Progress In Electromagnetics Research C, Vol. 13, 231–243, 2010

UTILIZING NONUNIFORM COUPLED TRANSMISSION LINES TO COMPACT MICROSTRIP CIRCUITS SUCH AS EDGE-COUPLED BANDPASS FILTERS

M. Khalaj-Amirhosseini and S. A. Hosseini-Tehrani

College of Electrical Engineering
Iran University of Science and Technology
Narmak, Tehran, Iran

Abstract—In this paper, we propose a method to reduce the length of the narrowband microstrip Uniform Coupled Transmission Lines (UCTLs), which has a general application to compact microstrip circuits. In this method, we use Nonuniform Coupled Transmission Lines (NCTLs) instead of UCTLs. To synthesize the desired NCTLs, their normalized width and gap are expanded as two truncated Fourier series. Then, the optimal values of the coefficients of the series are obtained through an optimization approach. The usefulness of the proposed method is verified using some examples. Also, an edged-coupled bandpass filter is compacted using the proposed method and then is fabricated and measured.

1. INTRODUCTION

It is required to compact microstrip circuits in many applications. On the other hand, many microstrip circuits contain one or some coupled transmission lines of specified length to work at a desired frequency. Therefore, one possible way to compact the microstrip circuits is to compact (to reduce the length of) the coupled transmission lines. Some efforts have been done to compact single transmission lines such as using DGS (Defected Ground Structures) [1], EBG (Electromagnetic Bandgap) [2], high impedance and meandering lines [3], fractal lines [4], stepped stubs [5] and nonuniform transmission lines [6, 7]. In this paper, the latter method is generalized to compact narrowband microstrip coupled transmission lines. In this way, we use Nonuniform Coupled Transmission Lines (NCTLs) instead of Uniform Coupled Transmission Lines (UCTLs). To synthesize the desired NCTLs of

Corresponding author: M. Khalaj-Amirhosseini (khalaja@iust.ac.ir).

arbitrary length, their normalized width and gap are expanded as two truncated Fourier series. Then, the optimal values of the coefficients of the series are obtained through an optimization approach. The usefulness of the proposed method is verified using some examples. Also, an edged-coupled bandpass filter at center frequency 1.5 GHz is designed and compacted using the proposed method and then is fabricated and measured.

2. NCTLS AS COMPACTED UCTLS

Figure 1 depicts a typical symmetrical microstrip Uniform Coupled Transmission Line (UCTL), whose width, gap, thickness, length and electric permittivity are w_0, s_0, h, d_0 and ε_r, respectively. Also, Fig. 2 depicts a typical symmetrical microstrip Nonuniform Coupled Transmission Line (NCTL), whose width, gap, thickness, length and electric permittivity are $w(z)$, $s(z)$, h, d and ε_r, respectively. We would like to design an NCTL so that its S parameters at frequency f_0 to be equal to those of a UCTL at the same frequency f_0. Furthermore, if we choose the length of the NCTL, d, smaller than that of the UCTL, d_0, the designed NCTL may be a considered as a compacted UCTL, in fact.

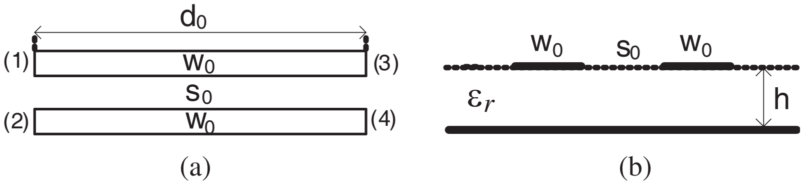

Figure 1. A typical microstrip uniform coupled transmission line, (a) longitudinal view, (b) the cross section.

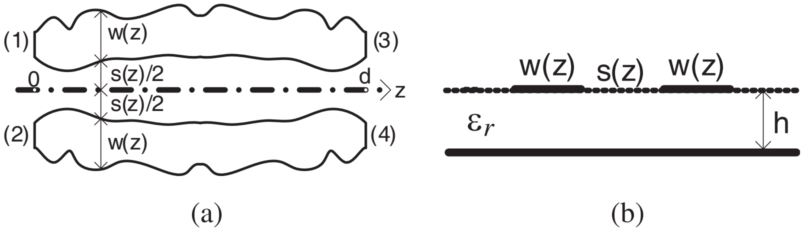

Figure 2. A typical microstrip nonuniform coupled transmission line, (a) longitudinal view, (b) the cross section.

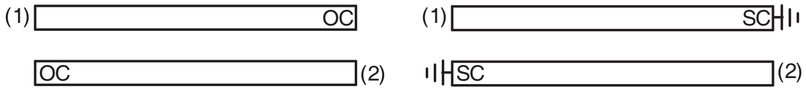

Figure 3. Longitudinal view of two special cases of coupled transmission lines (a) open circuited, (b) short circuited.

The well known *ABCD* parameters of UCTLs based on quasi-TEM assumption are given by [8]

$$\mathbf{T}_0 = \begin{bmatrix} \mathbf{A}_0 & \mathbf{B}_0 \\ \mathbf{C}_0 & \mathbf{D}_0 \end{bmatrix} = \exp\left(j\omega d_0 \begin{bmatrix} \mathbf{0} & \mathbf{L}_0 \\ \mathbf{C}_0 & \mathbf{0} \end{bmatrix} \right) \tag{1}$$

where $\mathbf{L}_0$ and $\mathbf{C}_0$ are the per-unit-length inductance and capacitance matrices of the UCTL, respectively. Also, there are some methods to obtain the *ABCD* parameters of NCTLs such as cascading many short sections [8, 9], finite difference [10], Taylor's series expansion [11], Fourier series expansion [12], the equivalent sources method [13], the method of Moments [14] and an approximate closed form solution [15]. Of course, the most straightforward to determine the *ABCD* parameters of NCTLs is dividing them into K uniform segments and then to use the following relation [8]

$$\begin{aligned}\mathbf{T} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} = \ & \exp\left(j\omega \frac{d}{K} \begin{bmatrix} \mathbf{0} & \mathbf{L}_1 \\ \mathbf{C}_1 & \mathbf{0} \end{bmatrix} \right) \cdots \exp\left(j\omega \frac{d}{K} \begin{bmatrix} \mathbf{0} & \mathbf{L}_k \\ \mathbf{C}_k & \mathbf{0} \end{bmatrix} \right) \\ & \cdots \exp\left(j\omega \frac{d}{K} \begin{bmatrix} \mathbf{0} & \mathbf{L}_K \\ \mathbf{C}_K & \mathbf{0} \end{bmatrix} \right) \end{aligned} \tag{2}$$

where $\mathbf{L}_k$ and $\mathbf{C}_k$ are the per-unit-length inductance and capacitance matrices of the k-th uniform segment, respectively. These matrices are related to the normalized width and gap functions $w(z)/h$ and $s(z)/h$ at the center of the k-th uniform segment and can be determined using some methods such as in [8, 16]. After finding the *ABCD* parameters (1) or (2), one can determine the S parameter matrix as follows

$$\mathbf{S} = \begin{bmatrix} \mathbf{S}_{11} & \mathbf{S}_{12} \\ \mathbf{S}_{21} & \mathbf{S}_{22} \end{bmatrix} \tag{3}$$

in which

$$\begin{aligned}\mathbf{S}_{11} &= \left[(\mathbf{A} + \mathbf{B}/Z_0)^{-1} + (\mathbf{C}Z_0 + \mathbf{D})^{-1} \right]^{-1} \\ &\quad \left[-(\mathbf{A} + \mathbf{B}/Z_0)^{-1} + (\mathbf{C}Z_0 + \mathbf{D})^{-1} \right] \end{aligned} \tag{4}$$

$$\begin{aligned}\mathbf{S}_{12} &= \left[(\mathbf{A} + \mathbf{B}/Z_0)^{-1} + (\mathbf{C}Z_0 + \mathbf{D})^{-1} \right]^{-1} \\ &\quad \left[(\mathbf{A} + \mathbf{B}/Z_0)^{-1}(\mathbf{A} - \mathbf{B}/Z_0) - (\mathbf{C}Z_0 + \mathbf{D})^{-1}(\mathbf{C}Z_0 - \mathbf{D}) \right] \end{aligned} \tag{5}$$

$$\mathbf{S}_{21} = 2[\mathbf{A} + \mathbf{B}/Z_0 + \mathbf{C}Z_0 + \mathbf{D}]^{-1} \tag{6}$$

$$\mathbf{S}_{22} = [\mathbf{A} + \mathbf{B}/Z_0 + \mathbf{C}Z_0 + \mathbf{D}]^{-1}[-\mathbf{A} + \mathbf{B}/Z_0 - \mathbf{C}Z_0 + \mathbf{D}] \tag{7}$$

where Z_0 is the assumed characteristic impedance. In the special cases such as open or short circuited shown in Fig. 3, which are two high applicable two-port circuits, the S parameter matrix (3) is reduced to the following ones, respectively.

$$\mathbf{S}_{\mathrm{OC}} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}_{\mathrm{OC}} = [\mathbf{S}_1 + \mathbf{S}_2 (\mathbf{I} - \mathbf{S}_4)]^{-1} \mathbf{S}_3 \tag{8}$$

$$\mathbf{S}_{\mathrm{SC}} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}_{\mathrm{SC}} = [\mathbf{S}_1 - \mathbf{S}_2 (\mathbf{I} + \mathbf{S}_4)]^{-1} \mathbf{S}_3 \tag{9}$$

in which $\mathbf{I}$ is a 2 by 2 identity matrix and also

$$\begin{bmatrix} \mathbf{S}_1 & \mathbf{S}_2 \\ \mathbf{S}_3 & \mathbf{S}_4 \end{bmatrix} \triangleq \begin{bmatrix} S_{11} & S_{14} & S_{12} & S_{13} \\ S_{41} & S_{44} & S_{42} & S_{43} \\ S_{21} & S_{24} & S_{22} & S_{23} \\ S_{31} & S_{34} & S_{32} & S_{33} \end{bmatrix} \tag{10}$$

3. SYNTHESIS OF NCTLS

In this section a general method is proposed to design symmetrical optimally microstrip NCTLs. First, we consider the following truncated Fourier series expansions for the normalized width and gap functions.

$$\ln\left(\frac{w(z)}{h}\right) = \sum_{n=0}^{N} C_n \cos(2\pi n z/d) \tag{11}$$

$$\ln\left(\frac{s(z)}{h}\right) = \sum_{n=0}^{N} S_n \cos(2\pi n z/d) \tag{12}$$

An optimal designed NCTL has to have the S parameters as close as possible to the S parameters of desired UCTL. Therefore, the optimal values of the unknown coefficients C_n and S_n in (11) and (12) can be obtained through minimizing the following defined error function.

$$\text{Error} = \sqrt{\frac{1}{4N^2} \sum_{i=1}^{2N} \sum_{j=1}^{2N} |\mathbf{S}(i,j) - \mathbf{S}_0(i,j)|^2} \tag{13}$$

In addition to above defined error function, one can use the following error functions for two special cases of open or short circuited shown in Fig. 3, respectively.

$$\text{Error} = \sqrt{\frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} \left| \mathbf{S}_{\text{oc}}(i,j) - \mathbf{S}_{0,\text{oc}}(i,j) \right|^2} \tag{14}$$

$$\text{Error} = \sqrt{\frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} \left| \mathbf{S}_{\text{sc}}(i,j) - \mathbf{S}_{0,\text{sc}}(i,j) \right|^2} \tag{15}$$

In (13)–(15), the subscript "0" refers to the UCTLs. Moreover, the above defined error functions should be restricted by some constraints such as easy fabrication and physical matching at two ends, like as the followings

$$\left(\frac{w}{h}\right)_{\min} \le \frac{w(z)}{h} \le \left(\frac{w}{h}\right)_{\max} \tag{16}$$

$$\left(\frac{s}{h}\right)_{\min} \le \frac{s(z)}{h} \le \left(\frac{s}{h}\right)_{\max} \tag{17}$$

$$\frac{w(0)}{h} = \frac{w(d)}{h} = \left(\frac{w}{h}\right)_{\text{end}} \tag{18}$$

where $(w/h)_{\min}$ and $(w/h)_{\max}$ are the minimum and maximum available normalized width as well as $(s/h)_{\min}$ and $(s/h)_{\max}$ are the minimum and maximum available normalized gap, respectively. Also, $(w/h)_{\text{end}}$ is desired normalized width at the ends of the microstrip lines, which can be choose as w_0/h.

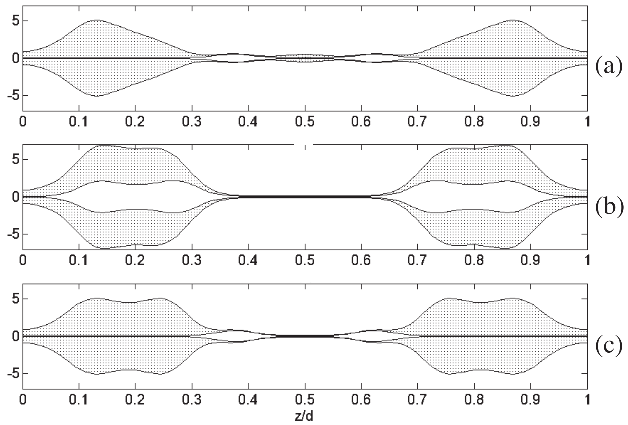

Figure 4. The top view of three designed microstrip NCTLs (up to down: No. 1, 2, 3).

It is mentionable that the aforementioned method can be employed for asymmetrical coupled transmission lines but with two relations similar to (11) for two dissimilar coupled lines.

To solve the above constrained minimization problem, we can use the fmincon.m file in the MATLAB program. fmincon uses a Sequential Quadratic Programming (SQP) method, in which a Quadratic Programming (QP) subproblem is solved at each of its iteration.

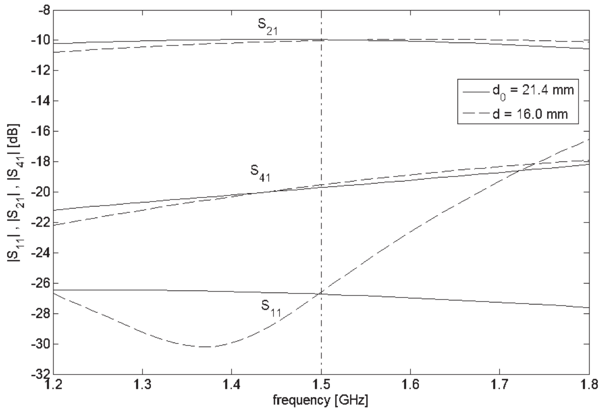

Figure 5. Comparing the magnitude of S parameters of designed NCTL No. 1 (4-Port, $d = 16\,\text{mm}$) with those of corresponding UCTL.

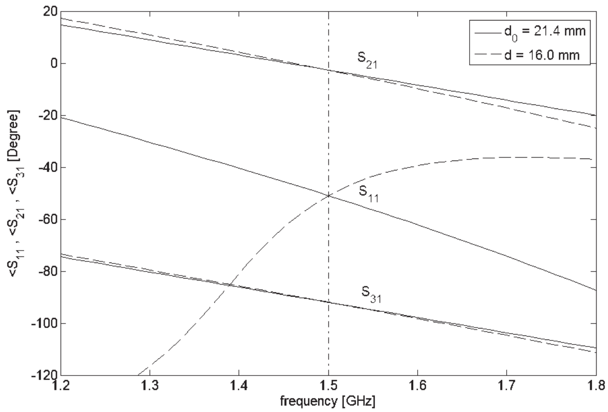

Figure 6. Comparing the angle of S parameters of designed NCTL No. 1 (4-Port, $d = 16\,\text{mm}$) with those of corresponding UCTL.

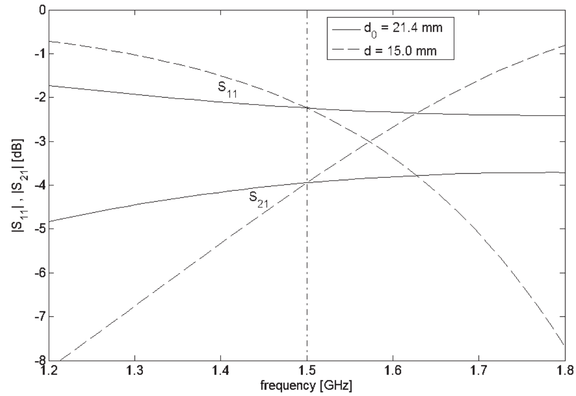

Figure 7. Comparing the magnitude of S parameters of designed NCTL No. 2 (O.C. 2-Port, $d = 15\,\mathrm{mm}$) with those of corresponding UCTL.

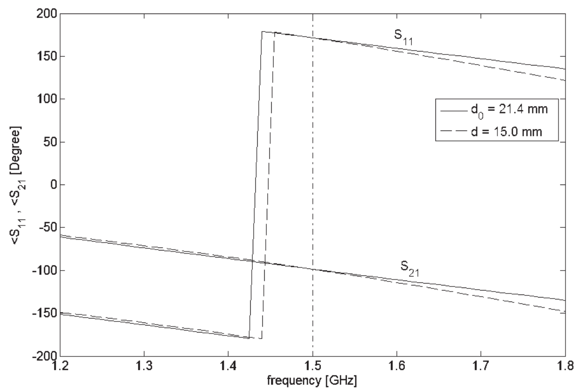

Figure 8. Comparing the angle of S parameters of designed NCTL No. 2 (O.C. 2-Port, $d = 15\,\mathrm{mm}$) with those of corresponding UCTL.

4. EXAMPLES AND RESULTS

Consider a microstrip UCTL with $\varepsilon_r = 9$, $w_0/h = 0.85$, $s_0/h = 0.25$ and $d_0 = 21.4$ mm, which acts as a 10 dB quarter wavelength coupler at frequency $f_0 = 1.5$ GHz with characteristic impedance $Z_0 = 50\,\Omega$. Three NCTLs have been optimally designed considering $N = 5$, $(w/h)_{\max} = (s/h)_{\max} = 6.3$, $(w/h)_{\min} = (s/h)_{\min} = 0.1$ and $(w/h)_{\text{end}} = w_0/h = 0.85$. Table 1 shows the unknown coefficients obtained from considering three defined errors in (13)–(15) assuming d

Table 1. The optimal values of the coefficients C_n and S_n for three designed NCTLs.

	n	0	1	2	3	4	5
4-Port	C_n	−0.2074	1.6706	−0.6052	−0.9515	0.3593	−0.4347
$d = 16$ mm	S_n	−2.0453	−0.9480	−0.0141	0.6887	−0.6856	0.3277
O.C. 2-Port	C_n	−0.0941	1.7827	−1.2885	−0.4530	0.2640	−0.3791
$d = 15$ mm	S_n	−0.3390	0.8752	−1.9878	−0.4589	−0.2562	−0.6604
S.C. 2-Port	C_n	−0.0131	1.8016	−1.3805	−0.3181	0.2384	−0.4974
$d = 15$ mm	S_n	−1.9357	−1.0693	0.0100	0.7668	−0.7493	0.3793

Table 2. The optimal values of the coefficients C_n and S_n for the bandpass filter.

	n	0	1	2	3	4	5
First and	C_n	0.3674	1.8933	−1.1729	−0.2056	0.2625	−0.3492
Fourth NCTLs	S_n	−1.611	−1.1357	0.9085	−0.6898	0.4723	−0.2457
Second and	C_n	0.3583	1.9010	−1.1837	−0.2006	0.2443	−0.3238
Third NCTLs	S_n	−0.9523	−2.1167	1.0140	−0.0746	−0.2729	0.1854

= 16 mm (25.2% compactness), $d = 15$ mm (29.9% compactness) and $d = 15$ mm (29.9% compactness), respectively. Also, Fig. 4 illustrates the top view of these three designed NCTLs. Furthermore, Figs. 5–10 compare the magnitude and angle of S parameters of these three designed NCTLs with those of corresponding UCTLs versus frequency. It is observed that the S parameters of designed NCTLs are very close to those of desired UCTLs. The resulted errors rs are 1.48×10^{-3}, 5.08×10^{-5} and 4.03×10^{-5}, respectively. Although the length of two-port open- and short-circuited NCTLs are chosen less than that of four-port NCTL but the errors of former ones are equal or even less than the error of the latter one.

In continuation, an edge-coupled 3-order bandpass filter with center frequency 1.5 GHz and bandwidth 100 MHz is designed using four open circuited UCTLs. The dimensions of the first and fourth UCTLs are $w_0/h = 1.78$, $s_0/h = 0.285$ and $d_0 = 30.20$mm. Also, the dimensions of the second and third UCTLs are $w_0/h = 2.17$, $s_0/h = 1.43$ and $d_0 = 29.87$ mm. This conventional BPF is compacted by replacing UCTLs with NCTLs of 30% compactness. Table 2 shows the unknown coefficients obtained from defined error in (14). Two conventional and compacted BPFs are then fabricated and measured. Fig. 11 shows the pictures of these BPFs, consisting of four microstrip coupled transmission lines on substrate with $\varepsilon_r = 3.5$. The normalized widths at the ends of all NCTLs have been chosen the same and equal

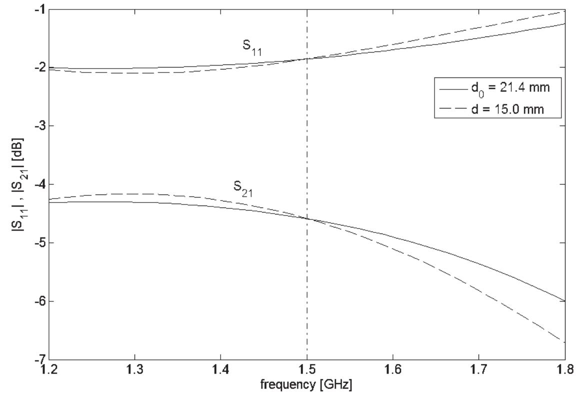

Figure 9. Comparing the magnitude of S parameters of designed NCTL No. 3 (S.C. 2-Port, $d = 15\,\mathrm{mm}$) with those of corresponding UCTL.

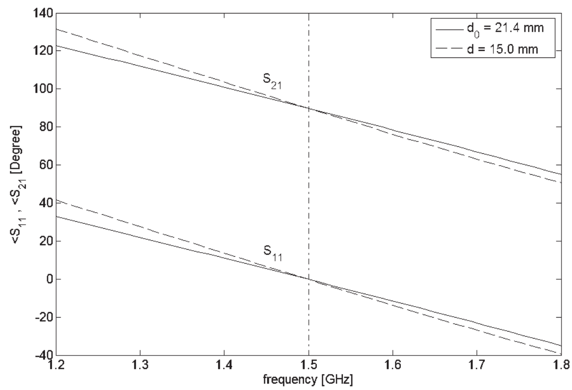

Figure 10. Comparing the angle of S parameters of designed NCTL No. 3 (S.C. 2-Port, $d = 15\,\mathrm{mm}$) with those of corresponding UCTL.

to that of a $50\,\Omega$ transmission line, i.e., $(w/h)_{\mathrm{end}} = 2.22$. Therefore, there is no discontinuity at the points of connection between two NCTLs. Figs. 12 and 13 illustrate the measured S parameters of two aforementioned BPFs. It is observed that the frequency response of compacted BPF is identical to that of conventional one regardless of a minus frequency shift. This frequency shift may be due to not considering the thickness of conductive strips, open-end effect and weak validation of quasi-TEM assumption at wide regions. To compensate the minus 200 MHz frequency shift, one may reduce the length of designed compacted BPF near to 15%, which gives totally 40% compactness for the final filter. It is interesting that the spurious responses existed in the conventional BPF have been weakened in the compacted one. The reason may be twofold; one is that the resonators based on NCTLs in this particular example is actually step impedance resonators, of with the harmonics are pushed away from the fundamental resonance. The other reason is the possible cancellation of coupling due to varied shape and therefore the spurious band cannot be constructed. It is worth of mention that some efforts such as in [17] to employing NCTLs for bandpass filters although not for the compacting purpose.

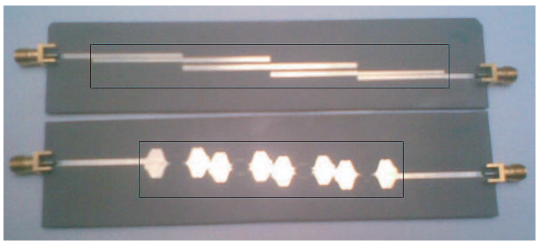

Figure 11. The picture of two edge-coupled BPF consisting of four UCTLs (up) and NCTLs (down).

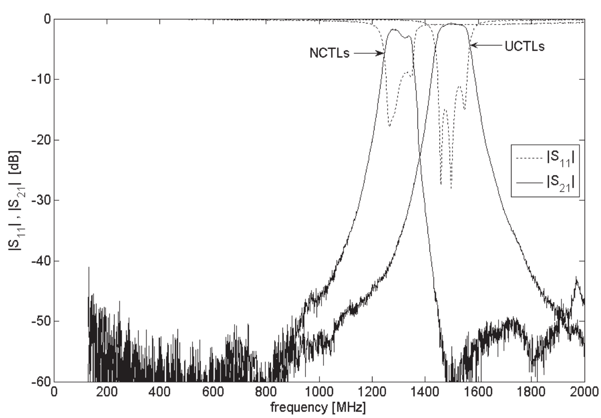

Figure 12. The measured S parameters of two conventional and compacted edge-coupled BPFs.

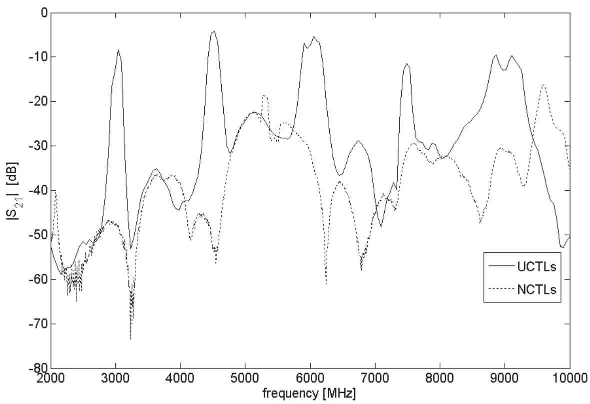

Figure 13. The measured S parameters of two conventional and compacted edge-coupled BPFs.

5. CONCLUSION

A new way to compact microstrip coupled transmission lines was proposed. In this way, we use Nonuniform Coupled Transmission Lines (NCTLs) instead of Uniform Coupled Transmission Lines (UCTLs). To synthesize the desired compact length coupled microstrip transmission lines, their normalized width and gap are expanded as two truncated Fourier series. Then, the optimal values of the coefficients of two series are obtained through an optimization approach. The usefulness of the proposed structures is verified using some examples. It was seen that the S parameters of the designed NCTLs could be close to those of the desired UCTLs. Finally, a 1.5 GHz edged-coupled bandpass filter is designed and compacted using the proposed method and then is fabricated and measured. It was seen that NCTLs do not have spurious responses in contrary to UCTLs.

REFERENCES

1. Dwari, S. and S. Sanyal, "Size reduction and harmonic suppression of microstrip branch-line coupler using defected ground structure," *Microwave and Optical Technology Letters*, Vol. 48, 1966–1969, 2006.
2. Falcone, F., T. Lopetegi, and M. Sorolla, "1-D and 2-D photonic bandgap microstrip structures," *Microwave and Optical Technology Letters*, Vol. 22, No. 6, 411–412, Sep. 1999.

3. Mandal, M. K., V. K. Velidi, A. Bhattacharya, and S. Sanyal, "Miniaturized quadrature hybrid coupler using high impedance lines," *Microwave and Optical Technology Letters*, Vol. 50, 1135–1137, May 2008.
4. Chen, W.-L. and G.-M. Wang, "Design of novel miniaturized fractal-shaped branch-line couplers," *Microwave and Optical Technology Letters*, Vol. 50, 1198–1201, May 2008.
5. Sakagami, I., M. Haga, and T. Munehiro, "Reduced branch-line coupler using eight two-step stubs," *IEE Proc.-Microw. Antennas Propag.*, Vol. 146, No. 6, 455–460, Dec. 1999.
6. Khalaj-Amirhosseini, M., "Nonuniform transmission lines as compact uniform transmission lines," *Progress In Electromagnetics Research C*, Vol. 4, 205–211, 2008.
7. Hosseini, F., M. Khalaj-Amirhosseini, and M. Yazdani, "A miniaturized Wilkinson power divider using nonuniform transmission line," *Journal of Electromagnetic Waves and Applications*, Vol. 23, No. 7, 917–924, June 2009.
8. Paul, C. R., *Analysis of Multiconductor Transmission Lines*, John Wiley and Sons Inc., 1994.
9. Khalaj-Amirhosseini, M., "Using linear sections instead of uniform ones to analyze the coupled nonuniform transmission lines," *Int. Journal of RF and Microwave Computer-aided Engineering*, Vol. 19, No. 1, 75–79, January 2009.
10. Khalaj-Amirhosseini, M., "Analysis of coupled or single nonuniform transmission lines using step-by-step numerical integration," *Progress In Electromagnetics Research*, PIER 58, 187–198, 2006.
11. Khalaj-Amirhosseini, M., "Analysis of coupled nonuniform transmission lines using Taylor's series expansion," *IEEE Trans. Electromagn. Compat.*, 594–600, Aug. 2006.
12. Khalaj-Amirhosseini, M., "Analysis of periodic and aperiodic coupled nonuniform transmission lines using the Fourier series expansion," *Progress In Electromagnetics Research*, PIER 65, 15–26, 2006.
13. Khalaj-Amirhosseini, M., "Analysis of coupled or single nonuniform transmission lines using the equivalent sources method," *IEEE Symp. Mic. Ant. Prop. EMC* (*MAPE 2007*), Hangzhou, China, August 14–17, 2007.
14. Khalaj-Amirhosseini, M., "Analysis of coupled or single nonuniform transmission lines using the method of moments," *Int. Journal of RF and Microwave Computer-aided Engineering*, Vol. 18, No. 4, 376–382, July 2008.

15. Khalaj-Amirhosseini, M., "A closed form analytic solution for coupled nonuniform transmission lines," *Progress In Electromagnetics Research C*, Vol. 1, 95–103, 2008.
16. Khalaj Amirhosseini, M., "Determination of capacitance and conductance matrices of lossy shielded coupled microstrip transmission lines," *Progress In Electromagnetics Research*, PIER 50, 267–278, 2005.
17. Le Roy, M., A. Perennec, S. Toutain, and L. C. Calvez, "Continuously varying coupled transmission lines applied to design band-pass filter," *Int. Journal of RF and Microwave Computer-aided Engineering*, Vol. 12, No. 3, 288–295, May 2002.