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RESONANCE WAVE SCATTERING BY A STRIP GRATING ATTACHED TO A FERROMAGNETIC MEDIUM

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Abstract—The diffraction of a uniform unit-amplitude E-polarized plane wave is considered in the case of its normal incidence on a strip periodic metal grating placed on the anisotropic hyrotropic ferromagnetic half-space boundary. The Dirichlet boundary conditions on the grating strips, the medium interface conjugation conditions, the Meixner condition that the energy is finite in any confined volume and the radiation condition are applied, and the boundary value diffraction problem in terms of Maxwell's (Helmholtz) equations is equivalently reduced to the dual system of functional equations with exponential kernel. The system is shown to be the Riemann-Hilbert problem in analytic function theory with the conjugation coefficient differing, in general, from "-1" and dependent on the incident wave frequency. An analytical regularization procedure based on the Riemann-Hilbert boundary value problem solution with the following use of the Plemelle-Sokhotsky formulas is suggested, resulting in the system of linear algebraic equations of the second kind with a compact operator. For these systems, the truncation technique possibility has been shown.

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Calculation algorithms and simulation packages in terms of C^{++} language have been developed. As a result, the reflection coefficient performance has been studied over sufficiently wide ranges of frequency and constitutive and geometrical parameters of the electrodynamical systems of interest. The frequency bands of the reflection coefficient resonant behavior have been established and examined. A numerical analytical model of these resonances has been proposed.

1. INTRODUCTION

There is increasing research activity on effects accompanying the electromagnetic wave propagation, diffraction and radiation when a boundary of such a medium as dielectric, ferromagnetic, chiral composite, meta-material, etc. is involved in the process [1–9]. The interest has been spurred by the growth in the synthesis of new artificial materials possessing unusual electromagnetic properties in the microwave band [6]. Another reason is pressing needs in both high-reflection and absorption structures with controllable scattering properties [4, 5].

According to [8, 9], a strip periodic grating backed by one of the above-mentioned materials gives rise to specific resonance effects. Furthermore, such familiar phenomena as nonreciprocity effect, Faradey's effect [10], etc. can go in unusual manner.

At present, the diffraction by a plane metal grating is understood well enough when the grating is in the free space environment which can additionally contain a homogeneous isotropic medium interface. As to the anisotropic medium boundary, it can be only involved when the medium of the kind is separated from the grating by an isotropic magnetodielectric layer. See, e.g., [11–14] with the references.

Work [15] addressed the wave diffraction by a strip grating located on a boundary of a medium whose permittivity and permeability were nondiagonal tensors. The initial boundary value diffraction problem was reduced to the singular integral equations that were algebraized by singular integral quadratures [16]. The resulting system of linear algebraic equations of the first kind had yet to be examined for the numerical solution stability, the rate of the condition number growth with quadrature order increase, etc. The only way to get rid of those troubles is, according to [17], an analytical regularization procedure to equivalently reduce the initial diffraction problem to the infinite system of linear algebraic equations of the second kind. Yet it must be mentioned that the system of dual series equations that the wave diffraction problem for a plane strip grating (or a finite collection of cylindrical screens) backed by an anisotropic (gyrotropic) medium is generally reduced to differs from the system appearing in the classical diffraction by gratings [11–13]. In that event, a question naturally arises on how the analytical regularization procedure can be built for them, too.

Publications on the subject date back to the 60-s of the last century (see [18]). The analytical regularization method suggested in [19, 20] for systems of dual series equations is based on the development of a closed-form solution to the "standard" dual equations and goes through the conjugation problem solution in the analytic function theory. This method was helpful to work the diffraction by partially screened plasma cylinders [21–23] and also by a plane strip grating attached to a "cold" magnetoactive plasma [24]. The development of the regularization procedure, which represents the Riemann-Hilbert problem method extension, was the subject of works [25, 26], where magnetostatic waves traveling along a strip grating laying on a ferrite layer boundary were treated.

The present paper concern is, first, the development of the analytical regularization procedure for dual series equations appearing in a wide class of problems for monochromatic plane wave diffraction by a strip grating backed by a gyromagnetic medium and, second, the performance of numerical experiments for studying specific features of the wave interaction with a ferromagnetic medium interface supporting a strip grating and placed in a magnetic field.

2. FORMULATION OF THE PROBLEM

A periodic grating is considered as a collection of z-parallel infinitely thin perfectly conducting strips placed in the plane $x = 0$. The grating slots are d wide, the grating period is l (see Fig. 1).

The half-space $\{(x, y) : |y| < \infty, x < 0\}$ is filled with a homogeneous ferromagnetic medium of the permittivity ε ($\varepsilon = \varepsilon' + i\varepsilon''$, $\varepsilon' > 1; \varepsilon'' \geq 0$) and the permeability like the tensor below (a constant magnetic field $\vec{H}_0$ is parallel to the oz axis)

$$\hat{\mu} = \left\| \begin{array}{ccc} \mu & i\mu_a & 0 \\ -i\mu_a & \mu & 0 \\ 0 & 0 & 1 \end{array} \right\|.$$

Here, $\mu = 1 - \frac{\chi_H \chi_M}{\chi^2 - \chi_H^2}$, $\mu_a = \frac{\chi \chi_M}{\chi^2 - \chi_H^2}$ with $\chi = \frac{\omega\, l}{2\pi c}$, $\chi_H = \frac{\omega_H l}{2\pi c}$, $\chi_M = \frac{\omega_M l}{2\pi c}$, where ω is the incident (primary) field frequency, $\omega_H = |\gamma|\, H_0$, $\omega_M = 4\pi M_0\, |\gamma|$ is the frequency characterizing the medium magnetization (γ is the gyromagnetic ratio for electrons, M_0 is the

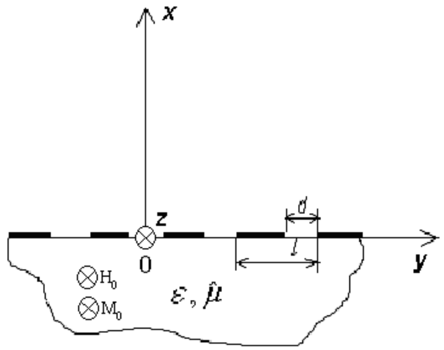

Figure 1. The cross-sectional view of the structure.

saturation magnetization, see. [10]), and c is the velocity of light in a vacuum.

In the half-space $x > 0$, a monochromatic E-polarized plane wave $E_z^i = e^{-ikx}$ travels along the $0x$ axis (normal incidence), where $k = {}^{\omega}\!/_{c}$. Throughout the paper, the time dependence described by the factor $e^{-i\omega t}$ will be omitted. The problem is to obtain the electromagnetic field of the wave diffraction by a strip grating attached to the boundary of the ferromagnetic half-space $x < 0$. One readily finds that, this electromagnetic field is E-polarized $(\vec{E} = (0, 0, E_z)\,,\ \vec{H} = (H_x, H_y, 0))$. Mathematically this case reduces to the following boundary value problem.

Functions $U_1(x, y)$ and $U_2(x, y)$ are required to determine in the half-spaces $x > 0$ and $x < 0$, respectively. The functions must meet

a) the Helmholtz equations

$$\begin{cases} \Delta U_1(x,y) + k^2 U_1(x,y) = 0, & x > 0; \\ \Delta U_2(x,y) + k^2 \varepsilon \mu_\perp U_2(x,y) = 0, & x < 0; \end{cases} \tag{1}$$

b) the periodicity condition

$$U_j(x, y \pm l) = U_j(x, y), \quad j = 1, 2; \tag{2}$$

c) the boundary conditions on the grating strips $|y + nl| > {}^{d}\!/_{2}$, $n = 0, \pm 1, \ldots$

$$\left(e^{-ikx} + U_1(x, y)\right)\Big|_{x=0} = 0, \quad U_2(0, y) = 0, \tag{3}$$

and the conjugation condition across the slots $|y+nl| < {}^{d}\!/_{2}$, $n = 0, \pm 1, \ldots$

$$\begin{aligned} 1+U_1(0,y) &= U_2(0,y), \\ k+i\frac{\partial U_1(0,y)}{\partial x} &= \frac{i}{\mu_\perp}\frac{\partial U_2(0,y)}{\partial x}+\frac{\tau}{\mu_\perp}\frac{\partial U_2(0,y)}{\partial y}, \end{aligned} \qquad (4)$$

d) the Meixner condition [27] in the vicinities of the grating strip edges
$\int\limits_M \left(|U_j|^2 + |\nabla U_j|^2\right) dxdy < \infty$ for any compact M from R^2, $j = 1, 2$;

e) the radiation conditions

$$\begin{aligned} U_1(x,y) &= \sum_{n=-\infty}^{+\infty} a_n e^{i\frac{2\pi n}{l}y} e^{iG_{1n}\frac{2\pi}{l}x},\ x>0, \\ U_2(x,y) &= \sum_{n=-\infty}^{+\infty} b_n e^{i\frac{2\pi n}{l}y} e^{-iG_{2n}\frac{2\pi}{l}x},\ x>0, \end{aligned} \qquad (5)$$

where $G_{1n} = \sqrt{\chi^2 - n^2}$ and $G_{2n} = \sqrt{\chi^2 \varepsilon \mu_\perp - n^2}$. The root branches are chosen along the following lines. When ε is a complex number,

$$\begin{aligned} &\chi \mathrm{Re} G_{1n} \geq 0, \quad \mathrm{Im} G_{1n} \geq 0, \\ &\chi \mathrm{Re} G_{2n} \leq 0, \quad \mathrm{Im} G_{2n} \geq 0 \quad \text{for } \chi_0 < \chi < \chi_+, \end{aligned}$$

and

$$\begin{aligned} &\chi \mathrm{Re} G_{1n} \geq 0, \quad \mathrm{Im} G_{1n} \geq 0, \\ &\chi \mathrm{Re} G_{2n} \geq 0, \quad \mathrm{Im} G_{2n} \geq 0 \quad \text{for } \chi < \chi_0 \ \text{ or } \chi > \chi_+, \end{aligned}$$

When ε is real,

$$\begin{aligned} &\chi \mathrm{Re} G_{1n} \geq 0, \quad \mathrm{Im} G_{1n} \geq 0, \\ &\chi \mathrm{Re} G_{2n} \geq 0, \quad \mathrm{Im} G_{2n} \geq 0. \end{aligned}$$

Here, $\mu_\perp = \frac{\chi^2 - \chi_+^2}{\chi^2 - \chi_0^2}$ is the effective permeability of the ferromagnetic medium, $\chi_0 = \sqrt{\chi_H(\chi_H + \chi_M)}$ is the normalized frequency of the ferromagnetic resonance, $\chi_- = \chi_H + 0.5\chi_M$, $\chi_+ = \chi_H + \chi_M$, and $\tau = \frac{\chi \chi_M}{\chi^2 - \chi_0^2}$.

The functions $U_1(x,y)$ and $U_2(x,y)$ are related to the sought diffraction field components as

$$E_z = \begin{cases} U_1(x,y), \ x > 0 \\ U_2(x,y), \ x < 0 \end{cases},$$

$$H_x = \frac{1}{ik} \begin{cases} \frac{\partial U_1}{\partial y}, \ x > 0 \\ \frac{1}{\mu_\perp}\left(\frac{\partial U_2}{\partial y} + i\tau\frac{\partial U_2}{\partial x}\right), \ x < 0 \end{cases}, \tag{6}$$

$$H_y = -\frac{1}{ik} \begin{cases} \frac{\partial U_1}{\partial x}, \ x > 0 \\ \frac{1}{\mu_\perp}\left(-i\tau\frac{\partial U_2}{\partial y} + \frac{\partial U_1}{\partial x}\right), \ x < 0 \end{cases}.$$

Upon radiation condition (5), relationships (6) yield the electromagnetic field components tangential to the medium interface as follows

$$E_z(x,y) = \begin{cases} e^{-ikx} + \sum\limits_{n=-\infty}^{\infty} a_n e^{i\frac{2\pi n}{l}y} e^{iG_{1n}\frac{2\pi}{l}x}, \ \ x > 0, \\ \sum\limits_{n=-\infty}^{\infty} b_n e^{i\frac{2\pi n}{l}y} e^{-iG_{2n}\frac{2\pi}{l}x}, \ \ x < 0, \end{cases}$$

$$H_y(x,y) = \begin{cases} e^{-ikx} - \frac{1}{\chi}\sum\limits_{n=-\infty}^{\infty} a_n G_{1n} e^{i\frac{2\pi n}{l}y} e^{iG_{1n}\frac{2\pi}{l}x}, \ \ x > 0, \\ \frac{1}{\chi\mu_\perp}\sum\limits_{n=-\infty}^{\infty} b_n (G_{2n} + in\tau) e^{i\frac{2\pi n}{l}y} e^{-iG_{2n}\frac{2\pi}{l}x}, \ \ x < 0. \end{cases} \tag{7}$$

Here, a_n and b_n are the unknown amplitudes of spatial harmonics of the diffraction field. In view of (3) and (4), these amplitudes are related as $b_0 = 1 + a_0, \ b_n = a_n, \ n \neq 0$.

3. REDUCTION OF THE BOUNDARY VALUE PROBLEM TO THE INFINITE SYSTEM OF LINEAR ALGEBRAIC EQUATIONS OF THE SECOND KIND

Using representations (7) and satisfying conditions (3) and (4), the system of dual series equations for the unknown amplitudes $(b_n)_{n=-\infty}^{+\infty}$ is obtained in the form

$$\begin{cases} \sum\limits_{n=-\infty}^{\infty} b_n e^{i\frac{2\pi n}{l}y} = 0, \qquad |y| > \frac{d}{2} \\ \sum\limits_{n=-\infty,\ n\neq 0}^{\infty} b_n \left(G_{2n} + \mu_\perp G_{1n} + in\tau\right) e^{i\frac{2\pi n}{l}y} = 2\chi\mu_\perp - b_0\chi\left(\mu_\perp + \sqrt{\varepsilon\mu_\perp}\right), \\ \qquad\qquad |y| < \frac{d}{2}. \end{cases} \tag{8}$$

Elementary manipulations (see [8]) reduce (8) to the equivalent system

$$\begin{cases} \sum\limits_{n=1}^{+\infty} nb_n e^{in\varphi} - b \sum\limits_{n=-\infty}^{-1} nb_n e^{in\varphi} = f\left(e^{i\varphi}\right), \ \ |\varphi| < \theta, \\ \sum\limits_{n=-\infty}^{+\infty} nb_n e^{in\varphi} = 0, \ |\varphi| > \theta, \\ \sum\limits_{n=-\infty}^{+\infty} (-1)^n b_n = 0, \ \varphi = \pi. \end{cases} \tag{9}$$

Here $\varphi = \frac{2\pi}{l} y$, $\theta = \frac{\pi d}{l}$, $b = \frac{1+\mu_\perp - \tau}{1+\mu_\perp + \tau}$, and the function $f\left(e^{i\varphi}\right)$ can be written in the Fourier series form

$$f\left(e^{i\varphi}\right) = \sum_{n=-\infty}^{+\infty} f_n e^{in\varphi}, \tag{10}$$

where $f_0 = \frac{i\chi}{1+\mu_\perp+\tau}\left[b_0\left(\mu_\perp + \sqrt{\varepsilon\mu_\perp}\right) - 2\mu_\perp\right]$, $\quad f_n = \delta_n b_n$, and $\delta_n = (1+\mu_\perp+\tau)^{-1}\left[|n|\left(1+\mu_\perp\right) + i\left(G_{2n} + \mu_\perp G_{1n}\right)\right]$.

The multiplication of the first and the second equations from (9) by $e^{im\varphi}$, $m = 0, \pm 1, \ldots$ and the integration within the limits $(-\theta, \theta)$ and $(-\pi, \theta)$ to (θ, π), respectively, yield the infinite system of linear algebraic equations, which is the functional equation of the first kind for unknowns $(b_n)_{n=-\infty}^{+\infty}$. All disadvantages of these equations, such as numerical instability of the finite system solution, the increase of the finite-system condition number as the truncation number increases, etc. have remained. Evidently whatever straightforward "algebraization" of Equation (9) is used, the result will be the same. Therefore the solution of (9) calls for a proper regularization procedure providing its equivalent reduction to the infinite system of linear algebraic equations of the second kind in the proper space of $(b_n)_{n=-\infty}^{+\infty}$ sequences.

Based on the results from [19–24], such a regularization procedure will be built now for coefficient b positive. It readily comes that $b > 0$, if the normalized frequency χ of the excitation wave meets the condition $\chi < \chi_-$ or $\chi > \chi_+$.

The central point of this regularization procedure is building a closed-form solution to (9) under the assumption that the Fourier coefficients of function (10) are available. To this end, system (9) is reduced to the boundary value conjugation (Riemann-Hilbert) problem in the analytic function theory.

Assume that $(b_n)_{n=-\infty}^{+\infty}$ is a desired solution. According to [11], define a function $B(w)$ of complex variable w by the formula

$$B(w) = \begin{cases} \sum\limits_{n=1}^{\infty} n b_n w^n, & |w| < 1, \\ -\sum\limits_{n=-\infty}^{-1} n b_n w^n, & |w| > 1. \end{cases} \quad (11)$$

As follows from the second equation of system (9), this function is analytic in a complex plane cut along the unit-circle arc L connecting the points $e^{-i\theta}$ and $e^{i\theta}$ through the point $w = 1$. Denote by $B^+(w)$ and $B^-(w)$ the limiting values of $B(w)$ on the arc L from the outside and inside of the circle $|w| < 1$, respectively. Then the first equation of system (9) becomes

$$B^+(w) + bB^-(w) = \sum_{n=-\infty}^{+\infty} f_n w^n, \quad w \in L. \quad (12)$$

So, we have arrived at the boundary value conjugation (Riemann-Hilbert) problem, which means building function $B(w)$ such that is analytic everywhere but on arc L. The $B(w)$ limiting values on arc L must meet condition (12). The problem solution will be sought in the class of functions admitting an intergrable singularity at the ends of arc L and decaying as $w \to \infty$. Owing to the methods suggested in [28], this problem solution is

$$B(w) = G(w)\left[\frac{1}{2\pi i}\int\limits_L \frac{f(t)\,dt}{G^+(t)(t-w)} + C\right], \quad (13)$$

where $f(t) = \sum\limits_{n=-\infty}^{+\infty} f_n t^n$ is function (10) and C is an arbitrary constant. The function $G(w)$ is the homogeneous Riemann-Hilbert problem solution ($f_n = 0,\ n = 0, \pm 1, \ldots$). It belongs to the indicated class of functions and can be written in the form

$$G(w) = \left(w - e^{i\theta}\right)^{-1/2 - i\beta}\left(w - e^{-i\theta}\right)^{-1/2 + i\beta}, \quad (14)$$

where $\beta = \frac{1}{2\pi}\ln b = \frac{1}{2\pi}\ln\frac{1+\mu_\perp-\tau}{1+\mu_\perp+\tau}$. The function $G^+(w)$ in (13) is the limiting value of $G(w)$ function on arc L from within the circle $|w| < 1$. It should be mentioned that the $G(w)$ behavior near the ends of arc L asymptotically coincides with the behavior of ∇U_1 and ∇U_2 functions near the grating strip edges and guarantees the Meixner condition fulfillment. It is easily seen that $\beta = 0$ when the ferromagnetic medium is absent ($\chi_M = 0$), which is the only case when the function $G(w)$

and, hence, the functions ∇U_1 and ∇U_2 possess a root singularity near the grating strip edge ($w = e^{\pm i\theta}$).

A straightforward calculation will show that $G(w)$ fits the following differential equation

$$\begin{aligned} \frac{dG(w)}{dw} &= \frac{2\beta\sin\theta + \cos\theta - w}{w^2 + 1 - 2w\cos\theta} G(w), \ w \neq e^{\pm i\theta}, \\ G(0) &= -e^{2\theta\beta}. \end{aligned} \tag{15}$$

In view of (15), $G(w)$ and $G^{-1}(w)$ are readily expanded into the series in terms of powers of complex variable w

$$G(w) = \begin{cases} -e^{2\beta\theta} \sum\limits_{n=0}^{\infty} P_n(\beta,\theta)\, w^n, & |z| < 1, \\ w^{-1} \sum\limits_{n=0}^{\infty} P_n(-\beta,\theta)\, w^{-n}, & |z| > 1, \end{cases} \tag{16}$$

$$G^{-1}(w) = \begin{cases} -e^{-2\beta\theta} \sum\limits_{n=0}^{+\infty} \Upsilon_n(-\beta,\theta)\, w^n, & |z| < 1, \\ w \sum\limits_{n=0}^{+\infty} \Upsilon_n(\beta,\theta)\, w^n, & |z| > 1. \end{cases} \tag{17}$$

Here, $P_n(\beta,\theta)$ are the Pollaczek polynomials [29] admitting the recurrent formulas

$$\begin{aligned} P_0(\beta,\theta) &= 1;\ P_1(\beta,\theta) = \cos(\theta) + 2\beta\sin(\theta); \\ P_n(\beta,\theta)|_{n\geq 2} &= \left(\left(2 - \frac{1}{n}\right)\cos(\theta) + \frac{2}{n}\beta\sin(\theta)\right) P_{n-1}(\beta,\theta) \\ &\quad - \left(1 - \frac{1}{n}\right) P_{n-2}(\beta,\theta); \\ P_{-n}(\beta,\theta) &= \exp(-2\beta\theta)\, P_{n-1}(-\beta,\theta). \end{aligned}$$

The functions $\Upsilon_n(\beta,\theta)$ are expressed in $P_n(\beta,\theta)$ terms as

$$\begin{aligned} &\Upsilon_0 = 1; \quad \Upsilon_1(\beta,\theta) = -\cos(\theta) + 2\beta\sin(\theta); \\ &\Upsilon_n(\beta,\theta) = P_n(\beta,\theta) - 2\cos(\theta)P_{n-1}(\beta,\theta) + P_{n-2}(\beta,\theta) \quad \text{for } n \geq 2. \end{aligned}$$

Notice that the Pollaczek polynomials $P_n(\beta,\theta)$ coincide with the Legendre polynomials if $\beta = 0$.

The further step in the solution of system (9) is making use of the Plemelle-Sokhotsky formulas [28] for the Cauchy-type integral in (13) to finally have that for $|\varphi| \leq \pi$

$$B^+\left(e^{i\varphi}\right) - B^-\left(e^{i\varphi}\right) = a\hat{f}\left(e^{i\varphi}\right) + \hat{G}\left(e^{i\varphi}\right)\left[\frac{1}{2\pi i}\int\limits_L \frac{f(t)\,dt}{G^+(t)\,(t - e^{i\varphi})} + C\right]. \tag{18}$$

Here, $a = -\frac{2\tau}{1+\mu_\perp-\tau}$, $\hat{G}(e^{i\varphi}) = G^+(e^{i\varphi}) - G^-(e^{i\varphi})$, $\hat{f}(e^{i\varphi}) = \begin{cases} 0, & |\varphi| > 0, \\ f(e^{i\varphi}), & |\varphi| < 0. \end{cases}$ And $G^-(e^{i\varphi}) = \lim_{\varepsilon\to+0} G(e^{i\varphi}(1+\varepsilon))$.

After the singular integral in (18) has been evaluated (the Cauchy residue theorem [30] is applied and the function $G^{-1}(w)$ from (17) is expanded in the power series),

$$\sum_{n=-\infty}^{+\infty} nb_n e^{in\varphi} = \hat{G}\left(e^{i\varphi}\right)\left[\sum_{n=-\infty}^{+\infty} f_n W_n\left(e^{i\varphi}\right) + C\right], \tag{19}$$

where

$$W_n(z) = \frac{1+\mu_\perp+\tau}{2(\mu_\perp+1)} \begin{cases} -\sum\limits_{s=0}^{n+1} \Upsilon_{n+1-s}(\beta,\theta)\, z^s : & n \geq 0 \\ e^{-2\beta\theta} z^{-1} - 1 : & n = -1 \\ e^{-2\beta\theta} \sum\limits_{s=0}^{-n-1} \Upsilon_{-n-1-s}(-\beta,\theta) z^{-s-1} : & n < -1 \end{cases},$$

f_n are the Fourier coefficients of function $f(e^{i\varphi})$.

Then, correlating the Fourier coefficients in (19) and using the third equation in (9) to determine the constant C, we eventually have

$$\begin{aligned} b_0 &= \hat{W}_0 \hat{f}_0 + \sum_{n\neq 0} n^{-1} \hat{V}_{n-1}^{-1}(\beta,\theta)\, \hat{f}_n, \\ b_m &= \sum_{n=-\infty}^{+\infty} m^{-1} \hat{V}_{m-1}^{n-1}(\beta,\theta)\, \hat{f}_n, \quad m = \pm 1, \pm 2, \ldots, \end{aligned} \tag{20}$$

where $\hat{f}_n = \left(1 + \frac{\tau}{1+\mu_\perp}\right) f_n$. The values $\hat{W}_0$ and $\hat{V}_{m-1}^{n-1}(\beta,\theta)$ are expressed via the Pollaczek polynomials $P_n(\beta,\theta)$ and the functions $\Upsilon_n(\beta,\theta)$ by the formulas below.

If $m = n$,

$$\hat{V}_{m-1}^{m-1}(\beta,\theta) = \frac{1}{2} \begin{cases} 0, & m = 0, \\ \sum\limits_{n=0}^{m} \Upsilon_{m-n}(\beta,\theta)\, P_{n-m}(-\beta,\theta), & m \geq 1, \\ -\sum\limits_{n=0}^{|m|} \Upsilon_{|m|-n}(-\beta,\theta)\, P_{n+m}(\beta,\theta), & m \leq -1. \end{cases} \tag{21}$$

If $m \neq n$,

$$\hat{V}_{m-1}^{n-1}(\beta,\theta)=\frac{1}{2}\begin{cases}\frac{e^{2\beta\theta m}}{m-n}\left[P_{m-1}(\beta,\theta)\,P_n(\beta,\theta)-P_m(\beta,\theta)P_{n-1}(\beta,\theta)\right], & n\neq 0\\ e^{2\beta\theta}P_{m-1}(\beta,\theta)-P_m(\beta,\theta), & n=0.\end{cases} \tag{22}$$

$$\hat{W}_0=-\frac{1}{2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\Big[e^{2\beta\theta}P_{n-1}(\beta,\theta) +e^{2\beta\theta}P_{n-1}(-\beta,\theta)+P_n(-\beta,\theta)+P_n(\beta,\theta)\Big]. \tag{23}$$

By definition, for the Pollaczek polynomials $P_n(\beta,\theta)$ in (21), (22) with index n negative, we have

$$P_n(-\beta,\theta)=e^{-2\beta\theta}P_{|n|-1}(-\beta,\theta), \quad n=-1,-2,\ldots. \tag{24}$$

So, under the assumption that $(f_n)_{n=-\infty}^{+\infty}$ are known, formulas (20)–(24) provide a closed form solution to dual series Equation (9). This solution reduces initial Equation (8) to the infinite system of linear algebraic equations of the second kind.

Indeed, replace f_n in (20) by its expression in b_n terms (see (10)). Finally, after some innocent manipulations we have

$$b_m+\sum_{n=-\infty}^{+\infty}A_{mn}b_n=Q_m, \quad m=0,\pm1,\pm2,\ldots,. \tag{25}$$

where the matrix elements A_{mn} and the right-hand side Q_m are

$$A_{mn}=\hat{\delta}_n\begin{cases}\hat{W}_0, & m=0,\ n=0,\\ -n^{-1}\hat{V}_{n-1}^{-1}(\beta,\theta), & m=0,\ n\neq 0,\\ -m^{-1}\hat{V}_{m-1}^{n-1}(\beta,\theta), & m\neq 0,\end{cases} \tag{26}$$

$$\hat{\delta}_n=|n|+i\frac{G_{2n}+\mu_\perp G_{1n}}{1+\mu_\perp},$$

$$Q_0=-\frac{i\chi 2\mu_\perp}{1+\mu_\perp}, \quad Q_m=-\frac{i\chi 2\mu_\perp}{(1+\mu_\perp)\,m}\hat{V}_{m-1}^{-1}(\beta,\theta). \tag{27}$$

Let us show that (25) is an infinite system of linear algebraic equations of the second kind. To this end, it will be suffice to prove that the matrix $\|A_{mn}\|_{m,n=-\infty}^{+\infty}$ produces a compact operator in the space of sequences $l_2=\left\{(b_n)_{n=-\infty}^{+\infty}:\ \sum_{n=-\infty}^{+\infty}|b_n|^2<\infty\right\}$, and the sequence $(Q_m)_{m=-\infty}^{+\infty}\in l_2$.

According to [17], the Pollaczek polynomials $P_n(\beta,\theta)$ asymptotically tend as $n \to +\infty$ to

$$P_n(\beta,\theta) = \frac{2e^{-\left(\frac{3\pi}{2}+\theta\right)\beta}}{\sqrt{2\sin(\theta)n}\,|\Gamma(0.5+i\beta)|^2}$$

$$\left(\begin{array}{l}\cos\left(\beta\ln(2n\sin\theta)-\left(n+{}^1\!/_2\right)\theta-\frac{3\pi}{4}\right)\mathrm{Re}\Gamma(0.5+i\beta)\\ +\sin\left(\beta\ln(2n\sin\theta)-\left(n+{}^1\!/_2\right)\theta-\frac{3\pi}{4}\right)\mathrm{Im}\Gamma(0.5+i\beta)\end{array}\right)\left(1+\mathrm{O}\left(\frac{1}{n}\right)\right), \quad (28)$$

where $\Gamma(z)$ is the gamma-function and $\mathrm{Re}(\ldots)$ and $\mathrm{Im}(\ldots)$ are its real and imaginary parts, respectively.

The $P_n(\beta,\theta)$ asymptotical estimate as $n \to -\infty$ comes from (28) in view of the relationship $P_{-n}(\beta,\theta) = e^{-2\beta\theta}P_{n-1}(-\beta,\theta)$. By virtue of (28), as $|m|,\ |n| \to \infty$, the inequalities

$$\left|\hat{V}_{m-1}^{n-1}(\beta,\theta)\right| < \frac{C\sqrt{|m|}}{|m-n|\sqrt{|n|}}, \quad (29)$$

$$\left|\hat{V}_{m-1}^{m-1}(\beta,\theta)\right| < C, \quad (30)$$

readily come from (22) when $m \neq n$ and from (21) for $m = n$. Here, C only depends on β and θ.

From the $\hat{\delta}_n$ expression, $\hat{\delta}_n = O\left(\frac{1}{|n|}\right)$. Therefore in view of (29) and (30), as $|m|,\ |n| \to \infty$, relationship (26) yields the inequalities

$$\begin{aligned}|A_{mn}| &< \frac{C}{|m-n|\,|n|^{3/2}\sqrt{|m|}},\\ |A_{mm}| &< \frac{C}{|m|}.\end{aligned} \quad (31)$$

Hence, the series $\sum\limits_{m,n=-\infty}^{+\infty}|A_{mn}|^2 < \infty$ is convergent, and matrix $\|A_{mn}\|_{m,n=-\infty}^{+\infty}$ sets a compact operator (the Hilbert-Schmidt operator) in the l_2 space [31]. The series convergence $\sum\limits_{m=-\infty}^{+\infty}|Q_m|^2 < \infty$ follows from (29).

We have shown that system (25) is an infinite system of linear algebraic equations of the second kind. This fact guarantees that a solution to system (25) can be constructed with any preassigned accuracy by truncation [31], which is the replacement of infinite system (25) by the system of N equations for N unknowns. This method provided the obtained numerical results discussed below.

4. NUMERICAL RESULTS

To numerically solve finite (truncated) system (25) of linear algebraic equations, corresponding algorithms were developed, delivering all characteristics of the diffraction field with any preassigned accuracy. The program realization on an IBM compatible PC was in programming language C^{++}. The analysis of the results for convergence and reliability was made to find out, for one, that the reflection coefficient (the zeroth harmonic module $|a_0| = |b_0 - 1|$) can be calculated with an error of 0.1% of the value provided that the truncation order of system (25) is chosen by the rule $N = [\chi\sqrt{|\varepsilon\mu_\perp|}+5]$ (here $[\dots]$ is the integral part of the number).

The numerical analysis was performed at the ferromagnetic medium characteristic frequencies $\omega_M = 31.1\,\text{GHz}$ and $\omega_H = 3.52\,\text{GHz}$, with the normalized frequencies being $\chi_M = \frac{\omega_M l}{2\pi c} = 0.27$ and $\chi_H = \frac{\omega_H l}{2\pi c} = 0.306$ for the grating period $l = 16.4\,\text{mm}$.

The numerical modeling corresponds to a lossy ferromagnetic medium of complex permittivity $\varepsilon = \varepsilon' + i\varepsilon''$, the real ε' and imaginary ε'' parts extended widely. The normalized frequency $\chi = \frac{\omega l}{2\pi c}$ of the E-polarized excitation wave is varied over the interval $\chi_0 < \chi < \chi_-$, where χ_0 is the ferromagnetic resonance normalized frequency and χ_- is the normalized surface-wave cutoff frequency of the ferromagnetic half-space.

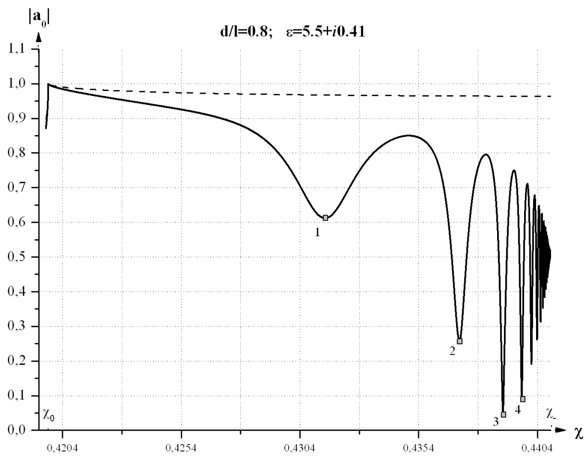

Figure 2. Frequency dependences of the reflection coefficient module for a grating backed by a ferromagnetic half-space and for a ferromagnetic half-space without a grating.

The frequency dependence of the reflection coefficient module $|a_0|$ of a grating attached to a ferromagnetic medium is presented in Fig. 2 for $d/_l = 0.8$, $\varepsilon = 5.5 + i0.41$ (solid line). There one also finds the analogous dependence (dashed curve) for $d/_l = 1.0$, $\varepsilon = 5.5 + i0.41$, which corresponds to the ferromagnetic half-space without a grating, the reflection coefficient explicitly determined by the expression $|a_0| = \left|\frac{\mu_\perp - \sqrt{\varepsilon\mu_\perp}}{\mu_\perp + \sqrt{\varepsilon\mu_\perp}}\right|$. A pronounced resonance character of the reflection coefficient module of the lossy ferromagnetic half-space with the periodic grating on its boundary is evident.

The frequency parameter χ can take a discrete set of values at which the considered structure either mostly reflects the excitation field energy $|a_0| > 0.5$ or mostly absorbs it $|a_0| < 0.5$. In the same frequency range, a ferromagnetic half-space without a grating (dashes) or a grating-loaded lossless ferromagnetic half-space ($\varepsilon'' = 0$) can fully reflect the incident plane wave, i.e., $|a_0| = 1$. The exception is some vicinity of the ferromagnetic resonance normalized frequency χ_0. There the frequency dependences of the reflection coefficient of the ferromagnetic half-space with and without a grating demonstrate a Wood's anomaly resonance. The resonance is caused by the wave appearing and travelling in the ferromagnetic medium at $\chi < \chi_0$.

To interpret an infinitely large number of resonances on frequency dependences of the reflection coefficient of a strip grating attached to a lossy ferromagnetic half-space, an approximate model is suggested. As known [3], when frequency parameter χ meets the condition $\chi_0 < \chi < \chi_-$, a slow surface wave can propagate along the ferromagnetic half-space boundary. Its phase velocity V_0 depends on the frequency parameter χ according to the formula

$$V_0 = 2c\sqrt{\frac{(\chi_+ - \chi_-)(\chi_-^2 - \chi^2)}{\alpha(\chi,\varepsilon) + \sqrt{\Delta(\chi,\varepsilon)}}}, \tag{32}$$

where

$$\alpha(\chi,\varepsilon) = \chi^2((\varepsilon+1)\chi_- - 2\chi_+) + \chi_+\chi_-\left[\chi_+(\varepsilon-1) - 2\varepsilon\chi_-\right],$$
$$\Delta(\chi,\varepsilon) = \alpha^2(\chi,\varepsilon) - [(\varepsilon-1)\chi^2 + \chi_+(\chi_+(\varepsilon+1) - 2\varepsilon\chi_-)]^2(\chi_-^2 - \chi^2),$$

c is the velocity of light in a vacuum.

As follows from (32), the χ_- value of the frequency parameter is the cutoff frequency of this surface wave, for its phase velocity vanishes at $\chi = \chi_-$. When a grating is present on the ferromagnetic half-space boundary, the diffraction field in the frequency range $\chi_0 < \chi < \chi_-$ is a superposition of an infinite number of spatial harmonics exponentially decaying away from the grating and traveling along it with the phase velocity $V_n = c\frac{\chi}{n}$, $n = \pm 1, \pm 2, \ldots$. (the exception is the $n = 0$

Table 1. Accuracy of the resonance frequency calculation.

n	$\chi_{\min_n}$	χ_n	Error, %
1	0.431057	0.423569	0.75
2	0.436721	0.432875	0.38
3	0.438558	0.436548	0.20
4	0.439350	0.438163	0.12
5	0.439757	0.438987	0.08
6	0.439994	0.439458	0.05
7	0.440142	0.439750	0.04
8	0.440242	0.439944	0.03
9	0.440312	0.440078	0.02
10	0.440363	0.440174	0.02

spatial harmonic). Upon the phase synchronism condition of these spatial harmonics with the surface wave on the ferromagnetic medium boundary, relation (32) yields the equation of the appearance

$$\chi = 2n\sqrt{\frac{(\chi_+ - \chi_-)(\chi_-^2 - \chi^2)}{\alpha(\chi,\varepsilon) + \sqrt{\Delta(\chi,\varepsilon)}}}, \tag{33}$$

where $n = 1, 2, \ldots$ are the numbers of the diffraction field spatial harmonics.

It can be shown that, within $\chi_0 < \chi < \chi_-$ for n given, Equation (33) has a unique root χ_n such that $\chi_n \to \chi_-$ as the spatial harmonic number n grows. In the other words, the normalized value of the cutoff frequency χ_- of the ferromagnetic half-space surface wave is the point of the accumulation of the roots of Equation (33).

Equation (33) has been solved numerically. The first ten roots χ_n, $n = 1, 2, \ldots, 10$ are given in Table 1 together with the frequency parameter $\chi_{\min}$ indicating the local minima of the reflection coefficient curve from Fig. 2. As seen, these frequency parameter values tend to the χ_n results coming from Equation (33) (the relative error is indicated in Table 1). The numerical analysis suggests that $\chi_{\min} \to \chi_n$ as the grating slot increases ($\frac{d}{l} \to 1$).

Thus, at a certain frequency parameter χ, a plane E-polarized electromagnetic wave incident on a strip grating backed by a ferromagnetic half-space can bring the phase velocity of the surface wave into the coincidence with the phase velocity of one of the spatial harmonics. After that the incident wave energy is

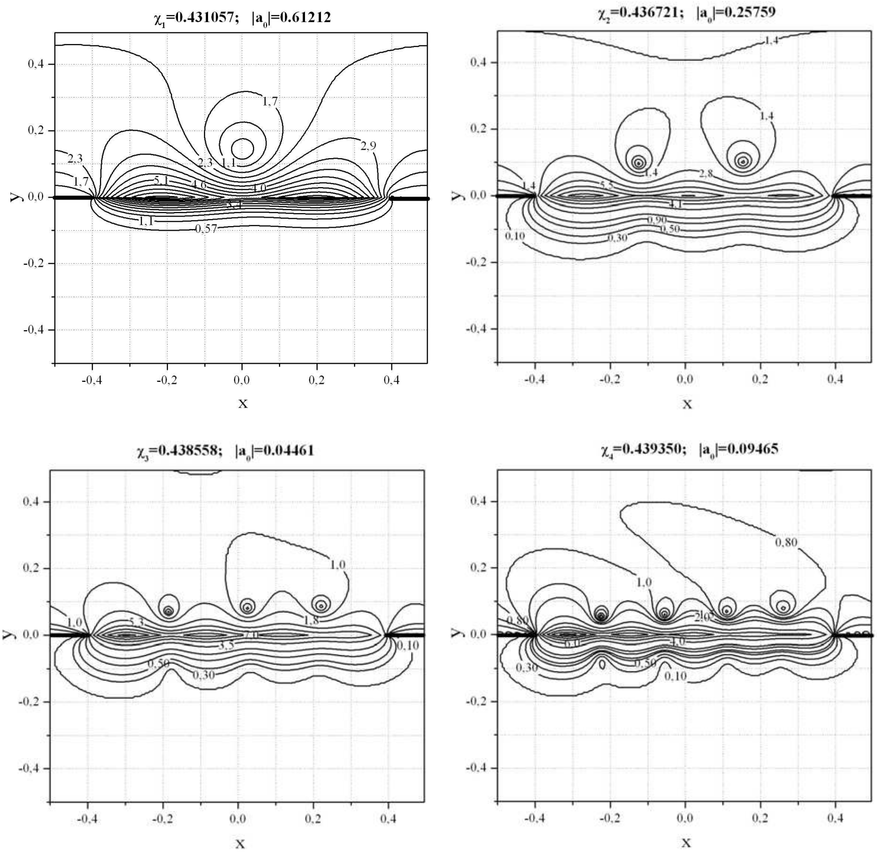

Figure 3. Equal amplitude lines $|E_z| = const$.

resonantly transferred to the surface electromagnetic field, which is a superposition of spatial harmonics.

In support, refer to Fig. 3 for the E_z equal amplitude lines ($|E_z| = const$) corresponding to the first four values of χ, where $|a_0|$ has a local minimum (see Fig. 2). One can see that the field is localized near the ferromagnetic half-space boundary, being, on this point, a surface field. Besides, as the resonance value of frequency parameter χ grows, a number of field variations within the structure period increases, which agrees well with the mentioned approximated model.

Here it should be noted that even though the excitation field is symmetrical (in view of the normal incidence of the wave $E_z = e^{-ikx}$) and the grating has geometrical symmetry about the oX axis, the diffraction field is clearly asymmetrical because of the ferromagnetic medium nonreciprocity.

The reflection coefficient $|a_0|$ versus the slot size (parameter d/l) of the strip grating is presented in Fig. 4, where equal amplitude

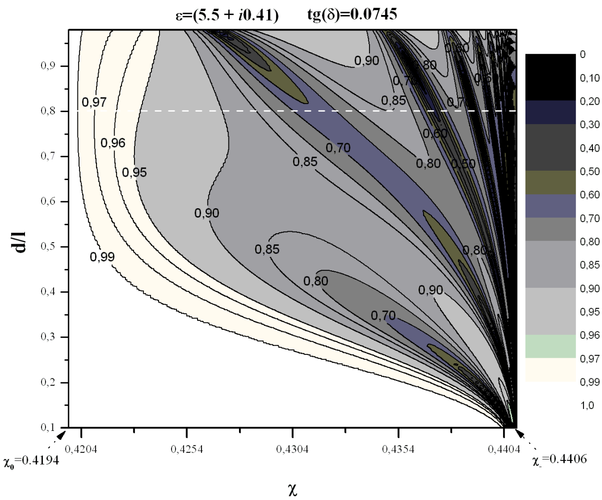

Figure 4. The reflection coefficient module versus frequency and grating slot size.

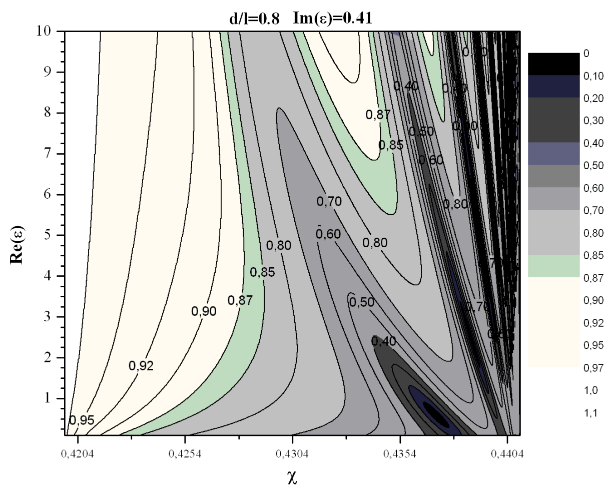

Figure 5. The reflection coefficient module versus frequency and the real part of the ferromagnetic medium permittivity.

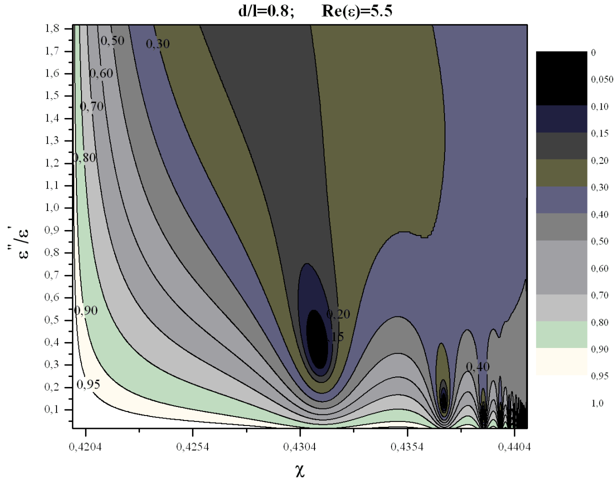

Figure 6. The reflection coefficient module versus frequency and ferromagnetic medium loss.

lines $|a_0|$ are presented as a function of χ and $^d/_l$. The same $|a_0|$ dependences on χ and $\mathrm{Re}(\varepsilon)$ are shown in Fig. 5 to follow the dynamics of the absorption resonances (the dark zones) under varying these parameters.

Of even more interest is the reflection coefficient behavior depending on the ferromagnetic medium loss described by the loss tangent $tg\delta = {}^{\varepsilon''}/_{\varepsilon'}$. Fig. 6 indicates that there exist optimum values of the loss tangent (the dark zones), where practically all the energy of the incident electromagnetic wave is transferred to one of surface harmonics of the periodic strip grating placed on the ferromagnetic half-space boundary.

5. CONCLUSION

In this paper, a method has been suggested for solving the diffraction problem of a plane E-polarized electromagnetic wave normally incident on a strip grating located on the ferromagnetic half-space boundary.

Based on this method, algorithms and computing programs were developed, and characteristic features of the E-polarized wave diffraction by a strip grating backed by a ferromagnetic half-space received their study in a wide range of geometrical and constitutive parameters. It has been found that the interval $\sqrt{\omega_H(\omega_H+\omega_M)} < \omega < \omega_H + \frac{\omega_M}{2}$ contains an infinite sequence of frequencies with a finite accumulation point coinciding with the surface-wave cutoff frequency of the ferromagnetic half-space. At these frequencies, a resonant absorption of the excitation wave energy holds.

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