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NEW FORMULAS FOR THE REFLECTION COEFFICIENT OF AN OPEN-ENDED RECTANGULAR WAVEGUIDE RADIATING INTO AIR INCLUDING THE EFFECT OF WALL THICKNESS OR FLANGE

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Abstract—New formulas are presented for the reflection coefficient at the open end of a rectangular waveguide radiating into air including the effect of wall thickness or flange. Existing formulas require significant amount of numerical calculations and do not cover the practical range of waveguide dimensions. Reflection coefficients of open-ended standard waveguides are simulated using commercial electromagnetic software and curve-fitted to derive new formulas. Proposed formulas also include the effect of the broad-to-narrow wall aspect ratio. The accuracy of proposed formulas is compared with existing analytical, numerical and experimental results.

1. INTRODUCTION

The accurate prediction of the complex reflection coefficient (Γ) of an open-ended rectangular waveguide (OERW) is required in the calculation of the gain of an OERW, which is used for the probe compensation in near-field antenna measurements [1–3]. Yaghjian [1] presented the measured magnitude and phase of the reflection coefficient of standard rectangular waveguides operating at C-, X- and Ku-bands. Selvan presented formulas for calculating the magnitude [4]

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and phase [5] of Γ. Selvan's formulas are complicated and do not give accurate results for the entire operating frequency range of the waveguide.

Rectangular apertures with metallic flanges find applications in structural inspection, large phased array systems, aeronautical communication, medical diagnosis and treatment, and material property measurement [6–8]. Analytical approaches to the reflection coefficient or equivalently the aperture admittance of a rectangular waveguide radiating into air in the presence of an infinite flange have been studied by many researchers [9–16], where modal expansion — mode matching and variational principles are among most frequently-used techniques. These authors provide equations for the reflection coefficient or the aperture admittance of an OERW, where complicated calculations such as series summation and numerical integration are required.

In this paper, we present a set of simple formulas for the magnitude and phase of the reflection coefficient of an OERW radiating into air. Effects of finite wall thickness or of a flange of finite size and the broad-to-narrow wall aspect ratio in standard rectangular waveguides are included. We derived new formulas by curve-fitting numerical simulation results. In the last decade there have been significant advances in electromagnetic simulation techniques. Most of recent commercial electromagnetic software suites are powerful and accurate enough that in many cases one can replace costly fabrication and measurement with simulation. The widely-used commercial electromagnetic softwares CST Microwave Studio (MWS) v.2009 and Ansoft HFSS v.11 are used for the numerical simulation. Proposed formulas, however, are no replacements for full-wave computational or rigorous analytical methods [9–15]. An approach similar to one presented in this paper can be applied to derive formulas for the reflection coefficient of an OERW radiating into a material half space.

2. REFLECTION COEFFICIENT OF AN OERW WITH A WALL OF FINITE THICKNESS

Figure 1 shows structures of an open-ended rectangular waveguide (OERW) investigated in this paper. The first one (Fig. 1(a)) is a standard waveguide with finite wall thickness and the second (Fig. 1(b)) has a flange of finite size. All of the vacant spaces inside and outside the waveguide are filled with air. It is assumed that the waveguide is excited with a dominant TE_{10} mode.

Dimensional parameters of an OERW are broad wall width a, narrow wall height b, wall thickness t, waveguide length L and square

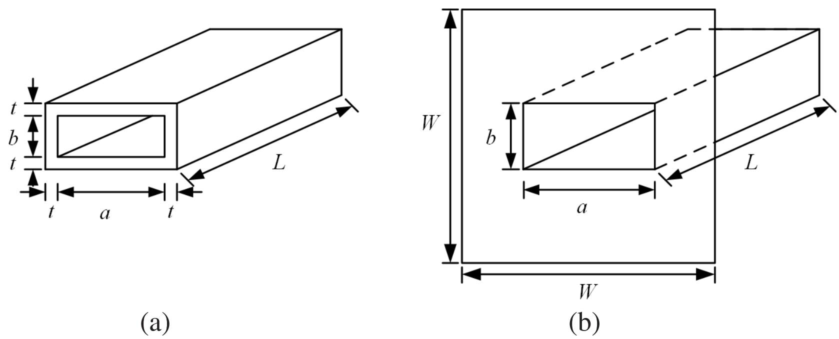


Figure 1. Structure of an open-ended rectangular waveguide. (a) With a wall of finite thickness, and (b) with a square flange of finite size.

flange size W. The waveguide length L has negligible effects on the reflection coefficient as long as it is sufficiently large. We let L be 10 times the broad wall width a.

The importance of an accurate formula for the aperture reflection coefficient can be seen in Yaghjian's classic gain formula [1] used in the probe compensation in near-field antenna measurements which reads

$$G = \frac{\pi k^2 ab}{8\left(1-|\Gamma|^2\right)\beta}\frac{k}{\beta}\left|\left[1+\frac{\beta}{k}+\Gamma\left(1-\frac{\beta}{k}\right)\right]\left(\frac{2}{\pi}\right)^2 + C_0\right|^2 \tag{1}$$

where Γ is the aperture reflection coefficient of the open-ended waveguide.

For $1.1 \leq f/f_c \leq 2.0$, we have simulated many cases of OERW's with a wall of finite thickness, from which we obtained following formulas for the magnitude and phase of the reflection coefficient at the waveguide aperture. Formula for the case of zero thickness were obtained using a commercial curve-fitting software Table Curve 2D$^{\text{TM}}$ by Systat Software Incorporated. Among many formulas generated by Table Curve 2D$^{\text{TM}}$, we have chosen following formulas considering the accuracy and the simplicity of expression. The correction term for the finite wall thickness is obtained by trial and error using Mathcad$^{\text{TM}}$ program by Parametric Technology Corporation.

$$|\Gamma| = \begin{cases} -8.023+24.083r-27.624r^2+15.747r^3 & \\ -4.484r^4+0.51r^5+t/(3.3a), & 0<t/a<0.11 \\ -4.31+0.496r+0.009r/\ln(r) & \\ +4.412(\ln(r))/r+3.849/r-t/\left(5.5ar^2\right), & 0.11 \leq t/a < 0.31 \end{cases} \tag{2}$$

$$\arg(\Gamma)^\circ=\begin{cases}10^6\times(-0.24777771+1.2444817r \\ -2.7265142r^2+34.0103072r^3-2.6418805r^4 \\ +1.3080474r^5-0.40310168r^6+0.070684631r^7 \\ -0.0053993664r^8)-\left(tr^9\right)/(6a), & 0<t/a<0.11 \\ -1721.82+213.89r-2.455/\ln(r)+1409.12/r \\ +1402.052\ln(r)/r+5tr^3/a, & 0.11\le t/a<0.31\end{cases} \tag{3}$$

where r is the normalized frequency given by

$$r=f/f_c \tag{4}$$

and f and f_c are the operating and cut-off frequencies of the waveguide. Waveguide dimensional parameters a, b and t are denoted in Fig. 1(a). Formulas (2), (3) are valid for the following parameter range.

$$0.40\le b/a\le 0.52,\quad 0<t/a\le 0.31,\quad 1.1\le f/f_c\le 2.0 \tag{5}$$

Although formulas (2), (3) do not explicitly contain the dependence on the aperture aspect ratio b/a, they give accurate results for all standard waveguides with a rectangular outer wall, ranging from WR-2300 (largest) to WR-4 (smallest). For these waveguides, the ratio b/a is between 0.406 and 0.512 and t/a between 0.0081 and 0.467.

Figures 2 to 4 show the magnitude and phase of the reflection coefficient at the open end of standard waveguides WR-90, WR-42 and WR-34 for which t/a is 0.056, 0.096 and 0.118, respectively. Here we present numerous graphs to show the accuracy of formulas (2), (3) for various values of b/a and t/a in standard rectangular waveguides.

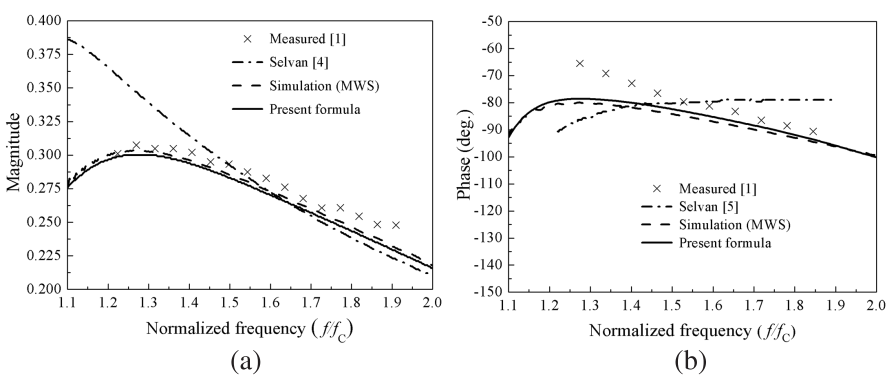


Figure 2. Aperture reflection coefficient at the open-end of the standard WR-90 waveguide without a flange. (a) Magnitude, and (b) phase.

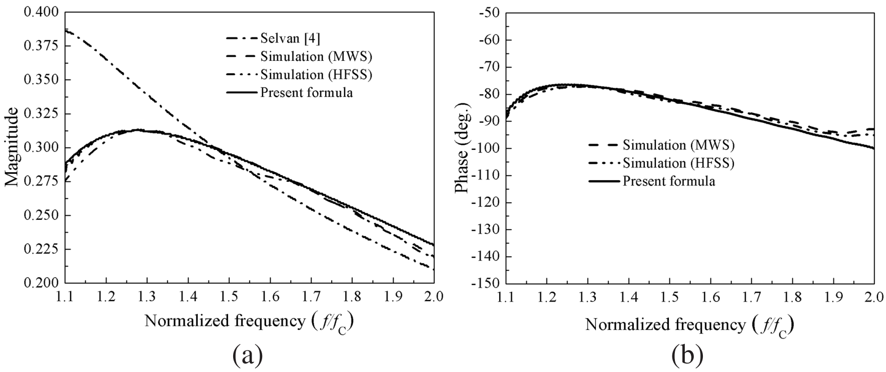


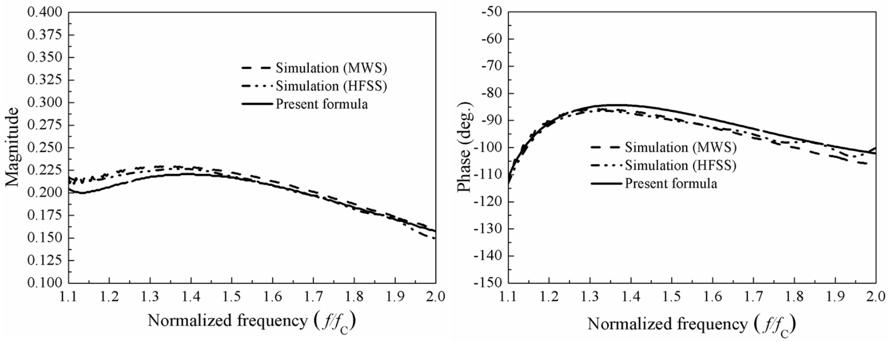
Figure 3. Aperture reflection coefficient at the open-end of the standard WR-42 waveguide without a flange. (a) Magnitude, and (b) phase.

In Figs. 2 to 4, the accuracy of formulas (2), (3) is compared with measurement [1], calculation by Selvan [4, 5], and with numerical simulation by MWS and HFSS. Proposed formulas (2), (3) agree well with numerical simulation for the entire operating frequency range of the waveguide. Selvan's formula does not include the effect of the wall thickness and deviates from numerical simulation and measurement of [1] especially at lower frequencies. Agreements between formulas (2), (3) and measurement [1] are good except at the lower frequency range. It appears that measurement in [1] contains small experimental errors. Fig. 5 shows the effect of waveguide wall thickness on the reflection coefficient. It shows that any accurate reflection coefficient formula should incorporate the effect of the waveguide wall thickness.

3. REFLECTION COEFFICIENT OF AN OEWG WITH A FLANGE

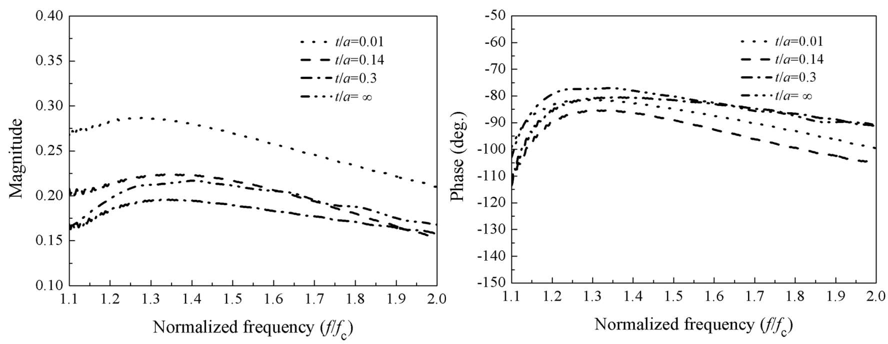
Figure 1(b) shows an open-ended rectangular waveguide with a flange of finite size. Flanges of standard rectangular waveguides from WR-19 to WR-112 are of square shape, where the flange width W ranges from 1.7 to 4.0 times the broad wall size a. The flange thickness ranges from 4.75 mm to 6.35 mm. For other standard waveguides, the flange is of rectangular or circular shape.

Figure 6 shows the simulated magnitude and phase of the reflection coefficient for flange sizes ranging from $2a$ to $10a$ obtained using the standard WR-34 waveguide. The effect of flange thickness is negligible so that we use 5 mm for the flange thickness. When the



(a) (b)

Figure 4. Aperture reflection coefficient at the open-end of the standard WR-34 waveguide without a flange. (a) Magnitude, and (b) phase.



(a) (b)

Figure 5. Effect of the waveguide wall thickness on the reflection coefficient. (a) Magnitude, and (b) phase.

flange width is increased beyond $4a$, there is almost no change in the reflection coefficient. For flange widths from $2a$ to $4a$, the magnitude and phase of the reflection coefficient change by 0.02 and 7 degrees, respectively. In these cases, one can use formulas for infinite flange in the approximate calculation of the reflection coefficient.

We obtained following formulas for the magnitude and phase of the aperture reflection coefficient of an OERW with an infinite flange by curve-fitting numerical results.

$$|\Gamma| = -738.057+367.292r+827.478r^{-1}-96.706r^2 -489.48r^{-2}+10.526r^3+119.1r^{-3}+0.12(a/b-2) \quad (6)$$

$$\arg(\Gamma)^\circ = 10^5 \times \big(0.046015-0.35995924r^{-1}+1.09304r^{-2}-1.64703r^{-3} +1.235497r^{-4}-0.369695r^{-5}\big)+50(a/b-2)r^{-3} \quad (7)$$

where r is the normalized frequency given by (4) and waveguide dimensions a and b are denoted in Fig. 1(b). Formulas (6), (7) are valid for

$$0.40 \le b/a \le 0.52, \quad 1.1 \le f/f_c \le 2.0, \quad W \ge 4a \quad (8)$$

Figures 7 and 8 show the magnitude and phase of the reflection coefficient at the open end of standard rectangular waveguides WR-90 and WR-34 with an infinite flange for which b/a is 0.444 and 0.5, respectively. In Fig. 7, the accuracy of formulas (6), (7) is compared analytical results by Tan [9] and Bois [10]. Fig. 8 compares proposed formulas with numerical simulation. In Figs. 7 and 8, we observe an excellent accuracy of the proposed formulas.

The normalized aperture admittance $\bar{Y}_a$ of an open-ended waveguide can be calculated from the reflection coefficient Γ using

$$\bar{Y}_a = \frac{Y_a}{Y_0} = \frac{1-\Gamma}{1+\Gamma} \quad (9)$$

where Y_a and Y_0 are the aperture admittance and the equivalent characteristic admittance of the waveguide, respectively. To find an un-normalized aperture admittance Y_a, one can use the following formula

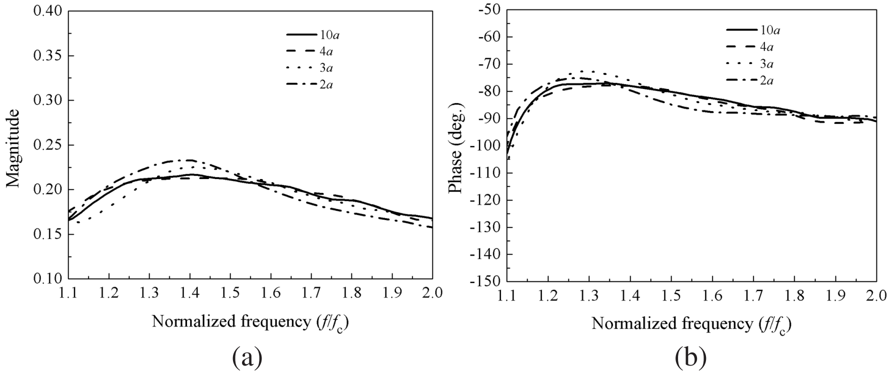


Figure 6. Effect of the flange size on the reflection coefficient. (a) Magnitude, and (b) phase.

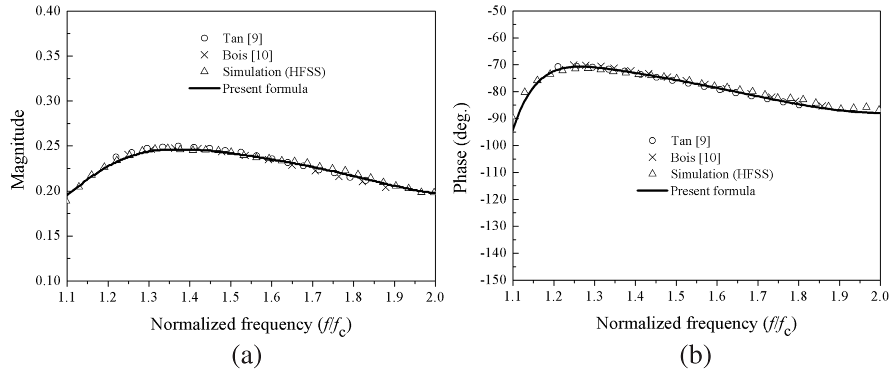


Figure 7. Aperture reflection coefficient of the open-ended WR-90 waveguide with an infinite flange. (a) Magnitude, and (b) phase.

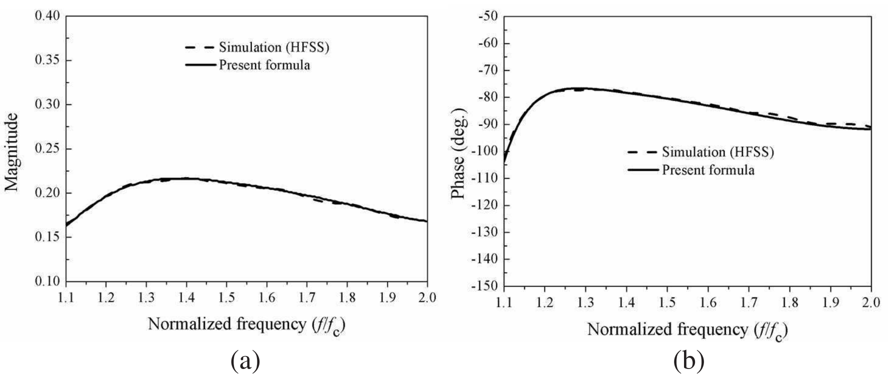


Figure 8. Aperture reflection coecient of the open-ended WR-34 waveguide with an infinite flange. (a) Magnitude, and (b) phase.

for the equivalent characteristic impedance of the TE_{10} mode in a rectangular waveguide.

$$Z_0 = \frac{1}{Y_0} = \frac{2b}{a} \frac{\sqrt{\mu_0/\varepsilon_0}}{\sqrt{1-(f/f_c)^2}} \tag{10}$$

In Fig. 9, we computed the aperture admittance of the WR-90 waveguide with an infinite flange and compared it with analytical and experimental results available in the literature [10–12]. Formulas (6), (7) agree well with analytical results by Bois [10], Bodnar [11], and Baudrand [12]. It appears that measurement by Bodnar [11] contains small experimental errors.

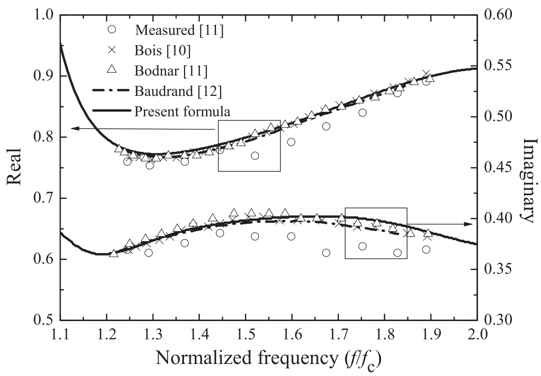


Figure 9. Normalized aperture admittance of the open-ended WR-90 waveguide with an infinite flange.

4. CONCLUSION

New formulas are presented for the aperture reflection coefficient of an open-ended rectangular waveguide (OERW) radiating into air including the effect of wall thickness or flange. The proposed formulas are obtained by curve-fitting numerical simulation by commercial electromagnetic software. The aperture aspect ratio b/a of the standard rectangular waveguide is also include. Proposed formulas agree well with results available in the literature. Proposed formulas are in simple form and significantly improve the accuracy of existing formulas. Formulas presented in this paper can be useful in many related fields such as in the calculation of the gain of an open-ended waveguide probe used in near-field antenna measurements.

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