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SELF-CONSISTENT APPROACH TO THE ELECTRODYNAMIC ANALYSIS OF THE CHIRAL STRUCTURES

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Abstract—Rigorous electrodynamic analysis is proposed to estimate the surface current density on the perfectly conducting chiral elements in the diffraction problems. It is reduced to the solution of the integral singular equations. The diffraction of plane electromagnetic wave on the cylindrical open ring is considered as an example.

1. INTRODUCTION

In the scientific world the concept of chirality is currently considered to be a universal property of the existence of objective reality of the world. Usually, chirality is a property of living (or non-living) object, which combines with its reflection in the plane mirror attached to a translation and rotation [1]. Chirality, in essence, is a manifestation of the asymmetry of the left and right in the wildlife and abiocoen.

Natural media, possessing chiral properties, were known since the XIX century in optics, which were called optically active. The phenomenon of optical activity was discovered in 1811 by French scientist François Arago in quartz [1, 2]. In 1860 the famous biologist Louis Pasteur explained the nature of the phenomenon of molecular asymmetry when the form of the molecules of left- and right-handed isomers relate to each other as mirror images.

Media possessing chiral properties in the microwave range can only be artificial. Chiral "molecules" in the microwave are artificial conducting one-, two- or three-dimensional microcells with mirror asymmetric shape, and their scales are much smaller than the length of the electromagnetic wave λ [3, 4].

We can use different pieces of a flat metal open rings as the "construction" material [5]. Fig. 1(a) shows the chiral structure formed

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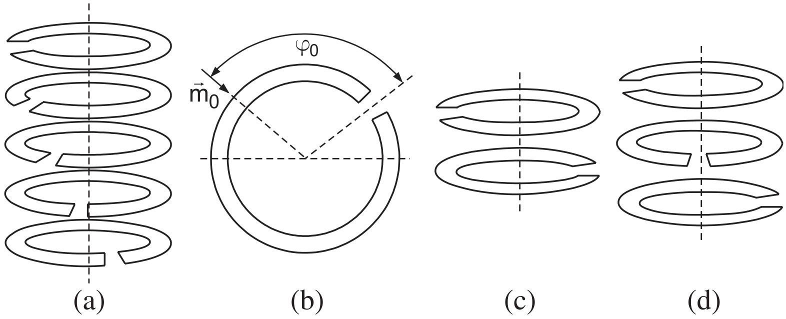

Figure 1. Chiral structure formed disconnected rings.

by this technology. We have intended to identify key tasks with which we can build a chiral structure (see Figs. 1(b)–(d)). The simplest key task is the diffraction of a plane electric wave *E*- and *H*-polarizations on the flat open ring (see Fig. 1(b)) [6]. φ_0 is the angle between the direction of the incident wave $\vec{m}_0$ and a line drawn from the center of the ring in the center of the gap. The system of the open rings with a certain spatial rotation of the gap is analogous to the helix.

We use self-consistent approach to resolve this problem.

2. RIGOROUS ALGORITHM FOR THE ANALYSIS OF THE CHIRAL STRUCTURES

Below, we give a rigorous self-consistent algorithm for analysis of inner problems (in terms of antenna terminology) of the chiral structures to accommodate the size of chiral elements [7]:

(i) The singular integral presentation (SIP) of the electromagnetic field is recorded through a surface current density on the antenna.

(ii) A singular integral equation (SIE) determines the surface current density. SIE is formed from the SIP during its consideration on the surface of the antenna.

(iii) SIE solving.

Main advantages of the algorithm are the establishment of the continuous transition: the surface current density $\leftrightarrow$ electric field strength that is absent in the standard method for calculating the electromagnetic field in the near field of the antennas [8]. Sometimes in dealing with inner problems, the first paragraph of this list can be skipped, and the SIE is deduced when the electromagnetic field is considered on the antenna.

3. DIFFRACTION OF A PLANE ELECTROMAGNETIC WAVE *H*-POLARIZATION ON THE PERFECTLY CONDUCTING CYLINDRICAL OPEN RING

Two-dimensional problem of diffraction of plane electromagnetic wave (PEMW) on an infinite perfectly conducting cylinder was resolved [9, 10]. A similar problem was solved for the dielectric cylinder [11]. We solved the problem of diffraction PEMW E- and H-polarizations on the dielectric circular cylinder with a perfectly conducting narrow metal strip of finite length on the lateral surface [12]. The field pattern of the diffracted field depends on the angle between incident wave and the metal strip on a dielectric cylinder. Below, we consider the problem of diffraction PEMW H-polarization on the perfectly conducting open ring. The vector of the magnetic field strength of the incident wave is in the ring plane and perpendicular to its axis z. Diffracted field is concentrated in the area of the gap when we select a certain amount of the gap.

The problem is explained in Fig. 2. PEMW falls at the perfectly conducting infinitely thin cylindrical open ring, located on an imaginary cylindrical surface $\rho = a$. φ_0 is the angle of incidence. $2h$ is the width of the strip. 2ξ is the angular length. $2l_s = 2\xi a$ is linear length. $2\Delta = 2\pi - 2\xi$ is the angular width of the gap. $2l_g = 2(\pi - \xi)\sin(\xi)/\xi$ is the linear width of the gap. The boundary conditions on the surface of the metal ring ($\rho = a$, $|\varphi| \le \xi$, $|z| \le h$) is

$$\vec{E}_\tau^{(\eta)} + \vec{E}_\tau^{(0)} = 0, \tag{1}$$

where $\vec{E}_\tau^{(\eta)}$ is the tangential electric field vector that is excited by the surface current density vector $\vec{\eta}$. $\vec{E}_\tau^{(0)}$ is the tangential electric field vector of the PEMW incident on the ring. φ_0 is the angle between the direction of incidence PEMW and the middle of the gap.

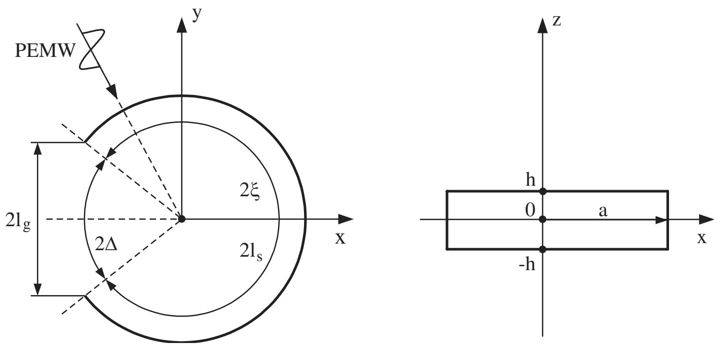

Figure 2. Geometry of the problem.

We introduce to simplify the following:

- The conductor is assumed perfectly conducting, infinitely thin, and fairly narrow ($2h \ll a$, $2h \ll \lambda$, where λ is the free-space wavelength), so we can consider only the longitudinal component of the surface current density $\eta_\varphi(\varphi, z)$.
- Surface current density equals zero at the ends of the strip ($\eta_\varphi(\varphi = |\xi|, z) = 0$).
- The distribution of surface current density is a quasi-static across the width in the first approximation.

$$\eta_\varphi(\varphi, z) = \frac{f(\varphi)}{\sqrt{1 - (z/h)^2}}, \tag{2}$$

where $f(\varphi)$ is unknown function describing the azimuthal distribution of the surface current density.

Let PEMW fall on the structure with angle φ_0. PEMW has three components $H_z^{(0)}, E_\rho^{(0)}, E_\varphi^{(0)}$. Volume current density on the strip with simplifications is

$$j_\varphi(\rho, \varphi, z) = \eta_\varphi(\varphi, z)\delta(\rho - a); \quad |z| \le h, \quad |\varphi| \le \xi. \tag{3}$$

We have two components of the vector potential in this formulation:

$$\left\{ \begin{array}{c} A_\rho(p) \\ A_\varphi(p) \end{array} \right\} = \int\limits_{V'} j_\varphi(q) \left\{ \begin{array}{c} \sin(\varphi - \varphi') \\ \cos(\varphi - \varphi') \end{array} \right\} G(p, q) dV', \tag{4}$$

where the Green function $G(p, q)$ is defined as follows:

$$G(p, q) = \frac{1}{4\pi} \frac{e^{-ikR}}{R}. \tag{5}$$

The distance R between the point source $q = \{\rho', \varphi', z'\}$ and observation point $p = \{\rho, \varphi, z\}$ in cylindrical coordinates is valid:

$$R = \sqrt{\rho^2 + \rho'^2 - 2\rho\rho' \cos(\varphi - \varphi') + (z - z')^2}.$$

We express electric field strength $\vec{E}$ and magnetic field strength $\vec{H}$ in terms of the vector potential (5) and substitute in (2). The SIE has the form (t, $t' \in [-1; 1]$):

$$\begin{aligned} \sigma E_\varphi^{(\eta)}(t) \;=\; & \int\limits_{-1}^{1} f(t') R_1(t, t') dt' - \int\limits_{-1}^{1} f'(t') R_2(t, t') dt' \\ & -\vartheta^2 \xi^2 \int\limits_{-1}^{1} f(t') \ln\left|t - t'\right| dt' + \int\limits_{-1}^{1} \frac{f'(t')}{t - t'} dt', \end{aligned} \tag{6}$$

where:

$$R_p(t,t') = \frac{1}{2\pi d\xi}\left[K_p(t,t') - \lim_{t\to t'} K_p(t,t')\right]; \quad p = 1, 2,$$

$\vartheta = ka$, $d = h/a$. $\sigma = 2\pi\vartheta\xi/(iW_c)$ is constant, $W_c = 2$.

SIE (7) is solved by the moments method. Unknown function $f(t')$ and its first-order derivative $f'(t')$ are presented as a polynomial series:

$$f(t') = \sum_{k=1}^{N} \sqrt{1-t'^2} A_k U_{k-1}(t'); \quad f'(t') = -\sum_{k=1}^{N} \frac{kA_k T_k(t')}{\sqrt{1-t'^2}}, \qquad (7)$$

where $T_k(x)$ is Chebyshev polynomial of the first kind of order k. $U_k(x)$ is Chebyshev polynomial of the second kind of order k. A_k is unknown coefficients of expansion.

The dependence of the normalized amplitude field pattern of the diffracted field is studied:

$$F_\varphi = \frac{|E_\varphi(\rho,\varphi)|}{|E_{\varphi\max}(\rho,\varphi_0)|}, \qquad (8)$$

where $E_\varphi(\rho,\varphi,z = 0)$ is intensity of the diffracted electric field in the far field ($k\rho \gg 1$), produced by surface currents on the ring, the direction of incidence wave and the geometry of the open ring. If the direction equals zero ($\varphi = 0$) and the ring size $L_e = 2l_g/\lambda \approx 1$ ($\Delta = 9°$), we can get one-way mode of distribution of the diffracted field (see Fig. 3).

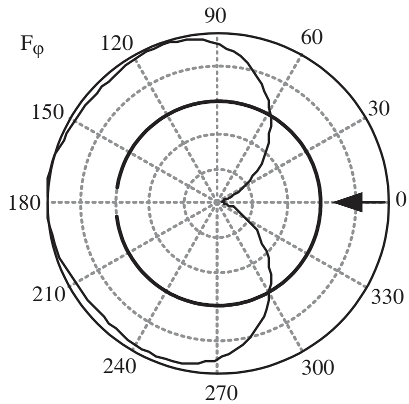

Figure 3. The distribution of $F_\varphi(\varphi)$ in the far field ($k\rho = 2000$). $\varphi_0 = 0$, $L_e = 1.15$ (geometry of the structure is shown directly on the plot).

As an example of the chiral structure on the basis of metal open rings, low reflecting conformal coating may be used, which reduces radar visibility of an object. This coating contains three layers: the first layer, which lies directly on the protected object, is an absorber made of radio absorbing material, and the second and third layers are orthogonal three-dimensional arrays in the form of identically oriented flat metal open rings embedded in a dielectric with relative static permittivity ε. The average radius of the plane rings a and the distance between the centers of neighboring rings d are determined from:

$$a = \frac{\lambda_n}{2\pi\sqrt{\varepsilon}}, \quad d \leq \frac{\lambda_n}{4\sqrt{\varepsilon}},$$

where λ_n is the central wavelength of the range of the electric waves that incident on the protected object (see Fig. 4).

Such a low reflecting coating may be used in the ground, water, aviation and space technics to reduce radar visibility of objects.

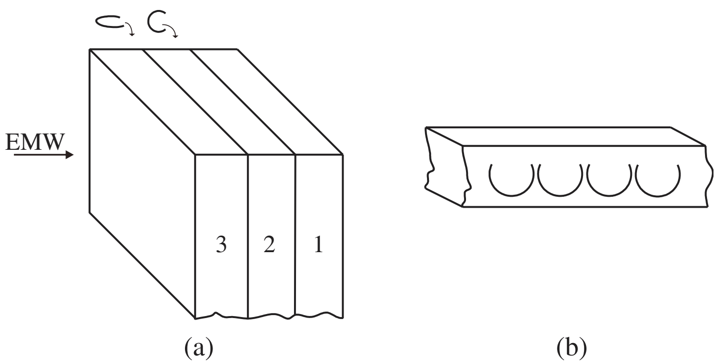

Figure 4. Low reflecting coating structure (a) and direct box with a dielectric array in the form of identically oriented flat open rings on one of its surfaces (b). 1 — absorber, 2 — layer composed of vertically glued dielectric parallelepipeds, 3 — layer that composed of horizontally bonded dielectric parallelepipeds (b).

4. CONCLUSION

The way to create artificial chiral structures on basis of metal open rings that are located on the dielectric layers was offered. For the electrodynamic modeling, such structures are encouraged to use self-consistent method based on a physical regularization of ill-posed electrodynamic problems [7]. The diffraction of plane electromagnetic wave on the cylindrical open ring is described as an example. These open rings are used for constructing low reflecting coating structures.

REFERENCES

1. Prohorov, A. M., *Encyclopedic Dictionary of Physics*, Bol'shaja Rossijskaja Jenciklopedija, Moscow, 1995.
2. Vol'kenshtejn, M. V., *Molecular Optics*, Nauka, Moscow-Leningrad, 1951.
3. Lindell, I. V., A. H. Sihvola, S. A. Tretyakov, and A. J. Viitanen, *Electromagnetic Waves in Chiral and Bi-isotropic Media*, Artech House, London, 1994.
4. Laktakia, A., V. K. Varadan, and V. V. Varadan, *Time-harmonic Electromagnetic Fields in Chiral Media. Lecture Notes in Physics*, Springer-Verlag, Berlin-Heidelberg-Boston, 1989.
5. Neganov, V. A., "Chiral metastructures on basis of plain flat metallic open rings," *Physics of Waves Propagation and Radio Systems*, Vol. 11, No. 3, 104–119, 2008.
6. Neganov, V. A., E. I. Prjanikov, and D. P. Tabakov, "Diffraction plane H-polarization electromagnetic wave on perfectly conducting open ring," *Physics of Waves Propagation and Radio Systems*, Vol. 11, No. 1, 22–29, 2008.
7. Neganov, V. A., *Physical Regularization of Ill-posed Problems of the Electrodynamics*, Sajns-Press, Moscow, 2008.
8. Sazonov, D. M., *Antennas and Microwave Devices*, Vysshaja Shkola, Moscow, 1988.
9. Markov, G. T. and A. F. Chaplin, *Excitation of Electromagnetic Waves*, Jenergija, Moscow-Leningrad, 1976.
10. Nikol'skij, V. V. and T. I. Nikol'skaja, *Electrodynamics and Propagation of the Waves*, Nauka, Moscow, 1989.
11. Neganov, V. A., O. V. Osipov, S. B. Raevskij, and G. P. Jarovoj, *Electrodynamics and Propagation of the Waves*, Radio i svjaz', Moscow, 2005.
12. Neganov, V. A. and A. A. Sarychev, "Calculation of field of plane electromagnetic wave reflected by perfectly conductive metal strip on dielectric cylinder," *Physics of Waves Propagation and Radio Systems*, Vol. 10, No. 1, 95–103, 2007.