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SEMI-INFINITE CHIRAL NIHILITY PHOTONICS: PARAMETRIC DEPENDENCE, WAVE TUNNELING AND REJECTION

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Abstract—The novel characteristics of wave transmission and reflection in one-dimensional semi-infinite chiral photonics have been investigated theoretically. Waves in each region have been formulated for both normal and oblique incidences. At a given incident angle, the transmission or reflection is found to be easily adjusted to be equal to 1 for the chiral photonics using chiral nihility media. The wave tunneling and rejection properties in chiral nihility photonics, as well as their parametric dependences on periodicity, chiral nihility and incident angles, have been explicitly presented theoretically and verified numerically.

1. INTRODUCTION

The rotation of the ellipse of light after passing through an isotropic chiral medium has been known [1]. In addition to those pioneering work [2–4], more recently, there is rapid development on the study of EM wave propagation in structured chiral media, e.g., chiral plate [5], Goos-Hänchen shift on chiral-dielectric interface [6], chiral slab [7], nonspherical chiral object [8], infinite chiral and gyrotropic chiral

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media and their Green's functions [9, 10], chiral duality [11] etc. A chiral medium is an object that cannot be brought into congruence with its mirror image by translation or rotation. The mirror image of a left-handed chiral object has right-handedness and vice versa. Many natural materials belong to the category of chiral media, such as diverse array of sugar, wire helix, and irregular tetrahedron. On the other hand, artificial structures have been proposed to mimic the optical activity and chirality by using arrays of achiral spheres [12] and metamaterials made of cutted metal strips/rings [13, 14].

Metamaterials that refract the wave negatively [15] have been demonstrated from microwave to optical regimes. For a chiral medium to be negatively refractive [16], the chirality parameter has to be sufficiently large compared with the product of the relative permittivity and permeability. In nature, large chirality parameters are not known to exist, and for artificial structures in microwave frequencies large chirality parameters have not been reported either [17]. Nevertheless, there are three possible ways to provide negative refraction from chiral media with fewer restrictions on the chirality.

The first approach is to achieve chiral nihility whose product of relative permittivity and permeability is close to zero while the chirality is still maintained at a finite value [18, 19]. The exotic phenomena inside a chiral nihility slab or at such an interface have been examined [19]. There have been a lot of revived interests of making use of chiral nihility to realize negative-index-related applications, such as surface wave modes in grounded chiral nihility waveguides [20, 21], fractional dual solution for chiral nihility metamaterials [22, 23], focusing [19, 24], chiral fibers [25], etc. The second solution is to make use of gyrotropic chiral media (all positive parameters) whose permittivity and permeability are tensors and have off-diagonal elements to alleviate the said restriction [26]. The third is to rely on gyrotropic-Ω materials (all positive parameters) where the chirality has off-diagonal elements [27]. The latter two ways can provide negative index without requiring permittivity and permeability being extremely small and chirality being large owing to the gyrotropic parameters.

We will focus on the isotropic chirality with the emphasis on the chiral nihility in chiral photonics. This configuration is finite and thus such systems do not have translation symmetry. For infinitely periodic chiral multilayers, plane wave theory and coupled-mode theory have been presented [28, 29] in which a 4×4 transfer matrix that includes all information about the stratified chiral medium is used. In contrast, our paper deals with a chiral photonic crystal with finite stacked mediums composed of alternating chiral nihility layers, and its solution is based on the 2×2 block representation transfer matrix formulation [30, 31] which is generalized from the 2×2 matrix approach [32]. Numerical results, parametric study, and discussion in the reflection and transmission spectra for two polarizations with the dependence on incident angles, chirality parameter, periodicity, and stack number are given in details.

2. PROBLEM FORMULATION

A periodic in the z-axis direction, structure of N identical basic elements (periods) is investigated (Fig. 1). Each of periods consists of two chiral layers with material parameters ε_j, μ_j, ρ_j and thicknesses d_j ($j = 1, 2$). The total length of the structure period is $L = d_1 + d_2$. The layers are unrestricted in the x- and y-directions. The input $z \leq 0$ and output $z \geq NL$ half-spaces are assumed to be free space with the parameters of ε_0 and μ_0.

Suppose that the incident field is a plane monochromatic wave of frequency ω with perpendicular (electric-field vector $\vec{E}$ is perpendicular to the plane of incidence) or parallel (electric-field vector $\vec{E}$ is parallel to the plane of incidence) polarization (s and p polarized waves). The direction of the wave propagation in the input isotropic medium $z \leq 0$ is defined by the angle φ_0 from the z-axis (through the paper a time

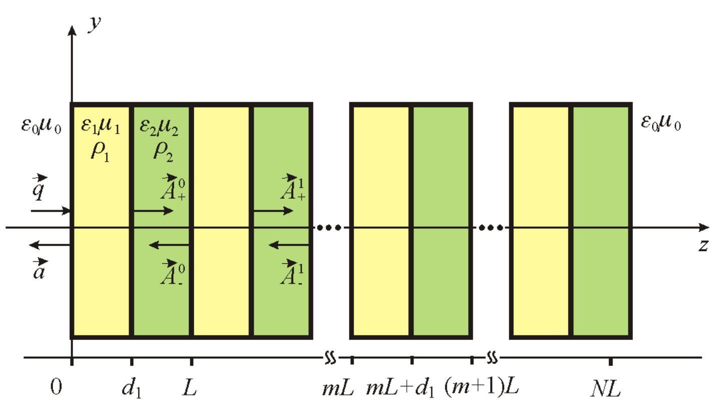


Figure 1. Chiral photonics with N stacks of alternating chiral slabs. Here $\vec{q}$ and $\vec{a}$ are the vectors of the incident and reflected field components, A_+^j and A_-^j are the amplitudes of the eigenwaves propagating in positive and negative direction, respectively, and $j = 0, 1, \ldots$ is the number of structure period.

conversion $\exp(-i\omega t)$ is assumed and omitted)

$$\begin{gathered}
\left\{ \begin{array}{c} E_{x0}^s \\ E_{y0}^p \end{array} \right\} = \pm \left\{ \begin{array}{c} A_0^s/\sqrt{Y_0^s} \\ iA_0^p/\sqrt{Y_0^p} \end{array} \right\} \exp[i(k_{y0}y + k_{z0}z)], \\
\left\{ \begin{array}{c} H_{y0}^s \\ H_{x0}^p \end{array} \right\} = \left\{ \begin{array}{c} A_0^s\sqrt{Y_0^s} \\ iA_0^p\sqrt{Y_0^p} \end{array} \right\} \exp[i(k_{y0}y + k_{z0}z)],
\end{gathered} \tag{1}$$

where $k_{y0} = k_0 \sin\varphi_0$ and $k_{z0} = k_0 \cos\varphi_0$ are the wavevector $\vec{k}$ components in the local coordinate system, $Y_0^s = Z_0^{-1}\cos\varphi_0$ and $Y_0^p = (Z_0 \cos\varphi_0)^{-1}$ are the wave admittances of the s and p polarized waves, respectively, $k_0 = \omega/c$ is the free-space wavenumber and $Z_0 = \sqrt{\mu_0/\varepsilon_0}$ is the wave impedance of the input half-space.

3. TRANSFER MATRICES

The characteristics of the reflected and transmitted fields of the structure under study can be determined on the basis of the generalized scattering matrix method [31]. The essence of this method consists in obtaining the reflection matrix of a semi-infinite periodic sequence of chiral layers which can be derived using specific shift symmetry of such structure. The notion of symmetry implies that the reflection properties of a semi-infinite structure will be unchanged if one or any finite number of layers next to the interface are removed. On the basis of the obtained reflection matrix of the semi-infinite structure the reflection and transmission fields as well as the inner field of a finite chiral structure can be easy derived.

It is well known that during the interaction of a linearly polarized plane wave with a periodic structure containing chiral layers the cross-polarized components appears in the reflected field. It is conveniently to describe the relation of the reflected and inner fields of a semi-infinite structure via some reflection and transmission matrices. The elements of these matrices are the amplitudes of the co-polarized and cross-polarized waves.

Let us first describe the method of solution related to a structure that consists of chiral layers separated by air gaps. Further it will be generalized on the case of an arbitrary periodical sequence of chiral layers.

Let $\mathbf{r}$ and $\mathbf{t}$ be the reflection and transmission matrices of a single chiral layer, and $\mathbf{R}$ is the reflection matrix of a semi-infinite discrete structure. The changes of the vector of complex amplitudes of the waves when they propagate through the air gap between chiral layers are described via the propagation matrix $\mathbf{u}$. Thus the relations of the field amplitudes in the input half-space and in the nearest region to the structure interface air gap are defined via the next conditions

$$\vec{A}^0_+ = \mathbf{t}\vec{q} + \mathbf{ru}\vec{A}^0_-; \quad \vec{A}^0_- = \mathbf{Ru}\vec{A}^0_+; \quad \vec{a} = \mathbf{R}\vec{q}; \quad \vec{a} = \mathbf{r}\vec{q} + \mathbf{tu}\vec{A}^0_-. \quad (2)$$

After the elimination of the vectors $\vec{A}^0_+$, $\vec{A}^0_-$, $\vec{a}$ from Eq. (2), the nonlinear equation for the unknown reflection operator $\tilde{\mathbf{R}}$ of the semi-infinite structure is obtained [31]

$$\tilde{\mathbf{R}} = \tilde{\mathbf{r}} + \tilde{\mathbf{t}}\tilde{\mathbf{R}}(\mathbf{I} - \tilde{\mathbf{r}}\tilde{\mathbf{R}})^{-1}\tilde{\mathbf{t}}, \quad (3)$$

where $\tilde{\mathbf{R}} = \mathbf{Ru}$, $\tilde{\mathbf{r}} = \mathbf{ru}$, $\tilde{\mathbf{t}} = \mathbf{tu}$, $\mathbf{I}$ is the identity matrix, and, in the presence of the wave polarization transformation, the matrices $\mathbf{R}$, $\mathbf{r}$, $\mathbf{t}$, and $\mathbf{u}$ are

$$\mathbf{R} = \begin{pmatrix} R^{ss} & R^{ps} \\ R^{sp} & R^{pp} \end{pmatrix}, \; \mathbf{r} = \begin{pmatrix} r^{ss} & r^{ps} \\ r^{sp} & r^{pp} \end{pmatrix}, \; \mathbf{t} = \begin{pmatrix} t^{ss} & t^{ps} \\ t^{sp} & t^{pp} \end{pmatrix}, \; \mathbf{u} = \begin{pmatrix} e_0 & 0 \\ 0 & e_0 \end{pmatrix}, \quad (4)$$

where $e_0 = \exp(-ik_{z0}d_2)$. In terms of the linearly polarized waves the matrix elements in Eq. (4) are the co-polarized (vv) and cross-polarized (vv') reflection and transmission coefficients ($v = s, p$) where, as it was mentioned above, the term s is related to the perpendicular polarization and the term p is related to the parallel polarization of plane electromagnetic waves. Eq. (3) containing the operator $\tilde{\mathbf{R}}$ can thus be rearranged

$$f(\mathbf{R}) = 0, \quad f(\tilde{\mathbf{R}}) = \tilde{\mathbf{R}} - \tilde{\mathbf{r}} - \tilde{\mathbf{t}}\tilde{\mathbf{R}}(\mathbf{I} - \tilde{\mathbf{r}}\tilde{\mathbf{R}})^{-1}\tilde{\mathbf{t}}. \quad (5)$$

The Newton method can be applied and series approximations to the solution are made accordingly

$$\tilde{\mathbf{R}}_j = \tilde{\mathbf{R}}_{j-1} - \left[f'(\tilde{\mathbf{R}}_{j-1})\right]^{-1} f(\tilde{\mathbf{R}}_{j-1}), \quad j = 1, 2, \ldots, \quad (6)$$

where $f'(\tilde{\mathbf{R}})$ is the derivative with respect to the argument of the matrix function, and $\tilde{\mathbf{R}}_0$ is some initial approximation (e.g., $\tilde{\mathbf{R}}_0 = \tilde{\mathbf{r}}$). Note that if the structure consists of achiral isotropic layers Eq. (5) is a quadratic equation related to the complex reflection coefficient and its solution is trivial. In such a form, Eq. (5) is repeatedly used to analyze the reflection from semi-infinite achiral structures with different compositions [31].

Now, the transfer matrix $\mathbf{T}$ can be obtained:

$$\vec{A}^0_+ = \mathbf{T}\vec{q}, \quad \mathbf{T} = (\mathbf{I} - \tilde{\mathbf{r}}\tilde{\mathbf{R}})^{-1}\mathbf{t}. \quad (7)$$

Provided that the vectors of amplitudes of eigenwaves propagating in a positive direction are denoted as A^j_+ (j is the number of structure period), the input and output vectors of the eigenwave amplitudes on the period boundaries are related

$$\vec{A}_{+}^{j+1} = \tilde{\mathbf{T}}\vec{A}_{+}^{j}. \tag{8}$$

On the other hand, in a periodic structure the fields in the neighboring periods differ only in a certain phase factor (the Floquet theorem)

$$\vec{A}_{+}^{j+1} = \exp(i\beta L)\vec{A}_{+}^{j}, \tag{9}$$

where the k-th eigenvalue $\exp(i\beta_k L)$ of the transfer matrix $\tilde{\mathbf{T}}$ (Bloch wavenumber) can be obtained from the next dispersion equation

$$\det\left[\mathbf{I} - \tilde{\mathbf{t}}\exp(-i\beta L) - \tilde{\mathbf{r}}[\mathbf{I} - \tilde{\mathbf{t}}\exp(i\beta L)]^{-1}\tilde{\mathbf{r}}\right] = 0. \tag{10}$$

Let now all half-space $z < 0$ be filled with a periodic structure of chiral layers possessing the same parameters as above. Consider the propagation of an eigenfield in an infinite layered structure which is incident from the half-space $z < 0$ through the free-space boundary in the plane $z = 0$. Let $\vec{A}_{\pm}$ and $\vec{B}_{\pm}$ be the vectors of the eigenfield amplitudes at this interval. Then the reflection $\boldsymbol{\zeta}$ and transmission $\boldsymbol{\tau}$ operators can be defined as follows

$$\vec{b} = \boldsymbol{\tau}\vec{A}_{+}, \quad \vec{B}_{-} = \boldsymbol{\zeta}\vec{A}_{+}. \tag{11}$$

The vectors of the eigenfield amplitudes satisfy the next conditions

$$\tilde{\mathbf{t}}(\vec{A}_{+}+\vec{B}_{+})=\vec{b},\ \ \vec{A}_{-}+\vec{B}_{-}=\tilde{\mathbf{r}}(\vec{A}_{+}+\vec{B}_{+}),\ \ \vec{A}_{-}=\tilde{\mathbf{R}}\vec{A}_{+},\ \ \vec{B}_{+}=\tilde{\mathbf{R}}\vec{B}_{-}. \tag{12}$$

From these equations the expressions for the reflection and transmission operators related to the eigenfield amplitudes are obtained as

$$\boldsymbol{\zeta} = (\mathbf{I} - \tilde{\mathbf{r}}\tilde{\mathbf{R}})^{-1}(\tilde{\mathbf{r}} - \tilde{\mathbf{R}}), \quad \boldsymbol{\tau} = \tilde{\mathbf{t}}(\mathbf{I} + \tilde{\mathbf{R}}\boldsymbol{\zeta}). \tag{13}$$

With the help of the operators introduced above it is easy to obtain the transmission and reflection matrices of a finite structure with N periods

$$\begin{aligned}\mathbf{t}_N &= \boldsymbol{\tau}\tilde{\mathbf{T}}^{N-2}\left(\mathbf{I} - \boldsymbol{\zeta}\tilde{\mathbf{T}}^{N-2}\boldsymbol{\zeta}\tilde{\mathbf{T}}^{N-2}\right)^{-1}\mathbf{T},\\ \mathbf{r}_N &= \mathbf{R} + \boldsymbol{\tau}\tilde{\mathbf{T}}^{N-2}\boldsymbol{\zeta}\tilde{\mathbf{T}}^{N-2}\left(\mathbf{I} - \boldsymbol{\zeta}\tilde{\mathbf{T}}^{N-2}\boldsymbol{\zeta}\tilde{\mathbf{T}}^{N-2}\right)^{-1}\mathbf{T}.\end{aligned} \tag{14}$$

To determine the reflection ($\mathbf{r}$) and transmission ($\mathbf{t}$) matrices of a single chiral layer the field in a homogeneous chiral media must be considered. Generally this field is characterized by the next displacement [1]

$$\vec{D} = \varepsilon\vec{E} + i\rho\vec{H}, \quad \vec{B} = \mu\vec{H} - i\rho\vec{E}, \tag{15}$$

that leads to the next coupled differential equations

$$\begin{aligned}\Delta_{\perp}E_x + k_0^2\left(n^2+\rho^2\right)E_x - 2ik_0^2\rho\mu H_x = 0,\\ \Delta_{\perp}H_x + k_0^2\left(n^2+\rho^2\right)H_x + 2ik_0^2\rho\varepsilon E_x = 0,\end{aligned} \tag{16}$$

where $n = \sqrt{\varepsilon\mu}$ is the refractive index of a chiral medium, and $\Delta_{\perp} = \partial^2/\partial y^2 + \partial^2/\partial z^2$ is the two-dimensional Laplacian.

The waves of the perpendicular and parallel linear polarizations can be presented as the superposition of two eigenwaves of a right $(\vec{Q}^{v+})$ and a left $(\vec{Q}^{v-})$ circular polarizations [1]:

$$\begin{aligned}E_x^s = Q^{s+} + Q^{s-}, \quad H_x^s = -iZ^{-1}\left(Q^{s+} - Q^{s-}\right),\\ E_x^p = iZ\left(Q^{p+} - Q^{p-}\right), \quad H_x^p = Q^{p+} + Q^{p-},\end{aligned} \tag{17}$$

where $Z = \sqrt{\mu/\varepsilon}$ is the wave impedance of a chiral medium. Such substitution transforms Eq. (16) into two independent Helmholtz equations:

$$\Delta_{\perp}Q^{v+} + \left(\gamma^{+}\right)^2 Q^{v+} = 0, \quad \Delta_{\perp}Q^{v-} + \left(\gamma^{-}\right)^2 Q^{v-} = 0. \tag{18}$$

Here $v = s, p$; $\gamma^{\pm} = k_0\sqrt{\varepsilon^{\pm}\mu^{\pm}} = k_0 n^{\pm} = k_0(n \pm \rho)$ are the propagation constants of the right (γ^{+}) (RCP) and left (γ^{-}) (LCP) circularly polarized eigenwaves, respectively, in the unbounded chiral media with the equivalent material parameters $\varepsilon^{\pm} = \varepsilon \pm \rho Z^{-1}$ and $\mu^{\pm} = \mu \pm \rho Z$. The relative impedance Z and index n are respectively defined as $\sqrt{\mu/\varepsilon}$ and $\sqrt{\mu\varepsilon}$. The general solutions of Eq. (18) for the RCP and LCP waves in a corresponding bounded chiral layer can be expressed by [30]

$$\begin{aligned}Q^{s\pm} &= \left(1/2\sqrt{Y^{s\pm}}\right)\left(A^{s\pm}\exp\left[i(k_{y0}y + \gamma_z^{\pm}z)\right] + B^{s\pm}\exp\left[i(k_{y0}y - \gamma_z^{\pm}z)\right]\right),\\ Q^{p\pm} &= \left(\sqrt{Y^{p\pm}}/2\right)\left(A^{p\pm}\exp\left[i(k_{y0}y + \gamma_z^{\pm}z)\right] + B^{p\pm}\exp\left[i(k_{y0}y - \gamma_z^{\pm}z)\right]\right),\end{aligned} \tag{19}$$

where $A^{v\pm}$, $B^{v\pm}$ denote the field amplitudes, $Y^{s\pm} = Z^{-1}\cos\varphi^{\pm}$, $Y^{p\pm} = (Z\cos\varphi^{\pm})^{-1}$ are the wave admittances, $\gamma_z^{\pm} = \gamma^{\pm}\cos\varphi^{\pm}$, and $\varphi^{\pm} = \sin^{-1}\left[n_0\sin\varphi_0/n^{\pm}\right]$ are the refracted angles of the two eigenwaves in a chiral medium. The substitution of Eq. (19) into Eq. (17) gives the field components of s and p polarizations.

Next, we consider a chiral-nihility medium ($\varepsilon = \mu = 0$, $\rho \neq 0$) [22]. The expressions related to $\varepsilon^{\pm}$ and $\mu^{\pm}$ are rearranged

$$\varepsilon^{\pm} = \pm\rho Z^{-1}, \quad \mu^{\pm} = \pm\rho Z. \tag{20}$$

It is obvious that here a situation when both material parameters ε^{+} and μ^{+} or ε^{-} and μ^{-} are negative is possible depending on the sign of the chirality parameter ρ. The backward-wave appears for the LCP wave when the chirality parameter ρ is a positive value and for the

RCP wave when ρ is negative. Since the propagation constants of the RCP (γ^+) and LCP (γ^-) waves in a chiral-nihility medium are equal in magnitude but opposite in sign to each other ($\gamma^\pm = \pm k_0\rho = \pm\gamma$), the solutions of Eq. (19) can be simplified

$$\begin{aligned} Q^{s\pm} &= \left(1/2\sqrt{Y^s}\right)\left(A^{s\pm}\exp[i(k_{y0}y\pm\gamma_z z)] + B^{s\pm}\exp[i(k_{y0}y\mp\gamma_z z)]\right), \\ Q^{p\pm} &= \left(\sqrt{Y^p}/2\right)\left(A^{p\pm}\exp[i(k_{y0}y\pm\gamma_z z)] + B^{p\pm}\exp[i(k_{y0}y\mp\gamma_z z)]\right), \end{aligned} \tag{21}$$

where $Y^{s+} = Y^{s-} = Y^s$, $Y^{p+} = Y^{p-} = Y^p$, $\gamma_z = \gamma\cos|\varphi^\pm|$, and the refraction angles are $\varphi^+ = -\varphi^- = \sin^{-1}[n_0\sin\varphi_0/\rho]$.

After the substitution of Eq. (21) into Eq. (17) and its further combination with Eq. (3) on the chiral-nihility layer boundaries, the reflection and transmission coefficients of the co-polarized (co) and cross-polarized (cr) field components can be expressed by

$$\begin{aligned} r^{co} &= r^{pp} = r^{ss} = G^{-1}\sin^2(\gamma_z d)\left(\cos^2\varphi_0 - \cos^2\varphi\right)\left(\cos^2\varphi_0 + \cos^2\varphi\right), \\ r^{cr} &= r^{ps} = r^{sp} = iG^{-1}\sin 2(\gamma_z d) \\ &\qquad \cos\varphi_0\cos\varphi\left(\cos^2\varphi_0 - \cos^2\varphi\right)\left(\cos^2\varphi_0 + \cos^2\varphi\right), \\ t^{co} &= t^{pp} = t^{ss} = 4G^{-1}\cos(\gamma_z d)\cos^2\varphi_0\cos^2\varphi, \\ t^{cr} &= t^{ps} = t^{sp} = -i2G^{-1}\sin(\gamma_z d)\cos\varphi_0\cos\varphi\left(\cos^2\varphi_0 + \cos^2\varphi\right), \end{aligned} \tag{22}$$

where $G^{-1} = 4\cos^2(\gamma_z d)\cos^2\varphi_0\cos^2\varphi + \sin^2(\gamma_z d)\left(\cos^2\varphi_0 + \cos^2\varphi\right)^2$, $\varphi = |\varphi^\pm|$ and $d = d_1$ is the thickness of the chiral-nihility layer.

4. REFLECTED AND TRANSMITTED FIELDS

4.1. Single Chiral-nihility Layer

Since the chiral-nihility condition is fulfilled only in the vicinity of a fixed frequency ω_0, the behaviors of the magnitudes of the reflection and transmission coefficients are investigated as functions of the angle of incidence, the chirality parameter and the refractive index. First, we consider optical properties of a single chiral-nihility layer. The angular dependence of the magnitudes of the reflected and transmitted fields of a chiralnihility layer ($\varepsilon = \mu = 0$, $\rho \neq 0$) and a convenient chiral layer ($\varepsilon \geq 1$, $\mu \geq 1$, $\rho \neq 0$) are given in the Fig. 2 for comparison. Note that in the second case the results are obtained using [7, 31]. In all numerical calculation a matching of Z is kept. In this case the magnitudes of the reflection and transmission coefficients of s- and p-polarized waves are equal to each other (Fig. 2).

The main difference is that an additional angle of co-polarized zero-transmission appears in the case of the chiral-nihility layers

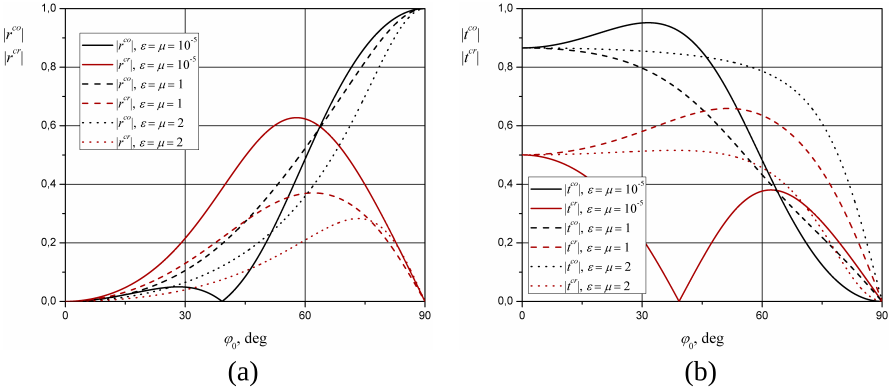


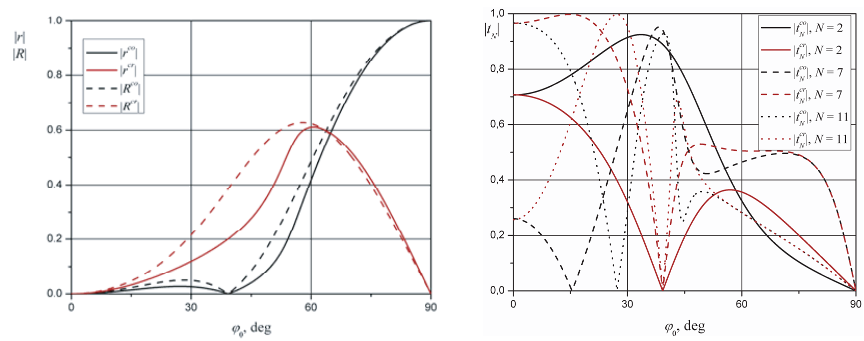
Figure 2. The magnitudes of the co-polarized and cross-polarized reflection (a) and transmission (b) coefficients of the matched ($Z_2 = 1$) chiral-nihility layer ($\varepsilon = \mu = 0$, $\rho \neq 0$) and convenient chiral layer ($\varepsilon \geq 1$, $\mu \geq 1$, $\rho \neq 0$) versus the angle of incidence, $\varepsilon_0 = \mu_0 = 1$, $\rho = 0.5$, $d = 5\,\text{mm}$, $f = 10\,\text{GHz}$.

($|r^{co}| = |t^{cr}| = 0$, $|r^{cr}| \neq 0$, $|t^{co}| \neq 0$) in contrast to a conventional chiral slab. From Eq. (22) this condition appears when $\cos\varphi_0 = (\cos\varphi)^*$ (the asterisk denotes the complex conjugation), and that is possible when $n_0 \sin\varphi_0/\rho \geq 1$. The other two conditions of the total reflection or transmission can be obtained from Eq. (22). The first condition is satisfied when $\varphi_0 = 0$ (the normally incident wave) leading to $\left(\cos^2\varphi_0 - \cos^2\varphi\right) = 0$. As a result, due to the impedance matching, no co-polarized and cross-polarized reflections appear ($|r^{co}| = |r^{cr}| = 0$) and a wave is completely transmitted through a layer with polarization transformation ($|t^{co}| \neq 0$, $|t^{cr}| \neq 0$). The polarization rotation is $\alpha = k_0\rho d$. The second condition is that, when $\varphi_0 = \pi/2$ (the sliding wave) that yields $\cos^2\varphi_0 = 0$, the total reflection appears for co-polarized waves ($|r^{co}| = 1$, $|r^{cr}| = |t^{co}| = |t^{cr}| = 0$).

4.2. Chiral Nihility Photonics with Finite Stacks

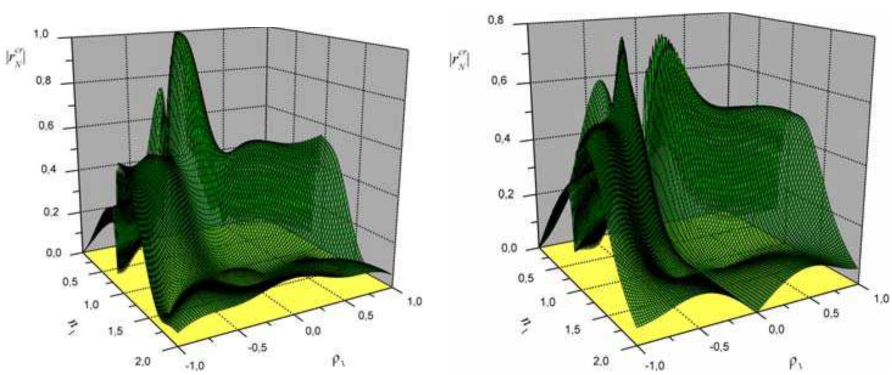
In the first case, the structure consists of a finite sequence of chiral layers separated by air gaps (Figs. 3 and 4).

Due to the unique behaviors of chiral nihility medium, the reflection coefficients of a single layer ($|r^{co}|$, $|r^{cr}|$) and their semi-infinite stack ($|R^{co}|$, $|R^{cr}|$) have similar characteristics (Fig. 3(a)). This is due to the nature of propagation of the two circularly polarized waves in the chiral-nihility layers and in the air gaps between them with different angles of refraction ($\varphi^+ = -\varphi^-$) [19]. Based on Eq. (14), the reflection ($|r_N^{co}|$, $|r_N^{cr}|$) and transmission ($|t_N^{co}|$, $|t_N^{cr}|$) coefficients of a finite multilayer structure can be obtained (Figs. 3 and 4). Finiteness of the structure leads to the interference effect that manifests itself in the form of oscillations of the magnitude and phase of the reflection and transmission coefficients as shown in Fig. 3. Similar to the case



(a) (b)

Figure 3. The magnitudes of the reflection (a) and transmission (b) coefficients of the finite and semi-infinite sequence of the matched chiral nihility layers ($Z_1 = 1$) separated with the air gap ($\varepsilon_0 = \mu_0 = \varepsilon_2 = \mu_2 = 1$, $\rho_2 = 0.0$) for different values of N versus the angle of incidence. $\varepsilon_1 = \mu_1 = 1 \times 10^{-5}$, $\rho_1 = 0.5$, $d_1 = d_2 = 5\,\mathrm{mm}$, $f = 10\,\mathrm{GHz}$.



(a) (b)

Figure 4. The magnitudes of the reflection coefficients of co-polarized (a) and cross-polarized (b) field components of the finite sequence of the matched chiral nihility layers ($Z_1 = 1$) separated with the air gap ($\varepsilon_0 = \mu_0 = \varepsilon_2 = \mu_2 = 1$, $\rho_2 = 0.0$) as a function of the chirality parameter ρ_1 and the refractive index $n_1 \cdot \phi_0 = 40°$, $N = 3$, $d_1 = d_2 = 5\,\mathrm{mm}$, $f = 10\,\mathrm{GHz}$.

of the single chiral nihility layer, there is no reflection at the normal angle of incidence, but the level of the polarization transformation changes in the transmitted field and depends on the value of N. As an example, the co-polarized and cross-polarized components of the transmitted field are equal to each other ($|t_N^{co}| = |t_N^{cr}|$) when $N = 2$. The cross-polarized ($|t_N^{co}| < |t_N^{cr}|$) and co-polarized ($|t_N^{co}| > |t_N^{cr}|$) components dominate in the transmitted fields when $N = 7$ and $N = 11$, respectively.

To calculate the reflection and transmission coefficients of the structure whose period consists of two chiral-nihility layers, it is necessary to make changes in the propagation matrix $\mathbf{u}$. If the propagation constant $\gamma_{z2} = \gamma_2 \cos|\varphi_2^{\pm}|$ of the second chiral-nihility layer is known, then the elements of the matrix $\mathbf{u}$ can be defined as $e_0 = \exp(-i\gamma_{z0} d_2)$.

It is obvious that for the proposed chiral photonics, the properties of the reflected and transmitted fields are determined by the relation between the chirality parameters ρ_j ($j = 1, 2$) of the adjacent layers. From Fig. 5, the conditions of complete wave tunneling and (or) rejection of the co-polarized and cross-polarized waves can be easily derived. Note that the maximal tunneling arises at $\rho_1 = -\rho_2$, and the maximum of the cross-polarized reflection occurs when the chirality parameters ρ_j have the same sign.

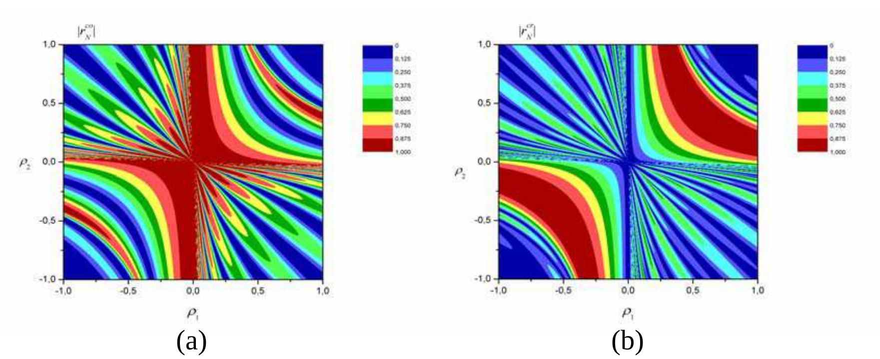


Figure 5. The magnitudes of the reflection coefficients of co-polarized (a) and cross-polarized (b) field components of the finite sequence of the matched chiral nihility layers ($Z_1 = Z_2 = 1$) as a function of the chirality parameter $\rho_j \cdot \phi_0 = 40°$, $N = 10$, $\varepsilon_0 = \mu_0 = 1$, $\varepsilon_j = \mu_j = 1 \times 10^{-5}$, $j = 1, 2$, $d_j = 5\,\text{mm}$, $f = 10\,\text{GHz}$.

5. CONCLUSION

In this paper, we have proposed a rigorous analytical approach to model the wave tunneling and rejection conditions for chiral nihility photonics. A semi-infinite periodical chiral structure consisting of alternative chiral nihility mediums has been considered, and the exotic wave properties have been studied and presented. It has been revealed that it is easier to realize an ideal photonic bandgap through adjusting the chirality in semi-finite chiral nihility photonics. The explicit interconnections between the exotic characteristics (in transmission/reflection) and the parameters (stack number, incident angle, polarization dependence, chirality in adjacent two chiral nihility stacks, etc) and the mutual effects between those parameters have been obtained and summarized. Such promising photonics can be applied to the design of chiral-nihility waveguides, fibers and polarization selectors.

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