^{-A_2}\right] A_{0LLR}(\xi)\frac{1}{\sqrt{n_2^2 - n_1^2\sin^2\gamma}} \times \left[\frac{R_1 R_2 b n_1 \cos\gamma_2}{abn_1 + a\sqrt{(R_1R_2)(n_2^2 - n_1^2\sin^2\gamma)} - bn_1R_1}\right]^{-1/2} \times \exp[-jk\{S_{0LLR} + n_2 t_1 + S_{exLLR}\}]d(2\alpha) \tag{15d}$$

Eqs. (15a)–(15d) are found by performing the integration numerically.

4. RESULTS AND DISCUSSIONS

Field pattern around the caustic of a Cassegrain system are determined using Eqs. (15a)–(15d) by using Maslov's method. Values for different parameters of Cassegrain system are: $kf = 170$, $ka = 40$, $kb = 60$, $kd = 50$, $kD = 150$. Limits of integration for Eqs. (15a)–(15d) are selected using the following relations [13].

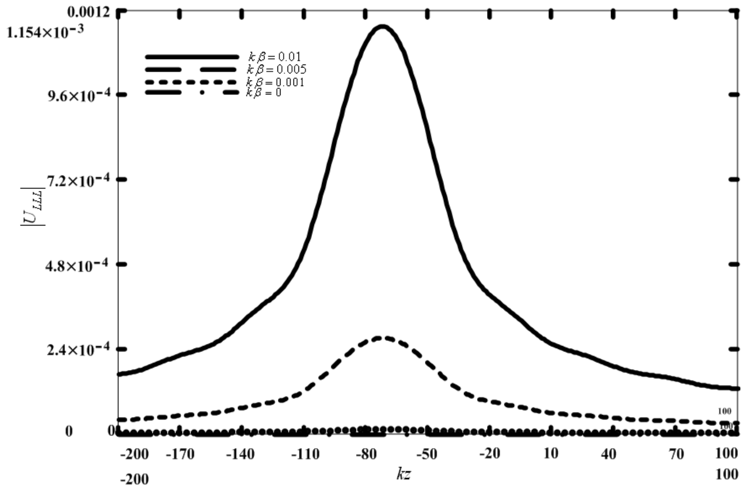
$$A_1 = 2\tan^{-1}\left(\frac{D}{2f}\right), \quad A_2 = \tan^{-1}\left(\frac{d}{2c}\right) \tag{16}$$

Equations of caustic for LLL and RRR rays are given by Eq. (15a) and Eq. (15b). These are similar in the case of ordinary medium [13].

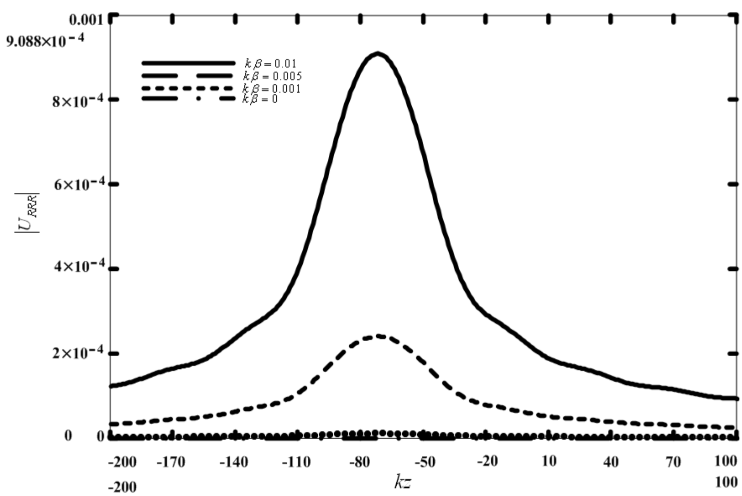
LLL and RRR rays coincide for all values of $k\beta$. As the value of $k\beta$ increases, magnitude of the field at caustic increases. This behavior is depicted in Figure 6. For $k\beta = 0$, $n_1 = n_2 = 1$ and

$$U_{LLL} = U_{RRR} = 0 \tag{17}$$

Equations of caustic for RRL and LLR rays are given by Eq. (15c) and Eq. (15d). As the value of $k\beta$ increases, the gap between the focal



(a)



(b)

Figure 6. Plots of Cassegrain system at $kx = 0$ for $k\beta = 0, 0.001, 0.005, 0.01$ for: (a) $|U_{LLL}|$, (b) $|U_{RRR}|$.

points of RRL and LLR rays increases as shown in Figure 7. It is due to the fact that enlarging the value of chirality parameter causes reduction in the phase velocity of LLR ray, i.e., it slows down. While by increasing the value of $k\beta$, phase velocity of RRL ray increases. This

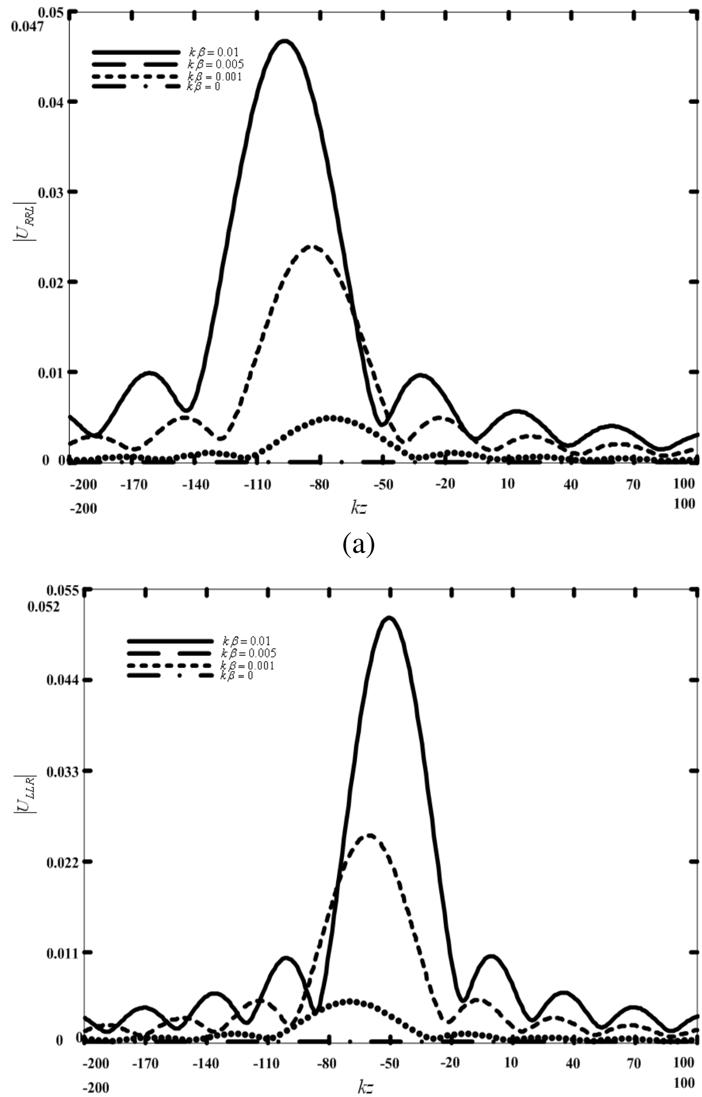


Figure 7. Plots of Cassegrain system at $kx = 0$ for $k\beta = 0, 0.001, 0.005, 0.01$ for: (a) $|U_{RRL}|$, (b) $|U_{LLR}|$.

is why the gap between RRL and LLR rays continues to increase with increased value of $k\beta$ as shown in Figure 7. For $k\beta = 0$, $n_1 = n_2 = 1$ and

$$U_{RRL} = U_{LLR} = 0 \tag{18}$$

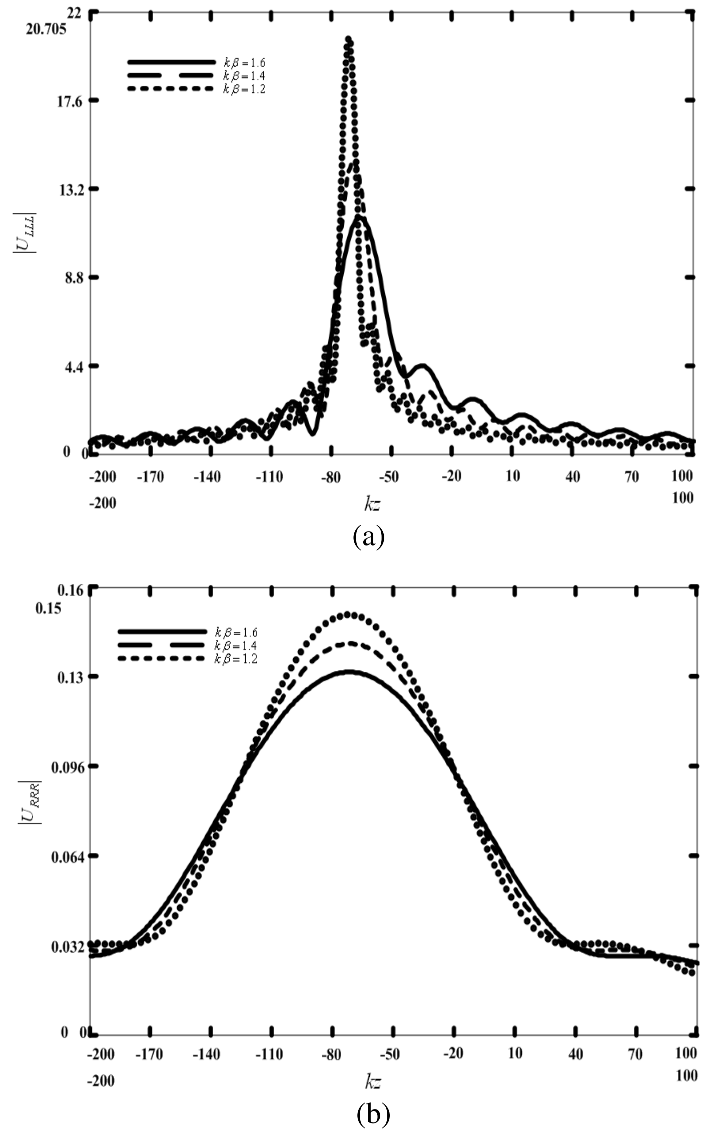


Figure 8. Plots of Cassegrain system at $kx = 0$ for $k\beta = 1.2, 1.4, 1.6$ for: (a) $|U_{LLL}|$, (b) $|U_{RRR}|$.

Eq. (17) and Eq. (18) explain that for zero chirality parameter, $\alpha_1 = \alpha$ and $\alpha_2 = \alpha$, i.e., LL and RR rays reduce to zero amplitude. Since LLL, RRR, LLR and RRL rays are reflected as the result of incidence of LL and RR rays on hyperbolic reflector, it is quite obvious that if LL and RR rays vanish at $k\beta = 0$ then LLL, RRR, RRL and LLR rays also have zero amplitude for zero chirality as shown in Figures 6–8. While other four rays LRL, LRR, RLR, RRL, caused due the incidence of RL and LR, will be as ordinary medium waves [13] for zero chirality case. Due to these properties, it can be advantageous in RF absorber and reflection controlling applications.

Plots of LLL and RRR rays for $k\beta > 1$ are given in Figure 8. In this case, LCP wave is traveling with NPV and RCP with PPV. Due to this, LLL and RRR rays are located at the same location as in the case of ordinary medium, while RRL and LLR waves diverge out and do not form a real focus. Hence for $k\beta > 1$, negative reflections occur, which can be applicable where invisibility is required.

5. CONCLUSION

It is found that the excitation of a Cassegrain system, placed in homogenous chiral medium, by plane wave yields eight rays, four of which converge and their field expressions are determined in this paper. Two of them, LLL and RRR, are focused at the same location as if the system was placed in an ordinary medium [13]. It is also observed that for PPV case other two rays, LLR and RRL, are focused on the opposite sides of the caustic locating in an ordinary medium location [13]. As $k\beta$ increases, the gap between the focal points of LLR and RRL rays increases. For NPV case, it is observed that the caustic for LLL and RRR rays does not change, while the caustics of LLR and RRL rays disappear because these two rays are now diverging.

APPENDIX A. EVALUATION OF $D(T)$

A.1. For LLL and RRR Rays

We will drive here for LLL ray which can be modified for RRR ray as well.

$$\begin{aligned} D(t)_{LLL} &= \frac{\partial(x_{LLL}, z_{LLL})}{\partial(\xi_1, t)} = \begin{vmatrix} \frac{\partial x_{LLL}}{\partial \xi_1} & \frac{\partial x_{LLL}}{\partial t} \\ \frac{\partial z_{LLL}}{\partial \xi_1} & \frac{\partial z_{LLL}}{\partial t} \end{vmatrix} \\ &= \frac{\partial(\xi_2 + P_{xLLL}t)}{\partial \xi_1} P_{zLLL} - P_{xLLL} \frac{\partial(\zeta_2 + P_{zLLL}t)}{\partial \xi_1} \end{aligned} \quad \text{(A1)}$$

where x_{LLL}, z_{LLL}, P_{xLLL} and P_{zLLL} are given in Eq. (7a), Eq. (7c) and Eq. (3a) respectively. By using relations $\frac{\partial\zeta_2}{\partial\xi_1} = \frac{\partial\zeta_2}{\partial\xi_2}\frac{\partial\xi_2}{\partial\xi_1} = \tan\psi\frac{\partial\xi_2}{\partial\xi_1}$, and simplifying we will get

$$D(t)_{LLL} = 2tn_1^2\left[\frac{\partial\alpha}{\partial\xi_1} - \frac{\partial\psi}{\partial\xi_1}\right] - n_1\frac{\cos(2\alpha-\psi)}{\cos\psi}\frac{\partial\xi_2}{\partial\xi_1} \tag{A2}$$

where

$$\frac{\partial\alpha}{\partial\xi_1} = \frac{\cos^2\alpha}{2f}, \quad \frac{\partial\psi}{\partial\xi_1} = \cos^2\psi\frac{a^4}{b^2\zeta_2^3}\frac{\partial\xi_2}{\partial\xi_1} \tag{A3}$$

The relation between (ξ_1, ζ_1) and (ξ_2, ζ_2) is as following

$$(\xi_2 - \xi_1) = -\tan 2\alpha(\zeta_2 - \zeta_1) \tag{A4}$$

Now differentiating both sides of Eq. (A4) with respect to ξ_1 and using Eq. (A3) we will get the following expression

$$\frac{\partial\xi_2}{\partial\xi_1} = \frac{\cos\psi}{\cos(2\alpha-\psi)}\left[1 - \frac{(\zeta_2-\zeta_1)\cos^2\alpha}{f\cos 2\alpha}\right] = \frac{\cos\psi R_2\cos^2\alpha}{f\cos(2\alpha-\psi)} \tag{A5}$$

Substituting Eq. (A5) and Eq. (A3) in Eq. (A2) and using relations $\zeta_2 = c - R_2\cos 2\alpha$ and $\zeta_1 = \frac{R_2\cos^2\alpha}{f}$ we will get

$$D(t)_{LLL} = \frac{\cos^2\alpha}{f}\left[\left(1 - \frac{2\cos^3\psi a^4 R_2}{\cos(2\alpha-\psi)b^2\zeta_2^3}\right)n_1^2 t - n_1 R_2\right] \tag{A6}$$

Now using $\cos 2\alpha = \frac{c-\zeta_2}{R_2}$ and $\sin 2\alpha = \frac{\xi_2}{R_2}$

$$\begin{aligned}\cos(2\alpha-\psi) &= \cos 2\alpha\cos\psi + \sin 2\alpha\sin\psi = \frac{c-\zeta_2}{R_2}\cos\psi + \frac{\xi_2}{R_2}\sin\psi \\ &= \frac{1}{R_2\sqrt{R_1R_2}}\left[\frac{b}{a}\zeta_2(c-\zeta_2) + \frac{a}{b}\xi_2^2\right] = \frac{b}{\sqrt{R_1R_2}}\end{aligned} \tag{A7}$$

Using Eq. (A7) in Eq. (A6) we will get

$$D(t)_{LLL} = n_1\frac{\cos^2\alpha}{f}R_2\left[1 - \frac{n_1 t}{R_1}\right] \tag{A8}$$

Similarly, $D(t)$ for RRR ray will be as following

$$D(t)_{RRR} = n_2\frac{\cos^2\alpha}{f}R_2\left[1 - \frac{n_2 t}{R_1}\right] \tag{A9}$$

A.2. Jacobian for LLR and RRL Rays

From Eq. (3d), it is seen that

$$\begin{aligned} D(t)_{LLR} &= \frac{\partial(x_{LLR}, z_{LLR})}{\partial(\xi_1, t)} = \begin{vmatrix} \frac{\partial x_{LLR}}{\partial \xi_1} & \frac{\partial x_{LLR}}{\partial t} \\ \frac{\partial z_{LLR}}{\partial \xi_1} & \frac{\partial z_{LLR}}{\partial t} \end{vmatrix} \\ &= \frac{\partial(\xi_2 + P_{xLLR}t)}{\partial \xi_1} P_{zLLR} - P_{xLLR} \frac{\partial(\zeta_2 + P_{zLLR}t)}{\partial \xi_1} \\ &= n_2^2 t \frac{\partial(\gamma_2 - \psi)}{\partial \xi_1} - n_2 \frac{\cos\gamma_2}{\cos\psi} \frac{\partial \xi_2}{\partial \xi_1} \end{aligned} \tag{A10}$$

Where

$$\begin{aligned} \frac{\partial(\gamma_2 - \psi)}{\partial \xi_1} &= \frac{\partial[\sin^{-1}(\frac{n_1}{n_2}\sin\gamma)]}{\partial \xi_1} - \frac{\partial \psi}{\partial \xi_1} \\ &= \frac{n_1 \cos\gamma}{\sqrt{n_2^2 - n_1^2 \sin^2\gamma}} \frac{\partial \gamma}{\partial \xi_1} - \frac{a^4 \cos^2\psi}{b^2 \zeta_2^3} \frac{\partial \xi_2}{\partial \xi_1} \end{aligned} \tag{A11}$$

Using Eq. (A11) in Eq. (A10) and simplifying we will get

$$\begin{aligned} D(t)_{LLR} &= -\left[\frac{n_2 R_2 \cos^2\alpha \cos\gamma_2}{f\cos\gamma}\right] + \frac{n_2^2 t \cos^2\gamma}{R_1 f \sqrt{n_2^2 - n_1^2 \sin^2\gamma}} \\ &\quad \left[n_1 R_1 \cos\gamma - a\left(\sqrt{n_2^2 - n_1^2 \sin^2\gamma} + n_1\cos\gamma\right)\right] \end{aligned} \tag{A12}$$

Similarly, for RRL ray

$$\begin{aligned} D(t)_{RRL} &= -\left[\frac{n_1 R_2 \cos^2\alpha \cos\gamma_1}{f\cos\gamma}\right] + \frac{n_1^2 t \cos^2\gamma}{R_1 f \sqrt{n_1^2 - n_2^2 \sin^2\gamma}} \\ &\quad \left[n_2 R_1 \cos\gamma - a\left(\sqrt{n_1^2 - n_2^2 \sin^2\gamma} + n_2\cos\gamma\right)\right] \end{aligned} \tag{A13}$$

APPENDIX B. DERIVATION OF $F = J(T)\frac{\partial P_Z}{\partial Z}$

B.1. Derivation for LLL and RRR Rays

From Eq. (7c)

$$z_{LLL} = \zeta_2 + \frac{P_{zLLL}}{P_{xLLL}}(x - \xi_2) = \zeta_2 + \cot(2\alpha - 2\psi)(x - \xi_2) \tag{B1}$$

Differentiating both sides with respect to ξ_2

$$\begin{aligned}\frac{\partial z}{\partial \xi_2} &= \frac{\partial \zeta_2}{\partial \xi_2} + \frac{\partial[\cot(2\alpha - 2\psi)(x - \xi_2)]}{\partial \xi_2} \\ &= \frac{1}{\sin(2\alpha - 2\psi)} \frac{\partial \xi_1}{\partial \xi_2} \left[2n_1 t \frac{\partial(\alpha - \psi)}{\partial \xi_1} - \frac{\cos(2\alpha - \psi)}{\cos \psi} \frac{\partial \xi_2}{\partial \xi_1} \right] \end{aligned} \quad \text{(B2)}$$

Now

$$\frac{\partial P_{zLLL}}{\partial \xi_2} = \frac{\partial[-n_1 \cos(2\alpha - 2\psi)]}{\partial \xi_2} = 2n_1 \sin(2\alpha - 2\psi) \frac{\partial \xi_1}{\partial \xi_2} \frac{\partial(\alpha - \psi)}{\partial \xi_1} \quad \text{(B3)}$$

$$\frac{\partial P_{zLLL}}{\partial z} = \frac{\partial \xi_2}{\partial z} \frac{\partial P_{zLLL}}{\partial \xi_2} \quad \text{(B4)}$$

Using Eq. (B2) and Eq. (B3) in Eq. (B4) we get following.

$$\begin{aligned}\frac{\partial P_{zLLL}}{\partial z} &= 2n_1 \sin^2(2\alpha - 2\psi) \frac{\partial(\alpha - \psi)}{\partial \xi_1} \\ &\quad \left[2n_1 t \frac{\partial(\alpha - \psi)}{\partial \xi_1} - \frac{\cos(2\alpha - \psi)}{\cos \psi} \frac{\partial \xi_2}{\partial \xi_1} \right] \end{aligned} \quad \text{(B5)}$$

Now

$$J(t)_{LLL} \frac{\partial P_{zLLL}}{\partial z} = 2n_1 \frac{\cos \psi \sin^2(2\alpha - 2\psi)}{\cos(2\alpha - 2\psi)} \frac{\partial \xi_1}{\partial \xi_2} \frac{\partial(\alpha - \psi)}{\partial \xi_1} \quad \text{(B6)}$$

Using Eq. (A3) and Eq. (A5) we will get

$$\begin{aligned}J(t)_{LLL} \frac{\partial P_{zLLL}}{\partial z} &= n_1 \frac{\cos \psi \sin^2(2\alpha - 2\psi)}{\cos(2\alpha - 2\psi)} \left[\frac{1}{R_2} - \frac{2a^4 \cos^3 \psi}{b^2 \zeta_2^3 \cos(2\alpha - 2\psi)} \right] \\ &= \frac{n_1 \sin^2(2\alpha - 2\psi)}{R_1} \end{aligned} \quad \text{(B7)}$$

Similarly, for RRR it will be

$$J(t)_{RRR} \frac{\partial P_{zRRR}}{\partial z} = \frac{n_2 \sin^2(2\alpha - 2\psi)}{R_1} \quad \text{(B8)}$$

B.2. Derivation for LLR and RRL Rays

From Eq. (7d)

$$z_{LLR} = \zeta_2 + \frac{P_{zLLR}}{P_{xLLR}}(x - \xi_2) = \zeta_2 + \cot(\gamma_2 - \psi)(x - \xi_2) \quad \text{(B9)}$$

Differentiating both sides with respect to ξ_2 and then simplifying we will get

$$\frac{\partial z}{\partial \xi_2} = \frac{1}{\sin(\gamma_2 - \psi)} \frac{\partial \xi_1}{\partial \xi_2} \left[n_2 t \frac{\partial(\gamma_2 - \psi)}{\partial \xi_1} - \frac{\cos \gamma_2}{\cos \psi} \frac{\partial \xi_2}{\partial \xi_1} \right] \quad \text{(B10)}$$

Now

$$\frac{\partial P_{zLLR}}{\partial \xi_2} = \frac{\partial[-n_2\cos(\gamma_2-\psi)]}{\partial \xi_2} = n_2\sin(\gamma_2-\psi)\frac{\partial \xi_1}{\partial \xi_2}\frac{\partial(\gamma_2-\psi)}{\partial \xi_1} \quad \text{(B11)}$$

$$\frac{\partial P_{zLLR}}{\partial z} = \frac{\partial \xi_2}{\partial z}\frac{\partial P_{zLLR}}{\partial \xi_2} \quad \text{(B12)}$$

Using Eq. (B10) and Eq. (B11) in Eq. (B12) we get following

$$\frac{\partial P_{zLLR}}{\partial z} = n_2\sin^2(\gamma_2-\psi)\frac{\partial(\gamma_2-\psi)}{\partial \xi_1}\left[n_2 t\frac{\partial(\gamma_2-\psi)}{\partial \xi_1} - \frac{\cos\gamma_2}{\cos\psi}\frac{\partial \xi_2}{\partial \xi_1}\right] \quad \text{(B13)}$$

Now

$$J(t)_{LLR}\frac{\partial P_{zLLR}}{\partial z} = n_2\frac{\cos\psi\sin^2(\gamma_2-\psi)}{\cos\gamma_2}\frac{\partial \xi_1}{\partial \xi_2}\frac{\partial(\gamma_2-\psi)}{\partial \xi_1} \quad \text{(B14)}$$

Using Eq. (A3) and Eq. (A5) we will get

$$J(t)_{LLR}\frac{\partial P_{zLLR}}{\partial z} = \left[\frac{n_2^2\sin^2(\gamma_2-\psi)\cos^2\gamma}{\cos\gamma_2\sqrt{n_2^2-n_1^2\sin^2\gamma}}\right] \left[\frac{n_1 bR_1 - a\sqrt{(R_1R_2)(n_2^2-n_1^2\sin^2\gamma)} - n_1 ab}{bR_1R_2}\right] \quad \text{(B15)}$$

Similarly, for RRL it will be

$$J(t)_{RRL}\frac{\partial P_{zRRL}}{\partial z} = \left[\frac{n_1^2\sin^2(\gamma_1-\psi)\cos^2\gamma}{\cos\gamma_1\sqrt{n_1^2-n_2^2\sin^2\gamma}}\right] \left[\frac{n_2 bR_1 - a\sqrt{(R_1R_2)(n_1^2-n_2^2\sin^2\gamma)} - n_2 ab}{bR_1R_2}\right] \quad \text{(B16)}$$

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