FREQUENCY-SELECTIVE ENERGY TUNNELING IN WIRE-LOADED NARROW WAVEGUIDE CHANNELS

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Abstract—Frequency-dependent energy tunneling that results in full transmission of electromagnetic energy through wire-loaded sharp waveguide bends is demonstrated by full-wave simulations. The frequencies at which the tunneling takes place is predicted by a numerical method that involves a variational impedance formula based on Green function of a probe-excited parallel plate waveguide. Analogous tunneling effects have also been previously observed in waveguide bends filled with epsilon-near-zero media. However, since the frequency response in the wire-loaded waveguides can be tailored by simply modifying the lengths of the wires, the phenomenon is scalable over a broad range of frequencies and can be potentially exploited in various filtering and multiplexing applications.

1. INTRODUCTION

Tunneling is referred to the phenomenon when the electromagnetic energy propagates in narrow channels along prescribed directions in highly dispersive media. The direction of propagation is determined by a particular relationship between the phase and group velocities. If the impedance is correctly matched in these directions, full transmission of energy is possible. Energy tunneling through narrow waveguide channels and bends filled with materials with near-zero electric

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permittivities have been explored in several recent studies [1–4]. One such arrangement, depicted in Fig. 1(a), is a 180° short-circuited bend in which two parallel-plate waveguides are connected through a narrow channel filled with an epsilon-near-zero (ENZ) material. In this waveguide configuration, the condition of full transmission is obtained by restricting the aperture A of the ENZ material to a small value [1].

More recently, analogous energy-tunneling effects have also been observed in waveguide bend loaded with wires, as shown in Fig. 1(b). The metallic wires, in such a configuration, are separated by a periodicity of T and are directed parallel to the electric (E) field [5]. The concept was inspired by the method of excitation of the waveguide cavities using probe antennas [6]. In such situations, the reflection coefficient at the entrance of the waveguide at a particular frequency can be minimized by optimizing various parameters of the probing system such as probe diameter $2r$, its length inside the waveguide l and separation from the conductor backing h. At the tunneling frequency, the energy is coupled from one waveguide to the other when the two probes operate at the impedance-matched condition. Numerically, this condition is calculated by writing the impedance in terms of the current density and the dyadic Green function of the probe exciting a parallel-

(a)   (b)   (c)   (d)
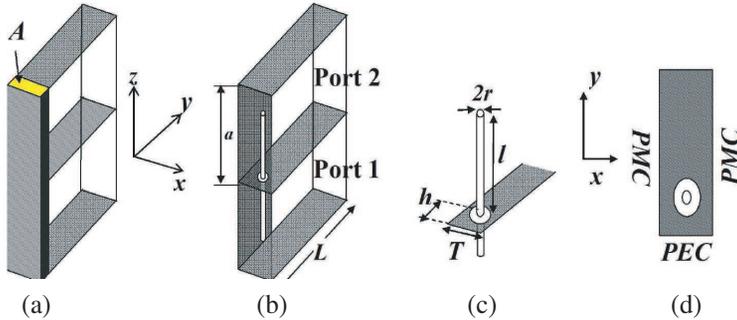
Figure 1. Unit cells of two different waveguide geometries that support electromagnetic energy tunneling (a) Two short-circuited wave guides connected by a thin layer of epsilon-near-zero material of cross-sectional area A and (b) the two waveguides connected with a cylindrical wire structure. The shaded walls are the perfectly electric conductors (PECs) and the un-shaded walls are perfectly magnetic conductors (PMCs) (c) various dimensions of the central conductor layer (d) top view of the central conductor layer showing the boundary conditions.

plate waveguide [6, 7]:

$$Z_{in} = -\frac{1}{I_{in}^2} \int_{S_o} J(r) \overline{G}(r|r') \cdot J(r') dS dS' \tag{1}$$

where S_o is the surface of the probe inside the waveguide. Assuming a y-directed uniform angular current density $J(r) = I(y)/2r\hat{y}$, the Green function is given by the following expression [7]:

$$\begin{aligned} G_{yy} &= \frac{jZ_o}{aTk_o} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{\varepsilon_{om}\varepsilon_{on}k_m^2}{\Gamma_{nm}} \cos\frac{n\pi x}{T} \cos\frac{n\pi x'}{T} \\ &\cdot \cos\frac{m\pi y}{T} \cos\frac{m\pi y'}{T} e^{-\Gamma_{nm}(z_>+h)} \sinh(\Gamma_{nm}z_< + h) \end{aligned} \tag{2}$$

where Z_o and k_o are the intrinsic impedance and wavenumber of the medium that fills the waveguide, ε_{on} is the Neumann factor which is equal to 0 for $n = 0$ and equals to 2 otherwise. $z_>$ is the larger of z and z' and $z_<$ is the smaller of the two. The rest of the variables are defined as:

$$k_m^2 = \left(\frac{m\pi}{a}\right)^2 - k_o^2 \tag{3}$$

$$\Gamma_{nm}^2 = \left(\frac{n\pi}{T}\right)^2 + k_m^2 \tag{4}$$

When simulated, the unit cell of the wire-loaded waveguide bend (Fig. 1(b)) mimics a parallel plate waveguide which extends infinitely along the $\pm x$-coordinates. The results can also be extended to the TE_{10} mode of a 3D waveguide by a simple transformation of the effective dielectric constant [4]. The numerical algorithm determines the tunneling frequency by searching the frequency at which the input impedance contains no imaginary part. The corresponding full-wave simulations of the transmission coefficient show a resonance peak at this frequency. Hence, the waveguide system acts as a cavity whose frequency response can be tuned by simply varying the length of the wires without affecting the size of the waveguide [5].

2. MULTI-CHANNEL FILTERING

The 180° waveguide bend of Fig. 1(b) can be designed to operate on multiple resonances by adding more wires of different resonant lengths. As a representative example, consider the short-circuited waveguide bend loaded with two wires of lengths 3 mm and 2 mm, as depicted in the inset of Fig. 2. The cavity resonates at two different frequencies which are estimated to be 26.8 and 32 GHz by the Green Function numerical method [7]. The unit cell structure is
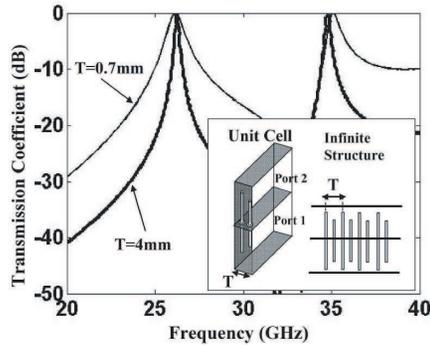
Figure 2. Transmission coefficients for dual-band cavities (shown in the inset) for two different periodicities. The inset shows the unit cell with the waveguide parameters given by $a = 3.56\,\text{mm}$ and $L = 5\,\text{mm}$ and the wire parameters: $r = 0.04\,\text{mm}$, $h = 0.178\,\text{mm}$, the lengths of the wire $l = 3\,\text{mm}$ and $2\,\text{mm}$. The boundary conditions, given in Fig. 1(d), compel the unit cell to behave as an infinitely extended structure along the x-axis.

also simulated on the finite-element full-wave simulator Ansoft's HFSS and the transmission coefficients (S_{21}) are evaluated for two different periodicities ($T = 0.7\,\text{mm}$ and $T = 4\,\text{mm}$). As shown in Fig. 2, the resonances for both the cases are observed at about 25.5 and 35 GHz.

This difference between the predicted the simulated resonant frequencies is partially related to the numerical approximations that are assumed in the Green function method [7]. Moreover, the numerical method does not take into account the mutual coupling between the two wires. The simulated electric field surface plots (not shown here) show that the lower frequencies are channeled through longer wires, which is consistent with the properties of resonance. The resonant wire is clearly identified by the enhanced surface fields. The field enhancement in individual wires is more when the periodicity increases because the electric field then channels through a fewer number of wires resulting in a narrower transmission bandwidth (as shown in Fig. 2 for $T = 0.4\,\text{mm}$).

Here, it is interesting to point out the difference between the tunneling mechanisms in ENZ-based and wire-loaded waveguides. The ENZ tunneling is characterized by uniform and much enhanced electric field distributions throughout the channel resulting in very small phase variations [4]. The tunneling in wire-loaded waveguides, on the other hand, is governed by resonances that are characterized by large phase changes and non uniform and enhanced electric and magnetic
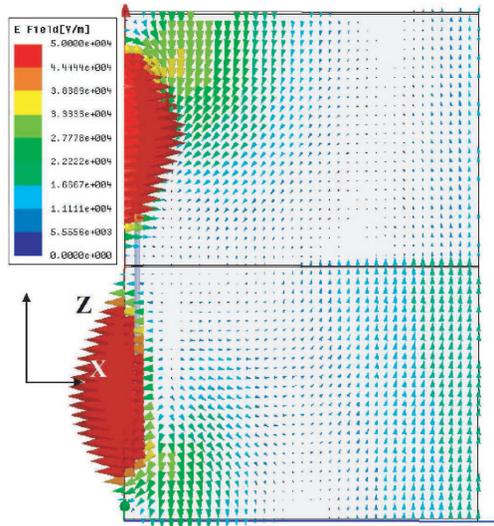
Figure 3. Transmission coefficients for dual-band cavities (shown in the inset) for two different periodicities. The inset shows the unit cell with the waveguide parameters given by $a = 3.56$ mm and $L = 5$ mm and the wire parameters: $r = 0.04$ mm, $h = 0.178$ mm, the lengths of the wire $l = 3$ mm and 2 mm. The boundary conditions, given in Fig. 1(d), compel the unit cell to behave as an infinitely extended structure along the x-axis.

field distributions. For example, consider the electric field vector distribution at resonance on one of the PMC's wall of the unit cell under study, depicted in Fig. 3. The strong non-uniform electric field experiences a 180° phase variation as it propagates through the channel at 25.5 GHz resonance.

3. FREQUENCY DISCRIMINATION

The concept of tunneling in the wire-loaded waveguides can be used to discriminate between multiple frequency channels (multiplexing or demultiplexing). To obtain a diplexer, a third waveguide of equal size is added to the previously discussed 180° waveguide and the two wires are placed in such a way that they overlap in the middle cavity (Fig. 4(a)). The input signal containing the two low and high frequency channels is fed at port 1. The two signals are separated in the middle cavity because each of them takes the minimum impedance path in reaching
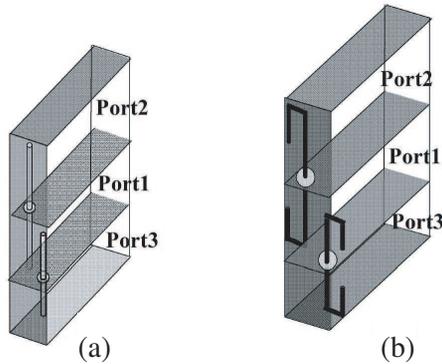
Figure 4. Two different geometries of a three waveguide arrangement used to discriminate two frequency channels: (a) with straight wires of lengths l_1 and l_2 and (b) with folded wires. The end sections of the wires overlap in the central cavity.
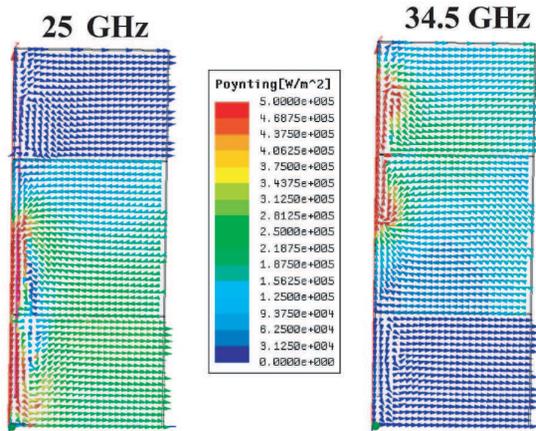
Figure 5. Power flow on the PMC walls of the three port device for the two different frequencies showing the channeling of the two channels to different outputs.

the output port. The separation of the two channels is elaborated in Fig. 5 by the aid of the Poynting Vector distribution that is observed on the PMC walls of the diplexer. As a result, the low frequency channel appears at port 2 after tunneling through the longer resonant wire and the higher frequency channel appears at port 3. The *s*-parameter plot (Fig. 6) shows acceptable transmission and reflection characteristics.

However, the inductive coupling between the two wires results in a poor isolation (S_{32}) of about -8 dB at port 2. The isolation can be improved by decreasing the length of the overlap (given by $l_1 + l_2 - a$) between the two wires in the middle guide. A way to decrease the overlap is to increase the size of the middle waveguides so that the two wires are moved vertically away from each other.
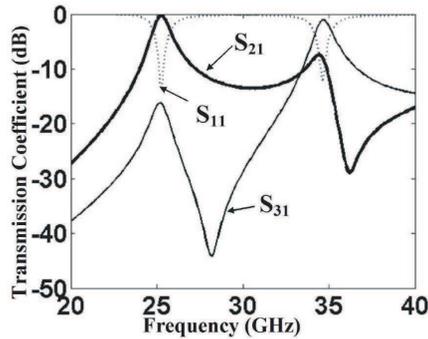
Figure 6. S-parameters for the three-port device of Fig. 4(a) showing the frequency discrimination. The isolation between the two channels is poor on port 3.
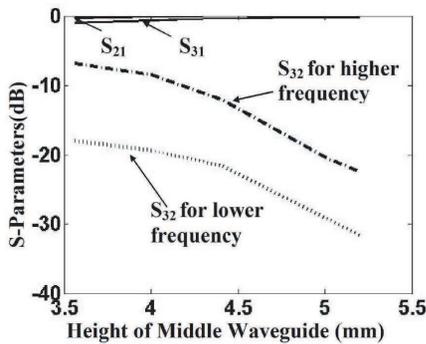
Figure 7. Effect of increasing the height of the middle waveguide on the s-parameters of the three-port device shown in Fig. 3(a). As the size of the waveguide increases, the overlap length between the two wires also increases resulting in better isolation at the output ports 2 and 3.
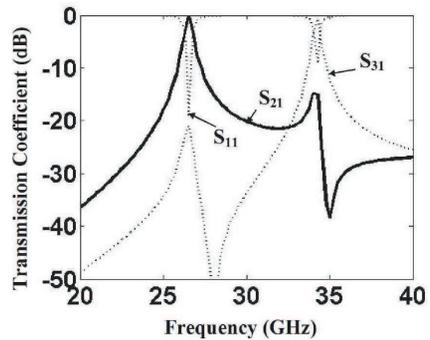
Figure 8. S-parameters for the diplexer of Fig. 4(b). The folded wire scheme renders less overlap between the wires resulting in better isolation between the output ports.

To demonstrate the improvement in isolation characteristics, the height of the middle waveguide is varied from 3.56 mm to 5.2 mm and the positions of the wires are adjusted so that the overlap length decreases from 2.44 mm to −0.2 mm. The results are summarized in Fig. 7. A considerable improvement in isolation (of about −10 dB) is noted on the output ports. The improvement in the isolation also results in decreased power leakage to from port 1 to port 2 resulting in better transmission characteristics. Another method of decreasing the overlap, which also has an advantage over the previous method as it renders equal heights of all waveguides, is to implement the wires in folded or meandered form as depicted in Fig. 4(b). As shown in Fig. 8, improved isolation parameters for both the channels are noted compared to results given in Fig. 6.

4. CONCLUSION

In this paper, we have studied the frequency-dependent tunneling of electromagnetic energy as it propagates through tight bends loaded with resonant wires. The full transmission between two waveguides connected in a 180° bend is obtained by simply loading it with wires of appropriate length. The wire in the two cavities act similar to matched probe antennas channeling maximum power at the resonant frequencies. The tunneling is characterized by non-uniform spatial field distributions and large phase changes that resemble a Fabry-Perot resonance. The frequency response of such wire-loaded cavities primarily depends on the lengths of the resonant wires and therefore can be tailored to any desired range of frequencies. Two potential applications namely a multiple channel waveguide filter and a diplexer are simulated using full-wave electromagnetic solver.

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